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Theory of the weak interaction vertices

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Abstract The electron field is constructed, showing explicitly the electromagnetic modes associated with the electric and magnetic fields of the electron. With a criterion taken from Dirac's work, it is observed that differentiation of the magnetic mode in the kinetic Hamiltonian generates an interaction, which exhibits the mathematical structure of the weak interactions. The result is then used to formulate a theory of the weak interaction vertices. The electron and its neutrino are described by means of the same Fermi field, in different representations. The intermediary boson $\vec{W}^{\pm}$ is introduced with the help of a complex combination of the electromagnetic field. The theory is compared with the theory of Weinberg-Salam.

Electromagnetic flux and mass are the only observable differences between the electron and the electron-neutrino. These particles in fact carry the same conserved lepton number, and, supposing that the difference in their masses is of electromagnetic origin, one may inquire on the possibility of describing them both with the same Fermi field.

In a previous paper[1], concerned with the explicit quantization of the electric and magnetic fields of the electron, I made a first attempt toward a unified treatment of the electron and its neutrino. My purpose here is to discuss a second unification approach, which has the virtue of being more conclusive, and closer to traditional schemes of particle physics, than the former one.

The quantization of the Coulomb field of the electron undertaken in ref. (1) is, in every regard, consistent with Dirac's method[2]. Dirac defines the physical electron field, multiplying a bare fermion field by a unitary operator e^{ieV_x}, which accounts for the Coulomb field mode. Then, after differentiation of this operator in the kinetic Hamiltonian, he establishes a link between the form of the local interaction in Quantum Elctrodynamics, and the electric flux of the electron.

A canonical method of flux quantization[1,3] allows one to reobtain Dirac's results.

I also propose a manner of introducing the magnetic field of the electron, by means of another flux operator factor, which I call the Ampère mode[1].

The explicit differentiation of the Ampère rnode in the kinetic Hamiltonian generates a second type of interaction, showing the mathematical striicture of the weak interactions. Further, I explore this result to build up a theory of the weak interaction vertices.

The proposed theory is then compared with the theory of Weinberg-Salam[4,5].

Given a neutral field $\psi_0^\dagger$, let the electron field be initially written as

$$\psi_e^\dagger(\vec{x}) = \mathcal{C}^\dagger(\vec{x})\mathcal{A}^\dagger(\vec{x})\psi_0^\dagger(\vec{x}) \ . \tag{1}$$

The operator factors $\mathcal{C}^\dagger$ and $\mathcal{A}^\dagger$ accounts respectively for the electric and magnetic fields produced by the electron.

$\vec{A}$ being the vector potential, the Coulomb rnode is the following unitary operator,

$$\mathcal{C}^\dagger(\vec{x}) = \exp ie\theta_\Gamma(\vec{x}) \quad ; \quad \text{with} \quad \theta_\Gamma(\vec{x}) = \int_\Gamma^{\vec{x}} \vec{A}.d\vec{\ell} \ , \tag{2}$$

where Γ is an open line ending at the point 2.

And if, as shown in fig. 1, S is a closed surface, and $\phi_{E,S}$ is the electric flux flowing through it, then the Coulomb mode shall be an eigenstate of $\phi_{E,S}$, with eigenvalue e, whenever $\vec{x}$ is inside S, or eigenvalue zero, if $\vec{x}$ is outside.

To define the Ampère mode, one has to deal with the auxiliary field $\vec{T}$, which is such that $\vec{E} = \vec{\nabla} \times \vec{T}$ and $\dot{\vec{T}} = \vec{B}$.

The $\vec{T}$ field is not independent from the $\vec{A}$ field. Rather, they correspond to two different representations of the electromagnetic field. I take both auxiliary fields to be complete, and assume the simultaneous validity of the local commutators below[1]

$$[E_i(\vec{x}), A_j(\vec{y})] = i\delta_{ij}(\vec{x} - \vec{y}) \quad ; \quad [B_i(\vec{x}), T_j(\vec{y})] = -i\delta_{ij}\delta(\vec{x} - \vec{y}) \ . \tag{3}$$

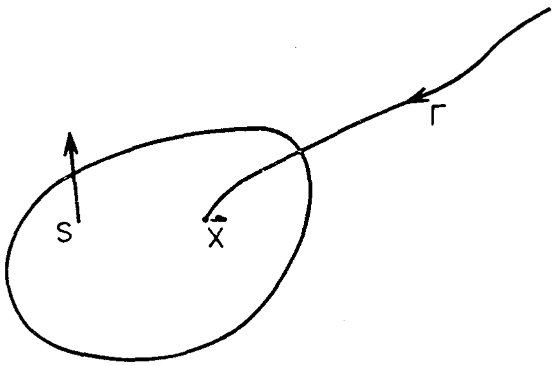

Fig. 1 - Closed surface S, and open line Γ. Appropriate topology for quantization of the Coulomb flux.

Let Γ_1 and Γ_2 be two closed lines, encircling the open surfaces S_1 and S_2, with the orientation and topology shown in **fig. 2.** If ϕ_B is the magnetic flux across S_1, and ϕ_E the electric flux through S_2, then the flux commutation law $[\phi_E, \phi_B] = i$ must hold[3]. And this flux commutator is simultaneously consistent wit the two local commutators of eq. **(3).** Then, the $\vec{T}$ field is a complete field, which can be **explicitly introduced in Quantum Electrodynamics, whenever it is needed.**

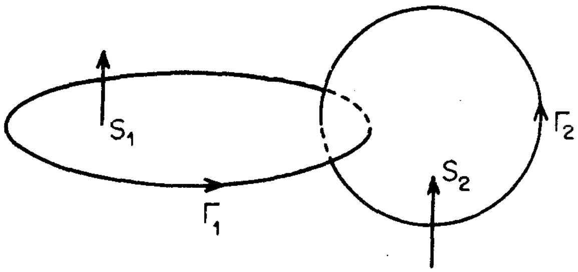

Fig. 2 - Closed lines Γ_1 and Γ_2. Appropriate topology for quantization of the magnetic flux.

Now take $\vec{J}_0$ to **be a neutral** current, forrned with **every neutral** field which **may receive** flux. $\vec{J}_0$ includes also the current : $\psi_0^\dagger \vec{\alpha} \psi_0$ **:,** associated with the conserved lepton number of the electron/electron-neutrino system. And define $\vec{\mathrm{D}}$ as the integral of $\vec{J}_0$ over the system's volume $\int d^3\vec{x}\vec{J}_0$.

The Ampère mode is then introduced by means of the following functional of the $\vec{T}$ field[1]

$$\mathcal{A}^\dagger(\vec{x}) = \exp ie\eta_\Gamma(\vec{x}) \quad , \quad \text{with} \quad \eta_\Gamma(\vec{x}) = \int_\Gamma^{\vec{x}} (\vec{D} \times \vec{T}).d\vec{\ell} \tag{4}$$

One notices that

$$\mathcal{A}^\dagger(\vec{x})\psi_0^\dagger(\vec{x})\mathcal{A}(\vec{x}) = \psi_0^\dagger(\vec{x})a^\dagger(\vec{x},\vec{\alpha}) \quad , \tag{5}$$

where the α_i are the Dirac matrices, and $a^\dagger(\vec{x},\vec{v})$ is the operator

$$a^\dagger(\vec{x},\vec{v}) = \exp ie\tilde{\eta}_\Gamma(\vec{x},\vec{v}) \quad , \quad \text{with} \quad \tilde{\eta}_\Gamma(\vec{x},\vec{v}) = \int_\Gamma^{\vec{x}} (\vec{v} \times \vec{T}).d\vec{\ell} \quad . \tag{6}$$

Given a positive energy test function $f_k(\vec{x})$, the one-electron state,

$$|k> = \int d^3\vec{x}\psi_e^\dagger(\vec{x})f_k(\vec{x})|\vee> = \int d^3\vec{x}\psi_0^\dagger(\vec{x})\mathcal{C}^\dagger(\vec{x})a^\dagger(\vec{x})f_k(\vec{x})|\vee> \quad , \tag{7}$$

has the register of the electric and magnetic fields of the electron. One can also formally[1] verify the Biot-Savart formula

$$< k|\vec{B}(\vec{y})|k> = \int d^3\vec{x}\frac{e}{4\pi}\vec{j}_k(\vec{x}) \times \frac{(\vec{y}-\vec{x})}{|\vec{y}-\vec{x}|^3} \quad , \quad \text{with} \quad \vec{j}_k = f_k^\dagger\vec{\alpha}f_k \quad , \tag{8a}$$

and the Coulomb law,

$$< k|\vec{E}(\vec{y})|k> = \int d^3\vec{x}\rho_k(\vec{x})\frac{e}{4\pi}\frac{(\vec{y}-\vec{x})}{|\vec{y}-\vec{x}|^3} \quad , \quad \text{with} \quad \rho_k = f_k^\dagger f_k \quad , \tag{8b}$$

At first sight, it seems that the two formulae above are contradictory with the principle of local causality, since they refer to the instantaneous fields produced by the particle. The contradiction is however only apparent, because each formula refers to a single mode of the electromagnetic field, whereas the question of causality can only be posed after considering the complete field.

We will verify that in fact[2], differentiation of the flux operator factors of the electron field, in the Hamiltonian kinetic term, generates local interaction, involving the complete electromagnetic field.

Theory of the weak interaction vertices

The field $\psi_0^\dagger = C\, A\psi_e^\dagger$ shall be taken to be the neutrino field. And, in order to couple the electron and the neutrino, one has to construct a charged boson field $\vec{W}^\pm$ designed to absorb the electron flux in the interaction vertices.

Then, I first define the complex field $\vec{Y}$, as a complex combination of the real field $\vec{A}$ and $\vec{T}$

$$\vec{Y} = \frac{1}{\sqrt{2}}(\vec{A} - i\vec{T}) \quad ; \quad \text{and} \quad \vec{Y}^* = \frac{1}{\sqrt{2}}(\vec{A} + i\vec{T}) \tag{9}$$

observing that the Hamiltonian of the transverse components of the electromagnetic field, $\frac{1}{2}(\vec{E}^2 + \vec{B}^2)$, can be rewritten with the transverse parts of the $\vec{Y}$ and $\vec{Y}^*$ fields, in the form

$$\mathcal{H} = \frac{1}{2}[\dot{\vec{Y}}^*\dot{\vec{Y}} + (\vec{\nabla} \times \vec{Y}^*)(\vec{\nabla} \times \vec{Y})] \ . \tag{10}$$

The $\vec{Y}$ field, although complex, is still a neutral field, since it has no flux factors. So, dressing the $\vec{Y}$ field with the flux factors, I define the vector field $\vec{W}^\pm$ as

$$\vec{W}^- = A\ C? \quad , \quad \text{and} \quad \vec{W}^+ = \vec{Y}^* C^\dagger A^\dagger \ . \tag{11}$$

By inverting these relations, and replacing $A^\dagger C^\dagger \vec{W}^-$ for $\vec{Y}$ in the Hamiltonian of eq. (10), or in the corresponding Lagrangian, one gets the $\vec{W}^\pm$ dynamics. And that dynamics has some properties of gauge field dynamics, as for example in the fact that the $\vec{W}^\pm$ Hamiltonian will also acquire quartic terms, coming from the differentiation of the flux operators.

One can verify the following relations between vacuum expectation values of products of field at different times

$$< T_i(x_1)T_j(x_2) > = < A_i(x_1)A_j(x_2) > \ , \tag{12a}$$

and

$$< T_i(x_1)A_j(x_2) > = - < A_i(x_1)T_j(x_2) > \ . \tag{12b}$$

The inversion of sign in the last expression is due to the relation $< B_i(x_1)A_j(x_2) > = < E_i(x_1)T_j(x_2) >$, which is just what one gets by differentiating deriving eq. (12b) with respect to t_1.

In order that a boson field be capable of receiving flux factors, it must be possible to construct conserved currents with its components. So, that boson field must be complex.

Now I discuss a theory of the weak interaction vertices, based upon flux quantization.

The problem is addressed from the viewpoint of the Hamiltonian, and I consider that, for every specific process, it should always be possible to write an associated Lagrangian, explicity showing every symmetry of the problem, in particular the Lorentz invariance.

Differently from the analysis of ref. (1), my criterion here is that the interaction should be generated by the differentiation of the flux modes attached to the fermion field, in the Hamiltonian kinetic term. This criterion is consistent with Dirac's work[2].

To illustrate Dirac's procedure we consider first the simpler case, when the electron has the Coulomb mode, but not the Ampère mode: $\psi_e = \psi_0 C$. We notice that the differentiation of the Coulomb mode in the gradient term of the Hamiltonian, generates the local interaction of Quantum Electrodynamics, in the gauge $A_0 = 0$

$$\int d^3\vec{x}(-i)\psi_e^\dagger\vec{\alpha}.\vec{\nabla}\psi_e = \int d^3\vec{x}[-i\psi_0^\dagger\vec{\alpha}.\vec{\nabla}\psi_0 - e\psi_0^\dagger\vec{\alpha}.\vec{A}\psi_0] \; . \qquad (13)$$

In scattering theory, the motion of the electron is given by the ψ_0 field dynamics, and not by the ψ_e field one[2].

If one begins with ψ_0 in the gradient term of the Hamiltonian in eq. (13), then it shows no interaction term, since ψ_0 commutes with the electromagnetic field. This means that the particle associated with ψ_0, which is an electron without flux, does not interact with the $\vec{A}$ field.

The next step is to include the Ampère mode. So, I define the field $\tilde{\psi}_0$ divided into two sets of modes, taking $\tilde{\psi}_0 = \psi_E + \psi_N$, where

$$\psi_E = \sum_{k_E} a_{k_E} f_{k_E} \quad , \quad \text{and} \quad \psi_N = \sum_{k_N} a_{k_N} f_{k_N} \; . \qquad (14)$$

The set $\{f_{k_E}\} \oplus \{f_{k_N}\}$ is a complete set of modes, and k_E # k_N, so that neither ψ_E, or ψ_N, are complete fields.

The specification of the set, $\{f_{k_E}\}$ or $\{f_{k_N}\}$, to which a given mode f_i belongs, will depend on the specific process one is studying. Those modes forming ψ_E will described the motion of the electron, whereas the ones making up ψ_N will account for the motion of the neutrinos.

With the purpose of treating together the electron-electron, neutrino-neutrino and electron-neutrino interactions, I first define a second auxiliary field ***i,*** :

$$\psi = \psi_E \mathcal{C} \mathcal{A}^{1/2} + \psi_N \mathcal{A}^{-1/2} \quad , \tag{15}$$

adding that:

(i) The flux difference between a state annihilated by ψ_E C $\mathcal{A}^{1/2}$, and another one annihilated by $\psi_N \mathcal{A}^{-1/2}$, is just the same flux difference between the electron and the neutrino, that is $\mathcal{C}^\dagger \mathcal{A}^\dagger$.

(ii) Thanks to the flux factors, the completeness of the ***i,*** field is only approximate,

$$\{\psi(\vec{x}), \psi^\dagger(\vec{y})\} = \delta(\vec{x} - \vec{y}) + O(e) \quad . \tag{16}$$

This however means no difficulty, since here I am concerned with the construction of a Hamiltonian up to oder $O(e)$, which is already sufficient for calculation of the main processes in the tree approximation.

(iii) The construction of the ***i,*** field, in eq. (15), is a particularization, which leads to a value of **30**" for the angle θ_W. If θ_W differs from that value, one can always redefine the ψ field in a suitable manner, and keeping the same difference in flux between the electron and the neutrino.

Then, I suppose that the dynamics of the electron-neutrino system is determined by the "free" Hamiltonian of the $ field:

$$H = \int d^3\vec{x} [\psi^\dagger(-i\vec{\alpha}.\vec{\nabla})\psi + m\bar{\psi}\psi] \ . \tag{17}$$

And again, I get the point interaction between the particles, by explicitly differentiating the flux modes attached to ψ, in the gradient term of H.

The flux operator factors in the definition of ψ are given by

$$\mathcal{A}^{-1/2} = \exp \frac{ie}{2} \int^{\vec{x}} (\vec{\alpha} \times \vec{T}).d\vec{\ell} \ , \tag{18}$$

and

$$C \ \mathcal{A}^{-1/2} = \exp \left\{ -ie \int^{\vec{x}} \vec{A}.d\vec{\ell} - \frac{ie}{2} \int^{\vec{x}} (\vec{\alpha} \times \vec{T}).d\vec{\ell} \right\} . \tag{19}$$

After differentiation, it follows that

$$-i\vec{\alpha}.\vec{\nabla}(\mathcal{A}^{-1/2}) = \mathcal{A}^{-1/2}(ei\gamma^5\vec{\alpha}.\vec{T} - i\vec{\alpha}.\vec{\nabla}) + O(e^2) \quad , \tag{20}$$

$$-i\vec{\alpha}.\vec{\nabla}(C \ \mathcal{A}^{1/2}) = C \ \mathcal{A}^{1/2}(-e\vec{\alpha}.\vec{A} - ei\gamma^5\vec{\alpha}.\vec{T} - i\vec{\alpha}.\vec{\nabla}) + O(e^2) \quad . \tag{21}$$

These equations have been obtained with the help of the relation $\vec{\alpha} \times \vec{\alpha} = 2i\gamma^5\vec{\alpha}$.

Finally, combining eqs. (17), (20) and (21), and frorn the definition of the $\vec{W}^{\pm}$ field, in eq. (11),one gets the local coupling between the particles

$$\mathcal{H}_{ee} = -e\psi_E^\dagger\vec{\alpha}.\vec{A}\psi_E - ie\psi_E^\dagger\gamma^5\vec{\alpha}.\vec{T}\psi_E \quad , \tag{22}$$

$$\mathcal{H}_{vv} = ie\psi_N^\dagger\gamma^5\vec{\alpha}.\vec{T}\psi_N \quad , \tag{23}$$

and

$$\mathcal{H}_{ve} = -\frac{e}{\sqrt{2}}\psi_N^\dagger(1-\gamma^5)\vec{\alpha}.\vec{W}^-\psi_E + \text{h.c.} \quad . \tag{24}$$

To obtain the Hamiltonian density of eq. (24), one must suppose that only left-handed components of the original field $\tilde{\psi}_0$ should take part in the composition of ψ_N, that is $(1+\gamma^5)\psi_N = 0$.

In $R_{,,}$ one recognizes the electron-neutrino charged current coupling of the weak interactions, with the angle $\theta_W = 30°$.

$\mathcal{H}_{vv}$ is, in turn, a pseudo-vector coupling, not symmetrical under charge conjugation; and the same is true for the second term of $\mathcal{H}_{ee}$. The pseudo-vector nature of these interactions is consistent with the fact that the $\vec{T}$ field is fornally a pseudo-vector, since $\vec{\mathrm{T}} = \vec{\mathrm{B}}$.

Theory of the weak interaction *vertices*

The breaking of charge conjugation is related to the existence of two different representations for the fermion field. The auxiliary field ψ has been defined as a mixture of representations. Recall in this regard that each one of the sets of modes, $\{f_{k_E}\}$ or $\{f_{k_N}\}$, is separately incomplete. And the interaction may transfer a particle from a subspace to the other, or from a representation to the other. That gives rise to a kind of instability, which explains the formal non-hermiticity of the Hamiltonian.

The relation given in eq. (12b) means that, in scattering processes, such as for instance $ev \to ev$, there is no contribution from the mixed propagator $< A_i(x_1)T_j(x_2) >$.

Considering the structure of the one-electron state,

$$\int d^3\vec{x}\psi_e^\dagger \tilde{f}_p|\vee> = \int d^3\vec{x}\psi_E^\dagger C^\dagger \; a^\dagger \tilde{f}_p|\vee> \quad , \tag{25}$$

one observes that, in scattering theory, and only there, what matters is the motion of the auxiliary field $\psi_E^\dagger$ (or $\tilde{\psi}_0^\dagger$), which describes the particle nucleus around which the Coulomb mode condensates.

However, as far as the motion of the $\psi_E^\dagger$ field is concerned, the important states are those with the form

$$\int d^3\vec{x}\psi_E^\dagger \tilde{f}_p|\vee> \quad . \tag{26}$$

Thus, the flux factors belonging to the electron field $\psi_e^\dagger$ must be absorbed by the very wave function $\tilde{f}_p$, leading to the formation of the wave function f_p :

$$C^\dagger (a^\dagger)^{1/2}\tilde{f}_p = f_p \quad , \quad \text{or} \quad \tilde{f}_p = Ca^{1/2}f_p \quad . \tag{27}$$

That is why the auxiliary field ψ is defined with a term $\psi_E C \; A^{1/2}$. The interpretation is that the flux factor $CA^{1/2}$ is dressing with flux the wave functions f_p, contained in ψ_E.

In Weinberg-Salam theory[4,5], and in the standard model, the electron and the neutrino are coupled to the neutral boson Z^0, with a pseudo-vector coupling. Then, one should identify the neutral pseudo-vectorfield $\vec{T}$, as being the Z_0 particle field.

A fair agreement between the present theory and Weinberg-Salam theory could be attained, in the predictions of elastic cross-sections, by taking $\theta_W = 27°$, and by introducing the electron and the neutrino fields as $\psi_E C\ A^{1/2-\epsilon}$ and $\psi_N A^{1/2-\epsilon}$, with $\epsilon \approx 0.05$. This modification must be followed by an appropriate alteration in the definition of the $\vec{Y}$-field, where the mixing angle[1] between $\vec{A}$ and $\vec{T}$ should be shlightly lowered.

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Resumo

Constróe-se o campo do elétron, explicitando os modos eletromagnéticos associados aos campos elétrico e magnético do elétron. Com um critério extraído do trabalho de Dirac, observa-se que a derivação do modo magnético, na Hamiltoniana cinética, gera uma interação que tem a estrutura matemática das interações fracas. Esse resultado é então utilizado para formular uma teoria dos vértices das interações fracas. O elétron e seu neutrino são descritos através do mesmo campo de Ferini, em representações diferentes. O boson intermediário $\vec{W}^{\pm}$ é introduzido como auxílio de uma combinação complexa do campo eletromagnético. A teoria é comparada com a teoria de Weinberg-Salam.