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On pseudodynamics of second order Lagrangian systems

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Abstract The classical dynamics of Lagrangian systems containing second derivatives of anticommuting variables is considered here. Examples of supersymmetric second order Lagrangians are presented. We apply Dirac's theory of constrained systems to these models and compare our results with those corresponding to first order Lagrangians proposed in the literature.

1. Introduction

Pseudodynamics, i.e. the dynamics of a system described by ordinary *c*-variables (even supernumbers) and by a-variables (odd supernumbers) is one of the most attractive subjects in the study of gauge supersymmetry and of its outstanding properties which have large application in theoretical physics[1-8]. The main motivation for thc analysis of systems of point particles containing odd variables which are invariant under supersymmetry transformations (superparticles, for short) is the desire to attain a better understanding of the more complex supersymrnetric string models. In fact, systems of superparticles can appear as limiting cases of superstrings. An example is the Brink-Schwarz superparticle[4] associated with ground states of the Green-Schwarz superstring[g]. The question of the accomplishment of a covariant quantization procedure for superstrings is of crucial importance and it motivates the investigation of quantum mechanics of superparticle models[10-13].

Besides these aspects superparticle systems have their own significance at the classical level. As pointed out by Galvão and Teitelboim[3], if we consider a system without spin we can introduce spin degrees of freedom on it, by means of *a*-variables provided that we supersymmetrize the corresponding action. In this way we get classical spinning particles. In terms of Dirac's Hamiltonian theory this is equivalent to taking the square root of the Hamiltonian generators of the spinless system. There is, of course, a close relationship between this result and the claim that supergravity is the square root of ordinary general relativity[14].

Another interesting problem concerning constrained Hamiltonian systems is that of the inclusion of higher order terms in the action and the possible modifications in the algebraic structure of the Poisson brackets of constraints. We can mention, for instance, alternative theories of gravitation with Lagrangians which are quadratic in the curvature tensor and/or its contractions[15–17]. The aim of such models, as is well known, is the obtention of a consistent quantum theory of the gravitational field. Recent works have also shown the relevance of squared terms in superstring actions[18–20]. These squared terms contain higher order derivatives and lead to a model without ghost particles in the low energy limit of the string theory.

Even the simpler case of point particle Lagrangians – constructed from usual c-variables – which depend at most on second derivatives of the dynamical variables seems to deserve a deeper investigation[21].

The purpose of this paper is to consider the pseudodynamics of second order Lagrangian systems, that is, the classical mechanics of supersymmetric Lagrangian models containing second derivatives of anticornmuting variables (a-variables). In Sec. 2 we recall the general features of second order Lagrangians. We also indicate the use of some prescriptions of Lagrangian and Hamiltonian formalisms in the context of models involving a-variables. Sec. 3 contains examples of super-particle second order systems. Comparison is made with some related first order Lagrangians encountered in the literature. In Sec. 4 we follow Dirac's method in order to derive Poisson and Dirac brackets for the constraints associated with the Lagrangians presented in Sec. 3. We also obtain, for each situation, the total angular momentum algebra and we discuss the interpretation of the spin variables. Some of our examples reveal the existence of a second order spin effect besides the cantribution to the spin vector due to the usual first order terms in the anti-commuting part of the Lagrangian. Sec. 5 closes that this note with some final remarks.

2. Preliminaries

Let us consider a pseudoclassical system described by the c-variables x_i ($i = 1, ..., n$) and by the a-variables θ_α ($\alpha = 1, ..., N$). Let L $(x_i, \dot{x}_i, \ddot{x}_i, \theta_\alpha, \dot{\theta}_\alpha, \ddot{\theta}_\alpha)$ be a second order Lagrangian from which we get the action principle

$$\delta \int_{t_A}^{t_B} L(x_i, \dot{x}_i, \ddot{x}_i, \theta_\alpha, \dot{\theta}_\alpha, \ddot{\theta}_\alpha) dt = 0 \ . \tag{2.1}$$

Recalling that there is an arbitrariness in the definitions of derivatives with respect of to a-variables we make the choice of left derivatives

$$\delta\phi(\theta_\alpha) \equiv \delta\theta_\beta \frac{\partial\phi}{\partial\theta_\beta}(\theta_\alpha) \ , \tag{2.2}$$

where ϕ is a function depending on a-coordinates[22].

The variation indicated in (2.1) leads to the equations of motion

$$\frac{\partial L}{\partial x_i} - \frac{d}{dt}\frac{\partial L}{\partial \dot{x}_i} + \frac{d^2}{dt^2}\frac{\partial L}{\partial \ddot{x}_i} = 0 \ , \tag{2.3}$$

$$\frac{\partial L}{\partial \theta_\alpha} - \frac{d}{dt}\frac{\partial L}{\partial \dot{\theta}_\alpha} + \frac{d^2}{dt^2}\frac{\partial L}{\partial \ddot{\theta}_\alpha} = 0 \ , \tag{2.4}$$

The canonical momenta are defined by

$$p^i = \frac{\partial L}{\partial \dot{x}_i} - \frac{d}{dt}\frac{\partial L}{\partial \dot{y}_i} \ , \tag{2.5}$$

$$\bar{p}^i = \frac{\partial L}{\partial \dot{y}_i} \ , \tag{2.6}$$

$$\pi^\alpha = \frac{\partial L}{\partial \dot{\theta}_\alpha} - \frac{d}{dt}\frac{\partial L}{\partial \dot{\eta}_\alpha} \ , \tag{2.7}$$

$$\bar{\pi}^\alpha = \frac{\partial L}{\partial \dot{\eta}_\alpha} \ , \tag{2.8}$$

with $y_i \equiv \dot{x}_i$ and $\eta_\alpha \equiv \dot{\theta}_\alpha$, and the canonical Hamiltonian is

$$
\begin{aligned}
&H(x_i, p^i, y_i, \bar{p}^i, \theta_\alpha, \pi^\alpha, \eta_\alpha, \bar{\pi}^\alpha) = \\
&= \dot{x}_i p^i + \dot{y}_i \bar{p}^i + \dot{\theta}_\alpha \pi^\alpha + \dot{\eta}_\alpha \bar{\pi}^\alpha + \\
&- L(x_i, y_i, \dot{y}_i, \theta_\alpha, \eta_\alpha, \dot{\eta}_\alpha) \ ,
\end{aligned} \tag{2.9}
$$

where we use Einstein's summation convention. Note that the order of the a-variables in (2.9) is compatible with our choice (2.2) for the derivativa. Then we can derive Hamilton's equations

$$
\dot{x}_i = \frac{\partial H}{\partial p^i} \ , \ \dot{p}^i = -\frac{\partial H}{\partial x_i} \ , \ \dot{y}_i = \frac{\partial H}{\partial \bar{p}^i} \ , \ \dot{\bar{p}}^i = -\frac{\partial H}{\partial y_i} \ , \tag{2.10}
$$

and

$$
\dot{\theta}_\alpha = \frac{\partial H}{\partial \pi^\alpha} \ , \ \dot{\pi}^\alpha = -\frac{\partial H}{\partial \theta_\alpha} \ , \ \dot{\eta}_\alpha = -\frac{\partial H}{\partial \bar{\pi}^\alpha} \ , \ \dot{\bar{\pi}}^\alpha = -\frac{\partial H}{\partial \eta_\alpha} \ , \tag{2.11}
$$

Now if we take a function $f(x_i, p^i, y_i, \bar{p}^i, \theta_\alpha, \pi^\alpha, \eta_\alpha, \bar{\pi}^\alpha)$ of even nature we obtain, with the aid of (2.10-11), the time derivative

$$
\begin{aligned}
\frac{df}{dt} = &\frac{\partial f}{\partial x_i}\frac{\partial H}{\partial p^i} - \frac{\partial f}{\partial p^i}\frac{\partial H}{\partial x_i} + \frac{\partial f}{\partial y_i}\frac{\partial H}{\partial \bar{p}^i} - \frac{\partial f}{\partial \bar{p}^i}\frac{\partial H}{\partial y_i} \\
&+ \frac{\partial f}{\partial \theta_\alpha}\frac{\partial H}{\partial \pi^\alpha} + \frac{\partial f}{\partial \pi^\alpha}\frac{\partial H}{\partial \theta_\alpha} + \frac{\partial f}{\partial \eta_\alpha}\frac{\partial H}{\partial \bar{\pi}^\alpha} + \frac{\partial f}{\partial \bar{\pi}^\alpha}\frac{\partial H}{\partial \eta_\alpha} \ ,
\end{aligned} \tag{2.12}
$$

The right-hand side member in (2.12) corresponds to the Poisson bracket $\{f, H\}$ so that we have the definition of Poisson brackets between quantities of even nature as follows:

$$
\begin{aligned}
\{E_1, E_2\} = &\frac{\partial E_1}{\partial x_i}\frac{\partial E_2}{\partial p^i} - \frac{\partial E_2}{\partial p^i}\frac{\partial E_2}{\partial x^i} + \frac{\partial E_1}{\partial y_i}\frac{\partial E_2}{\partial \bar{p}^i} + \\
&- \frac{\partial E_1}{\partial \bar{p}^i}\frac{\partial E_2}{\partial y_i} + \frac{\partial E_1}{\partial \theta_\alpha}\frac{\partial E_2}{\partial \pi^\alpha} + \frac{\partial E_1}{\partial \pi^\alpha}\frac{\partial E_2}{\partial \theta_\alpha} + \frac{\partial E_1}{\partial \eta_\alpha}\frac{\partial E_2}{\partial \bar{\pi}^\alpha} + \frac{\partial E_1}{\partial \bar{\pi}^\alpha}\frac{\partial E_2}{\partial \eta_\alpha} \ .
\end{aligned} \tag{2.13}
$$

From (2.13) we immediately obtain the brackets between two variables of odd nature and those involving one even quantity and one odd quantity, which are respectively

$$
\begin{aligned}
\{O_1, O_2\} = &\frac{\partial O_1}{\partial x_i}\frac{\partial O_2}{\partial p^i} - \frac{\partial O_1}{\partial p_i}\frac{\partial O_2}{\partial x_i} + \frac{\partial O_1}{\partial y_i}\frac{\partial O_2}{\partial \bar{p}^i} - \frac{\partial O_1}{\partial \bar{p}^i}\frac{\partial O_2}{\partial y_i} + \\
&- \frac{\partial O_1}{\partial \theta_\alpha}\frac{\partial O_2}{\partial \pi^\alpha} - \frac{\partial O_1}{\partial \pi^\alpha}\frac{\partial O_2}{\partial \theta_\alpha} - \frac{\partial O_1}{\partial \eta_\alpha}\frac{\partial O_2}{\partial \bar{\pi}^\alpha} - \frac{\partial O_1}{\partial \bar{\pi}^\alpha}\frac{\partial O_2}{\partial \eta_\alpha}
\end{aligned} \tag{2.14}
$$

and

$$\{E,0\} = \frac{\partial E}{\partial x_i}\frac{\partial 0}{\partial p^i} - \frac{\partial E}{\partial p^i}\frac{\partial 0}{\partial x_i} + \frac{\partial E}{\partial y_i}\frac{\partial 0}{\partial \bar{p}^i} - \frac{\partial E}{\partial \bar{p}^i}\frac{\partial 0}{\partial y_i} + \\ + \frac{\partial E}{\partial \theta_\alpha}\frac{\partial 0}{\partial \pi^\alpha} + \frac{\partial E}{\partial \pi^\alpha}\frac{\partial 0}{\partial \theta_\alpha} + \frac{\partial E}{\partial \eta_\alpha}\frac{\partial 0}{\partial \bar{\pi}^\alpha} + \frac{\partial E}{\partial \bar{\pi}^\alpha}\frac{\partial 0}{\partial \eta_\alpha} \tag{2.15}$$

Equations (2.12-15) are extensions of those presented in Ref. 1, where several properties concerning Poisson brackets of even-even, even-odd and odd-odd variables were demonstrated as well.

An obvious application of (2.13-15) yieds the following nonvanishing Poisson brackets:

$$\{x_i, p^j\} = \delta_i^j = \{y_i, \bar{p}^j\}\ ,\ \{\theta_\alpha, \pi^\beta\} = -\delta_\alpha^\beta = \{\eta_\alpha, \bar{\pi}^\beta\}\ . \tag{2.16}$$

The Dirac bracket for systems with a-variables is given by

$$\{A,B\}^* = \{A,B\} - \{A,\xi_a\}(C^{ab})^{-1}\{\xi_b,B\}, \tag{2.17}$$

where A and B can have even or odd nature. As in the case of usual c-coordinate systerns, Dirac brackets are introduced in order to eliminate the second class constraints from the theory, so that the correspondence principle can be applied and a commutator algebra of operator associated with dynamical variables can be constructed. Also, as mentioned in Ref. 3, the existence of a set of second class constraints guarantees the existence of $(C^{ab})^{-1}$ in (2.17).

The extension to field theory of the results sketched above will not be treated here. We only note that for superfield Lagrangians the matrix representing the Poisson brackets of the constraints happens to be singular[2*]. This is a consequence of the existence in supersymmetric field theories of constraints which are different in character from those encountered in c-variable field theories. In this latter case[25] and also for a-variable models of point particles one can ensure that the matrix C^{ab} of Poisson brackets of constraints is non-singular. In the former case, in spite of the impossibility of inverting C^{ab}, one can define uniquely - at least for some simple situations - a C^{-1} matrix which gives the correct form of the Dirac brackets (see Ref. **24).**

3. Selected examples

We proceed by presenting some examples of second order Lagrangians. The supersymmetric systems suggested here are simple but may serve as a starting point for the study of similar Lagrangian models involving fields.

Nonrelativistic free superparticle

Inspired by the first order Lagrangian proposed in Ref. 3, p. 1864, we construct the following second order action

$$S_1 = \int_{t_1}^{t_2} L_1 dt + \frac{m}{2}[x(2)\cdot\dot{x}(2) - x(1)\cdot\dot{x}(1)] + \frac{i}{2}\theta(1)\cdot\theta(2) + \\ + k_{20}[\dot{\theta}(1)\cdot\theta(1) - \dot{\theta}(2)\cdot\theta(2)] \ , \tag{3.1}$$

with

$$L_1 = -\frac{m}{2}x\cdot\ddot{x} + \frac{i}{2}\dot{\theta}\cdot\theta + k_{20}\theta\cdot\theta \ , \tag{3.2}$$

where the simplified notation $x \equiv (x_j)$, $\theta \equiv (\theta_\alpha)$ is adopted, j,a $= 1,2,3, \theta(1) \equiv \theta(t_1), \theta(2) \equiv \theta(t_2)$, etc., with t_1 and t_2 representing initial and final times in the action principle, and k_{20} being an even constant. The Lagrangian (3.2) can be viewed as a particular case of the more general expression

$$\bar{L}_1 = -mx\cdot x\ddot{x}/2 + \sum_{A>B} k_{AB} \overset{(A)}{\theta} \cdot \overset{(B)}{\theta} \tag{3.3}$$

with the indices over the 9-vasiables denoting their derivative order. By taking $K_{10} = i/2$, K_{20}=arbitrary even constant, and other $K_{AB} = 0$ we obtain (3.2). We remark that the motivation for the inclusion of boundary terms in (3.1) is the maintenance of the equivalence between the number of boundary conditions and that of the order of the differential equations of motion. This need for supplementary boundary terms was discussed in detail in the context of a-variable first order Lagrangians.[3] The equation of motion derived from (3.1) are

$$\ddot{x} = 0 \ , \tag{3.4a}$$

$$\dot{\theta} = 0 \ , \tag{3.4b}$$

with extremization of S under the conditions

$$\delta x(1) = 0 = \delta x(2) \ , \ \delta\theta(1) + \delta\theta(2) = 0 \tag{3.5}$$

The first two conditions given in (3.5) correspond to the (second) order of the differential equation (3.4a), while the third condition in (3.5) is consistent with the first order equation (3.4b). Thus (3.2) describes a nonrelativistic free particle.

Using the supersymmetry transformations

$$\delta x = \frac{i}{m}\epsilon\theta \ , \ \delta\theta = \epsilon\dot{x} \ , \ \delta y = \frac{i}{m}\epsilon\eta \ , \ \delta\eta = \epsilon\dot{y} \ , \tag{3.6}$$

where y $\equiv \dot{x}$, $\eta \equiv \dot{\theta}$ and ϵ is an odd constant, one can show that the action (3.1) is invariant under supersymmetry, that is $\delta S = 0$ under (3.6), with the conserved quantity $\dot{x} \cdot 8$.

In passing we point out that (3.2) and (3.6) could be expressed in a more concise manner with the aid of the superfields

$$X(t,\tau) \equiv x(t) + i\tau\theta(t), \ \mathrm{Y(t,r)\ E}\ y(t) + i\tau\eta(t) = \dot{X}(t,\tau) \ , \tag{3.7}$$

where τ is an odd parameter. Introducing the operators

$$\nabla_0 \equiv \partial/\partial t \ , \mathrm{V} \equiv \partial/\partial\tau + i\tau\ \partial/\partial t \ , \ \mathrm{V} \equiv \partial/\partial\tau - i\tau\ \partial/\partial t \tag{3.8}$$

we can rewrite (3.6), with m = 1, as

$$\delta X = \epsilon\tilde{\nabla}X \quad , \quad \delta Y = \epsilon\tilde{\nabla}Y \ , \tag{3.9}$$

and the action (3.1) takes the form

$$S_1(X,Y) = \int dt d\tau \left\{ \frac{1}{2i}\Big[\nabla(\nabla_0 X)\Big] \cdot X - K_{20}\nabla_0 Y \cdot \nabla X \right\} + \\ + \text{boundary terms} \tag{3.10}$$

To regain (3.1) from (3.10) we make use of the standard integrals $\int d\tau = 0, \int d\tau \cdot \tau = 1$.

The superfield formulation[28,26] is particularly useful in the case of the relativistic superparticle, since it allows a unified description of reparametrization invariance and local supersymmetry.

The invariance of (3.1) under rotations and the Dirac bracket algebra of the spin vector components will be discussed in the next section.

Accelerated superparticle

Let us consider the action

$$S_2 = \int_{t_1}^{t_2} dt \Big\{ -\frac{m}{2} x \cdot \ddot{x} + mcx + K_{21} \ddot{\theta} \cdot \dot{\theta} \Big\} + \\ + \frac{m}{2}[x(2) \cdot \dot{x}(2) - x(1) \cdot \dot{x}(1)] + K_{21} \dot{\theta}(1) \cdot \dot{\theta}(2) \ , \tag{3.11}$$

where K_{21} and c are even constants.

From S_2 we obtain the equations of motion

$$\ddot{x} = c \ , \tag{3.12a}$$

$$\dddot{\theta} = 0 \ , \tag{3.12b}$$

the latter irnplying i) = a, where a is an anticommuting constant.

In the variational principle we have adopted the conditions

$$\delta x(1) = 0 = \delta x(2) \ , \tag{3.13a}$$

corresponding to two conditions for the second order eq. (3.12a), and

$$\delta\theta(1) = 0 = \delta\theta(2) \ , \ \delta\dot{\theta}(1) + \delta\dot{\theta}(2) = 0 \ , \tag{3.13b}$$

three conditions associated with the third order eq.(3.12b).

We can check the invariance of (3.11) under the transformation

$$\delta x = \frac{\epsilon}{m} \ddot{\theta} \quad , \quad \delta\theta = -\epsilon \dot{x} / 2K_{21} \ ,$$

with the conservation of $(\dot{x}\ddot{\theta} - c\dot{\theta})$.

Super harmonic oscillator

Now we take the action

$$S_3 = \int_{t_1}^{t_2} dt \Big\{ -\frac{1}{2}(m x \cdot \ddot{x} + K x^2) + \frac{i}{2}\dot{\theta} \cdot \theta + K_{21} \ddot{\theta} \cdot \dot{\theta} \Big\}$$
$$+ \frac{1}{2} m [x(2) \cdot \dot{x}(2) - x(1) \cdot \dot{x}(1)] - \frac{i}{2}\theta(1) \cdot \theta(2) + K_{21} \dot{\theta}(1) \cdot \dot{\theta}(2) \ , \quad (3.14)$$

where K and K_{21} are even constants.

The equations of motion are

$$m\ddot{x} + Kx = 0 \ , \qquad (3.15a)$$

$$-i\dot{\theta} + 2K_{21} \dddot{\theta} = 0 \ . \qquad (3.15b)$$

From eq. (3.15b) we have: $-i\theta + 2K_{21}\ddot{\theta} = A$, where $A \equiv (A_\alpha)$ is a vector whose components are odd constants. In the application of the variational method to (3.14) the conditions

$$\delta x(1) = 0 = \delta x(2) \ , \ \delta\theta(1) = \delta\theta(2) = 0 \ , \ \delta\dot{\theta}(1) + \delta\dot{\theta}(2) = 0 \qquad (3.16)$$

where used.

As in the preceding examples the boundary terms were chosen to provide a consistent extremization procedure.

The supersymmetry invariance of (3.14) can be verified using $\delta x = \frac{i}{m}\epsilon\theta$, $\delta\theta = -\epsilon\dot{x}$ if the additional conditions $A = 0$ and $K_{21} = -i/2\omega^2$ are imposed, with $\omega^2 = K/m$. In this case (3.14) describes a supersymmetric harmonic oscillator. The bosonic part of the action (which contains the x-dependent terms) leads to ordinary harmonic oscillations. The fermionic (θ-dependent) terms generate oscillations of the superparticle in the space of a-coordinates.

Extended Bose-Fermi oscillator

It is not difficult to transfer to the context of second order Lagrangians a combined Bose-Fermi model examined in Ref. 23, p. 285. The configuration space of this system is $I\!R_c \times I\!R_a^2$, where $I\!R_c$ and IR, are the real spaces of c-variables and of a-variables, respectively. The action is

$$S_4 = \int_{t_1}^{t_2} dt \left\{ \frac{1}{2}(\dot{x}^2 - \omega^2 x^2) + \frac{i}{2}(\tilde{\theta} \cdot \dot{\theta} + \omega \tilde{\theta} \Lambda \theta) \right\} . \tag{3.17}$$

The frequency w is the **same** in both fermionic and bosonic **terms.** Thís choice **assures** the supersymmetry invariance. The matrix A is defined as

$$\Lambda = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \tag{3.18}$$

and $\tilde{\theta}$ denotes the transposed of the vector $\theta =$)

We propose the following extension of (3.14)

$$\begin{aligned} \bar{S}_4 = S_4 + \int_{t_1}^{t_2} dt(\alpha \tilde{\dot{\theta}} \Lambda \ddot{\theta} + \beta \dot{\tilde{\theta}} \Lambda \dot{\theta}) + \frac{1}{2} \tilde{\theta}(1) \cdot \theta(2) + \\ + \alpha[\dot{\tilde{\theta}}(1) \Lambda \theta(2) - \tilde{\theta}(1) \Lambda \dot{\theta}(2)] + \beta[\tilde{\theta}(1) - \tilde{\theta}(2)] \times \\ \times [\Lambda(\dot{\theta}(1) + \dot{\theta}(2))] , \end{aligned} \tag{3.19}$$

where a and β are even parameters. Concerning the bosonic part we adopt the customary action **principle** with conditions $\delta x(1) = 0 = \delta x(2)$. The fermionic part produces the two second order equations of motion

$$i(\omega \Lambda \theta + \dot{\theta}) + 2(\alpha - \beta) \Lambda \ddot{\theta} = 0 . \tag{3.20}$$

The four boundary conditions

$$\delta\theta(1) + \delta\theta(2) = 0 , \tag{3.21a}$$

$$\delta\dot{\theta}(1) + \delta\dot{\theta}(2) = 0 \tag{3.21b}$$

have been used. Note that the imposition of the more familiar conditions $\delta\theta(1) = 0 = \delta\theta(2)$, $\delta\dot{\theta}(1) = 0 = \delta\dot{\theta}(2)$ instead of (3.21) implies the inconsistency of eight requirements on the θ's and $\dot{\theta}$'s for the two second order equations (3.20). The additional boundaries in *(3.19)* were chosen in order to yield *(3.20)* for $\delta S_4 = O$ under *(3.21)*. By the way, the first order action S_4 of Ref. *23* must be supplemented by a boundary term as well, namely $i\tilde{\theta}\cdot\theta(2)/2$. Then its Euler-Lagrange equations $\omega\Lambda\theta + \dot{\theta} = 0$ are consistently derived under the requirements $\delta\theta(1) + \delta\theta(2) = 0$.

When $a = \beta$ one obtains from *(3.20)* the habitual dynamical equations

$$\ddot{\theta} + \omega^2\theta = 0 \ . \tag{3.22}$$

In this case only (3.21a) is needed in the extremization procedure, since *(3.20)* reduces to a set of two first order equations.

One can check the invariance of $\bar{S}_4$ under the supersymmetry transformations

$$\delta x = i\theta E \quad , \quad 68 = (\dot{x}I\!I - \omega x\Lambda)E \ , \tag{3.23}$$

with

$$E = \begin{pmatrix} \epsilon_1 \\ \epsilon_2 \end{pmatrix} \quad , I\!I \equiv \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \tag{3.24}$$

ϵ_1 and ϵ_2 being odd infinitesimal parameters.

It follows that the quantity $Q \equiv (51 + \omega x\Lambda)\theta$ is conserved. This Q is, in fact, the generator of *(3.23)*.

The action S_4 admits a simple generalization leading to a nonlinear system.[23] In a similar manner we can generalize $\bar{S}_4$ by making the substitutionof S_4 by

$$S_4' = \int \frac{1}{2}\{\dot{x}^2 - [V'(x)]^2 + i[\tilde{\theta}\dot{\theta} + V''(x)\tilde{\theta}\Lambda\theta]\}dt \ . \tag{3.25}$$

This nonlinear $\bar{S}_4'$ gives the equations of motion

$$\ddot{x} + V'(x)V''(x) - \frac{i}{2}V'''(x)\tilde{\theta}\Lambda\theta = 0 \ . \tag{3.26a}$$

$$i(V''(x)\Lambda\theta + \dot{\theta}) + 2(\alpha - \beta)\Lambda\ddot{\theta} = 0 \ . \tag{3.26b}$$

The supersymmetry transformations which leave the generalized $\bar{S}_4'$ invariant are

$$\delta x = \dot{x}\xi + i\tilde{\theta}E \ , \ 68 = \dot{\theta}\xi + [\dot{x}\Pi - V'(x)\Lambda]E \ , \tag{3.27}$$

where ξ is an even parameter. One regains (3.20) and (3.23) for $V(x) = \omega x^2/2$.

Relativistic free superparticle

Here we take

$$\begin{aligned} S_5 = \int_{t_1}^{t_2} & \left\{ \frac{Z^2}{2N} + \frac{i}{2}(\dot{\theta}\cdot\theta + \dot{\theta}_5\cdot\theta_5) + K_{20}(\ddot{\theta}\cdot\theta + \right. \\ & \left. + \ddot{\theta}_5\cdot\theta_5) - \frac{1}{2}Nm^2 - iMm\theta_5 \right\} + \frac{i}{2}[\theta(1)\cdot\theta(2) + \\ & \theta_5(1)\cdot\theta_5(2)] + K_{20}[\dot{\theta}(1)\cdot\theta(1) + \dot{\theta}_5(1)\cdot\theta_5(1) + \\ & + \theta(2)\cdot\dot{\theta}(2) + \theta_5(2)\cdot\dot{\theta}_5(2)] \ , \end{aligned} \tag{3.28}$$

where $(\theta,\theta_5) \equiv ((\theta_\mu),\theta_5)$, $x \equiv (x_\mu)$, $Z_\mu \equiv \dot{x}_\mu - iM\theta_\mu$, $\mu = 0,\ldots,3$, M and N are odd and even Lagrange multipliers, respectively, and again appropriate surface terms must be included.

Extremization of S_5 with respect to x_μ, θ_μ, θ_5, M and N gives

$$\frac{d}{dt}\left(\frac{Z^\mu}{N}\right) = 0 \ , \ \frac{M}{N}Z^\mu - \dot{\theta}^\mu = 0 \ , \tag{3.29}$$

under the condition

$$\delta x_\mu(1) = 0 = \delta x_\mu(2) \ , \ \delta\theta_\mu(1) + \delta\theta_\mu(2) = 0 \ , \delta\theta_5(1) + \delta\theta_5(2) = 0 \ . \tag{3.30}$$

The action S_5 is invariant under the supersyrnmetry transformations

$$\delta x = i\epsilon\theta \ , \ \delta\theta = \epsilon\ Z/N \ , \ \delta\theta_5 = m\epsilon \ , \ \delta M = \dot{\epsilon} \ , \ \delta N = 2i\epsilon M \ , \tag{3.31}$$

where $\epsilon = \epsilon(t)$ is an odd parameter.

The relativistic superparticle described by (3.28) has also reparametrization invariance, i.e. invariance under the transformations

$$\delta x = \xi\dot{x} \ , \ \delta\theta = \xi\dot{\theta} \ , \ \delta\theta_5 = \xi\dot{\theta}_5 \ , \ \delta M = (\xi M)^{\cdot} \ , \ \delta N = (\xi N)^{\cdot} \ , \tag{3.32}$$

$\xi(t)$ being an even parameter.

In the massless limit of (3.28), that is

$$\bar{S}_5 = \int_{t_1}^{t_2} dt \left\{ \frac{Z^2}{2N} + \frac{i}{2}\dot{\theta}\cdot\theta + K_{20}\ddot{\theta}\cdot\theta \right\} + \frac{i}{2}\theta(1)\cdot\theta(2) + \\ + K_{20}[\dot{\theta}(1)\cdot\theta(1) + \theta(2)\cdot\dot{\theta}(2)] \tag{3.33}$$

we can get an equivalent superfield action which exhbits both reparametrization and local supersymmetry invariances united by means of a compact set of transformations. The result is

$$\tilde{S}_5(X,Y) = \int dt d\tau \left\{ Q(t,\tau)\left[\frac{1}{2i}\nabla_0 X\cdot\nabla X - K_{20}\nabla_0 Y\cdot \nabla X\right]\right\} + \\ + \text{surface terms}\ , \tag{3.34}$$

where the derivatives are defined in (3.8), and $Q(t,\tau) \equiv N(t) + i\tau M(t)$. The corresponding first-order action is presented in Ref. *8*. The unified super-reparametrization is expressed by the transformations $t' = t + \alpha(t,\tau)$ and $\tau' = \tau + \beta(t,\tau)$, where $\alpha(t,\tau)$ and $\beta(t,\tau)$ are appropriately defined functions which will not be explicitly given here.

4. Hamiltonian formalism

In this section we are concerned with the application to our examples of Dirac's prescriptions for constrained systems[25]. In all these examples, after the calculation of the Poisson brackets we are left with second class sets of constraints. Hence we compute the correspondent Dirac brackets in order to obtain first class algebras which can supply adequate structures for canonical quantization.

The Dirac brackets of angular momentum components are also derived here. The spin components result to be functions of odd variables.

Nonrelativistic free superparticle

The fermionic part of Lagrangian (3.2) gives rise to the primary constraints

$$\phi_\alpha = \pi_\alpha - \frac{i}{2}\theta_\alpha + K_{20}\dot{\theta}_\alpha \approx 0\ , \tag{4.1a}$$

$$\xi_\alpha = \bar{\pi}_\alpha - K_{20}\theta_\alpha \approx 0 \ , \tag{4.1b}$$

where π^α and $\bar{\pi}^\alpha$ are obtained from (2.7-8) for the Lagrangian (3.2); $\alpha = 1,2,3$.

The total Hamiltonian is

$$H_T = \frac{\bar{\bar{p}}^2}{2m} + M^\alpha\Big(\pi_\alpha - \frac{i}{2}\theta_\alpha + K_{20}\eta_\alpha\Big) + N^\alpha(\bar{\pi}_\alpha - K_{20}\theta_\alpha) \ , \tag{4.2}$$

with $\bar{\bar{p}}_j = m\dot{x}_j$, and M^α, N^α are anticommuting Lagrange multipliers. The bosonic part can be treated as a nonsingular first order system with the canonical pair $(x_j, \bar{\bar{p}}_j)$, yielding the usual results.[21]

We have no secondary constraints, since

$$\dot{\phi}_\alpha = \{\phi_\alpha, H_T\} \approx 0 \quad , \quad \dot{\xi}_\alpha = \{\xi_\alpha, H_T\} \approx 0 \quad . \tag{4.3}$$

The Poisson brackets between constraints are

$$\{\phi_\alpha, \phi_\beta\} = i\delta_{\alpha\beta} \ , \ \{\xi_\alpha, \xi_\beta\} = 0 = \{\phi_\alpha, \xi_\beta\} \ , \tag{4.4}$$

hence the ϕ_α are second class and we apply (2.17) to determine the Dirac brackets between the odd canonical variables,

$$\{\theta_\alpha, \theta_\beta\}^* = i\delta_{\alpha\beta} \ , \ \{\eta_\alpha, \eta_\beta\}^* = 0 = \{\theta_\alpha, \eta_\beta\}^* \ , \tag{4.5}$$

After the introduction of the Dirac brackets the π_α become auxiliary quantities proportional to the dynamical variables θ_α since $\phi_\alpha = \pi_\alpha - i\theta_\alpha/2 = 0$ can now be viewed as a strong equation.

From Noether's theorem one obtains the expression for the total angular momentum of a second order superparticle. In the absence of surface terms in the action principle the result is

$$J_{ij} = (x_i p_j - x_j p_i + \dot{x}_i \bar{p}_j - \dot{x}_j \bar{p}_i) + (\theta_i \pi_j - \theta_j \pi_i + \dot{\theta}_i \bar{\pi}_j - \dot{\theta}_j \bar{\pi}_i) \ , \tag{4.6}$$

with $i, j = 1, ..., N$.

The first term in parentheses in (4.6) contains the first order $(L_{(1)ij})$ and the second order $(L_{(2)ij})$ angular momentum, with $L_{ij} = L_{(1)ij} + L_{(2)ij}$, while the second term gives the spin part $S_{ij} = S_{(1)ij} + S_{(2)ij}$, also with contributions of first and second order. When the extremization involves boundary terms the correct procedure is to take ***ab initio*** the variation of the action including boundaries. By rearranging terms in δS and by requiring $\delta S = 0$ under rotations ***of*** canonical variables one then derives the consemation of the angular momentum.

Applying this prescription to the action **(3.1)** we obtain

$$J_{ij} = x_i \bar{\bar{p}}_j - x_j \bar{\bar{p}}_i + i\theta_i\theta_j \quad , \quad i,j = 1,2,3 \ . \tag{4.7}$$

In this calculation the variations related to terms containing K_{20} cancel out, so that our result does not differ from that obtained for the first order particle[3]. Moreover, the Dirac brackets between the generators J_{ij} given by **(4.7)** exhibit the usual algebra associated with the three-dimensional rotation group, as can be easily checked.

Accelerated superparticle

For the bosonic part we have the constraints

$$\xi_j = p_j - \frac{m}{2} y_j \approx 0 \quad , \quad \lambda_j = \bar{p}_j + \frac{m}{2} x_j \approx 0 \quad ; i,j = 1,2,3 \ . \tag{4.8}$$

The Poisson brackets between these constraints are

$$\{\xi_i, \xi_j\} = 0 = \{\lambda_i, \lambda_j\} \ ; \ \{\xi_i, \lambda_j\} = -m\delta_{ij} \ , \tag{4.9}$$

leading to the Dirac brackets

$$\{x_i, x_j\}^* = 0 = \{\dot{x}_i, \dot{x}_j\}^* \ ; \ \{x_i, \dot{x}_j\}^* = \frac{\delta_{ij}}{m} \ , \tag{4.10}$$

In view of **(4.8)** the $p_j, \bar{p}_j$ can be considered as nondynamical quantities after the computation of Dirac brackets. Note that from **(4.10)** we derive $\{x_i, \bar{p}_j\}^* = \delta_{ij}$, the ordinary result for the first order accelerated particle.

The fermionic part yields

$$\phi_j = \bar{\pi}_j - K_{21}\dot{\theta}_j \approx 0 \ , \tag{4.11}$$

$$\{\phi_i, \phi_j\} = 2K_{21}\delta_{ij} \ , \tag{4.12}$$

$$\{\theta_i,\theta_j\}^* = 0 = \{\theta_i, \dot\theta_j\}^* = \{\theta_i,\pi_j\}^* = \{\pi_i,\pi_j\}^* \ ,$$
$$\{\dot\theta_i,\dot\theta_j\}^* = -\frac{\delta_{ij}}{2K_{21}} \ , \{\theta_i,\pi_j\}^* = -\delta_{ij} \ . \qquad (4.13)$$

The equation $\pi_j = -2K_{21}\ddot\theta_j = -2K_{21}\dot\eta_j$ propagates the η's, so that we have in this case the dynamical variahles $\theta_j, \dot\theta_j, \pi_j$ and the auxiliary $\bar\pi_j$.

We note that with the aid of the equations of motion we can write the Hamiltonian in the form

$$H_T = \dot x p + c\bar p - mcx/2 + \dot\theta.\pi + a.\bar\pi - K_{21}a.\dot\theta + M^j\xi_j + N^j\lambda_j + R^j\phi_j \ .$$

From the consistency conditions on the primary constraints it follows then that $M^j = 0 = N^j = R^j$. Therefore we have no secondary constraints.

The total angular momentum is

$$J_{ij} = x_i\bar{\bar{p}}_j - x_j\bar{\bar{p}}_i + 2K_{21}(\dot\theta_i\dot\theta_j - \theta_i\ddot\theta_j + \theta_j\ddot\theta_i) \ . \qquad (4.14)$$

Therefore a second order effect is manifest in S_{ij}. Introducing, as usual, the spin vector

$$S_i \equiv \frac{1}{2}\epsilon_{ijk}S_{jk} = \epsilon_{ijk}K_{21}(\dot\theta_j\dot\theta_k - 2\theta_j\ddot\theta_k) \ , \qquad (4.15)$$

where ϵ_{ijk} is the three-dimensional Levi-Civita symbol, we obtain the customary algebra

$$\{S_i, S_j\}^* = \epsilon_{ijk}S_k \ . \qquad (4.16)$$

Super harmonic oscillator

In this case the primary constraints are

$$\xi_j = p_j - \frac{m}{2}y_j \approx 0 \ , \qquad (4.17a)$$

$$\lambda_j = \bar p_j + \frac{m}{2}x_j \approx 0 \ , \qquad (4.17b)$$

$$\sigma_j = \bar\pi_j - K_{21}\eta_j \approx 0 \ . \qquad (4.17c)$$

The formula $\pi_j = \frac{i}{2}\theta_j - 2K_{21}\dot\theta_j$ propagates the dynamical variable θ_j η_j but it also exhbits a relation between $\dot\theta_j$ and π_j. If we use

$$\ddot\theta_j = \frac{1}{2K_{21}}(A_j + i\dot\theta_j) \ , \tag{4.18}$$

derived from (3.15b), we get the additional constraint

$$\tau_j = \pi_j + \frac{1}{2}\theta_j + A_j \approx 0 \ . \tag{4.17d}$$

In this way we are left with the set of true dynamical variables $(x_i, y_i, \theta_i, \eta_i)$ with Dirac brackets

$$\{x_i, x_j\}^* = 0 = \{y_i, y_j\}^* \ ; \ \{x_i, y_j\}^* = \frac{\delta_{ij}}{m} \ ; \tag{4.19a}$$

$$\{\theta_i, \cdot_I \quad -i\delta_{ij} \ ; \ \{\eta_i, \eta_j)^* = -\frac{\delta_{ij}}{2K_{21}} \ ; \{\theta_i, \theta_j\}^* = 0 \cdot \tag{4.19b}$$

The absence of secondary constraints can be verified by imposing $\{\phi, H_T\} \approx 0$, with $\phi \equiv (\xi_j, \lambda_j, \sigma_j, \tau_j)$,

$$H_T = \dot x.p - \omega^2 x.\bar p + \dot\theta.\pi + A.\Big(\frac{\bar\pi}{2K_{21}}\Big) + i\theta.\Big(\frac{\bar\pi}{2K_{21}} - A.\Big(\frac{\dot\theta}{2}\Big) + F.\phi \ ,$$

$A \equiv (A_j)$, $F \equiv (M^j, N^j, R^j, T^j)$.

The spin vector in this case turns out to be

$$S_i = \epsilon_{ijk}\Big[\frac{1}{2}\theta_j\theta_k - \frac{i}{2\omega^2}(\dot\theta_j\dot\theta_k - 2\theta_j\ddot\theta_k)\Big] \ . \tag{4.20}$$

Extended Bose-Fermi oscillator

The c-variable terms of *(3.19)* are associated with the usual harmonic oscillations. The novelty is the fermionic part, which gives the constraints

$$\Pi + \frac{i}{2}\theta - (2\beta - \alpha)\Lambda\dot\theta \approx 0 \ , \ \bar\Pi - \alpha\Lambda\theta \approx 0 \ . \tag{4.21}$$

The nonvanishing brackets between the dynamical variables θ and $\dot\theta$ are

$$\{\dot\theta_1, \dot\theta_1\}^* = -\frac{i}{4(\alpha-\beta)^2} = \{\dot\theta_2, \dot\theta_2\}^* \ ;$$

$$\{\theta_1, \dot{\theta}_2\}^* = \frac{1}{2(\alpha - \beta)} = -\{\theta_2, \dot{\theta}_1\}^* \tag{4.22}$$

We are assuming $\alpha \neq \beta$. When $\alpha = \beta$ equation (3.20) reduces to $\dot{\theta} + \omega\Lambda\theta = 0$, which implies $\ddot{\theta} + \omega^2\theta = 0$. In this particular case the Dirac brackets between canonical variables become

$$\{\theta_\mu, \theta_\nu\}^* = -i\delta_{\mu\nu} \; ; \; \{\dot{\theta}_\mu, \dot{\theta}_\nu\}^* = 0 = \{\theta_\mu, \dot{\theta}_\nu\}^* \quad ; \quad \mu\nu = 1,2 \; . \tag{4.23}$$

The spin is given by the antisymmetric matrix

$$S = \tilde{\theta}\Lambda\Pi + \tilde{\bar{\theta}}\Lambda\bar{\Pi} \; ,$$

which has the components

$$S_{12} = -S_{21} = -i\theta_1\theta_2 + 2(\alpha - \beta)(\theta_1\dot{\theta}_1 + \theta_2\dot{\theta}_2) \; , \tag{4.24}$$

and $S_{\mu\nu} = -i\theta_\mu\theta_\nu$, if $\alpha = \beta$. Besides the existence of a difference in sign between this $S_{mu\nu}$ and the S_{ij} of (4.7) we also remark that the first brackets in (4.5) and (4.23) have opposite signs, with the consequence that the same algebra (4.16) is obeyed in both situations.

Relativistic free superparticle

The primary constraints

$$\phi_\mu = \pi_\nu - \frac{i}{2}\theta_\mu + K_{20}\dot{\theta}_\mu \approx 0 \; , \; \xi_\mu = \bar{\pi}_\mu - K_{20}\theta_\mu \approx 0 \; ,$$

$$\xi = \pi_5 - \frac{i}{2}\theta_5 + K_{20}\dot{\theta}_5 \approx 0 \; , \; \lambda = \bar{\pi}_5 - K_{20}\theta_5 \approx 0 \; , \tag{4.25}$$

have the same algebra of Poisson brackets as the corresponding constraints of the first order relativistic particle.[3] Therefore we get for the real variables the standard nonvanishing brackets

$$\{x_\mu, p_\nu\}^* = \eta_{\mu\nu} \; ; \; \{\theta_\mu, \theta_\nu\}^* = i\eta_{\mu\nu} \; , \{\theta_5, \theta_5\}^* = i \; , \tag{4.26}$$

The dynamics is determined by the Hamiltonian constraints

$$\mathcal{H} = m^2 + p^2 \approx 0 \quad , \quad \varsigma = m\theta_5 + \theta_\mu p^\mu \approx 0 \; , \tag{4.27}$$

which obey the algebra

$$\{\varsigma,\varsigma\} = i\mathcal{H} \quad ,\{\varsigma,\mathcal{H}\} = 0 = \{\mathcal{H},\mathcal{H}\} \; . \tag{4.28}$$

$\mathcal{H}$ is the generator os reparametrizations and ς (associated with the spin degrees of freedom) is the generator of supersymmetry transformations. Eq. **(4.28)** shows that ς can be viewed as the square root of $\mathcal{H}$, i.e. the supersymmetry invariance generated by ς is obtained by taking the square root of the effective Hamiltonian H of the spinless system .[3,6] This H can be written as H = MX, M being a Lagrange multiplier, while the Hamiltonian for the spinning particle is a linear combination of $\mathcal{H}$ and ς.

The invariance of the action **(3.33)** under the Poincaré transformations

$$\delta x_\mu = \omega_\mu{}^\nu x_\nu + \epsilon_\mu \; , \; \delta p_\mu = \omega_\mu{}^\nu p_\nu \; , \; \delta\theta_\mu = \omega_\mu{}^\nu .\theta_\nu \; , \; \delta\theta_5 = 0 \; , \tag{4.29}$$

leads to the conservation of

$$J_{\mu\nu} = x_\mu p_\nu - x_\nu p_\mu + i\theta_\mu\theta_\nu \; . \tag{4.30}$$

Consequently, in both relativistic and nonrelativistic cases the total angular momentum of the first order model and that of the second order model coincide. This result illustrates the ambiguity associated with the existence of different Lagrangians describing the same dynamics and giving rise to the same interactions of the spin with externa1 fields.

5. Final remarks

In summary, we have considered second order Lagrangian models which possess a first order counterpart. The prescription of supersymmetry invariance relating commuting and anticommuting variables and that of a consistent formulation of the variational principle have been adopted in the construction of our examples. Along the lines of previous investigations by other authors on the subject of first order supersymmetric Lagrangians, we have studied the pseudodynamics of second order systems. The existence of constraint relations has imposed the use of

Dirac's approach in order to derive the algebraic structures describing the evolution of these systems. We notice that our free particle Lagrangians (relativistic and nonrelativistic) of second order differ from their first-order counterparts by a total derivative, giving rise to the same equations of motion and to identical expressions for the angular momentum, this latter result implying the same behaviour in presence of external fields. The canonical treatment for this class of second-order Lagrangians was developed in Ref. 21.

In the cases representing the accelerated particle and the oscillators we have found a second-order effect which is manifest in the spin vector, even though the algebra of Dirac brackets between components of the spin vector remains the same as that of first order similar models. It is interesting to mention that for the super harmonic oscillator the magnitude of the second order effect of spin depends on the value of the frequency w, which, from the requirement of supersymmetry invariance, is the same for oscillations in c-coordinate and in a-coordinates spaces. Thus, in this example, even if the variables which originate the spin belong to the space of odd coordinates, a quantity w related to oscillations in ordinary R, space does affect the interactions mediated by the spin.

Another point is the comparison between the introduction of the spin via a-variables and the spin effect produced either by properties of the particle itself or by properties of its environment. This latter situation is exemplified by Papapetrou's method for the derivation of the equations of spinning test particles in curved space[27]. On the other hand, the spin effect due to the point particle itself appears for instance in the worldline limit of the Polyakov model of strings with extrinsic curvature[28], or in its generalized version[29]. In these classes of models the so-called rigidity of the string furnishes a measure of the influence of the extrinsic curvature on the motion of the point particle, in the one-dimensional limit. Owing to this implicit influence geodesics are not in general straight lines. Other models associate the spin tensor with classical spinors representing internal dynamical variables[30]. This suggestion is closer to that of the utilization of a-variables, in the sense that in both situations supplementary coordinate spaces are called to intervene with the purpose of creating an appropriate classical scenario for the spin.

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Resumo

Considera-se a dinâmica clássica de sistemas contendo derivadas segundas de variáveis que anticomutam. Apresenta-se exemplos de lagrangeanos supersimétricos de segunda ordem. A teoria de vínculos de Dirac é aplicada a esses modelos, comparando-se os resultados com aqueles correspondentes a lagrangeanos de primeira ordem propostos na literatura.