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Lifetime in ⁹¹Zr

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Abstract Nuclear lifetimes of five levels up to an excitation energy of $\approx 2.5 \text{ meV}$ in ⁹¹Zr were measured by the Doppler Shift Attenuation Method (DSAM). The experimental results were compared with theoretical predictions based on the shell model assuming ⁸⁸Sr to be the inert core.

1. Introduction

The lifetimes of the first five excited levels have been measured by several different methods^{1,2,3} but not all by the same method. There is also inelastic deuteron scattering data available⁴ but the extraction of a lifetime from the measured cross section is model dependent. The present experiment was performed with a view to provide the first complete set of lifetime measurements of the first five excited states with one technique (Doppler Shift Attenuation Method) in order to minimize possible systematic errors as evidenced by the conflicting reported values for the lifetime of the first excited state.

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Previous calculations of the low-lying characteristics of ${}^{91}Z_r$ have relied, with few exceptions, on increasingly elaborate shell model schemes. The first calculations by Talmi⁵ considered only $(d_{5/2})n$ configurations and were aimed at reconstructing the spectrum of levels. More recent calculations^{6,7} consider the $p_{1/2}$ and $g_{9/2}$ orbitals for the protons and $d_{5/2}$ and $s_{1/2}$ for the neutrons. In addition Chuu et al.⁶ included the $d_{3/2}$ and $g_{7/2}$ neutron orbitals in the calculations. These works studied not only the spectrum of levels but calculations of the spectroscopic factors for one-particle pickup and stripping and, in a few cases, transition probabilities^{6,7,8,9}.

The limited number of calculated transition probabilities and the lack of published wave functions have led us to recalculate the nuclear structure of ⁹¹Zr based on the model of Chuu et al.⁶. A comparison of all available lifetime measurements with our theoretical predictions is presented.

2. Experimental method

Levels in ⁹¹Zr up to an excitation energy of $\simeq 2.5$ MeV were populated by means of the ⁸⁸Sr($\alpha, n\gamma$)⁹¹Zr reaction using a-particle beams of (9.75 - 10.8) MeV incident energy from the Pelletron tandem accelerator of the University of São Paulo. A thick (35 mg/cm²) target was made of pressed metallic natural strontium. Gamma-ray spectra were obtained at several angles between $0^0 - 90^0$ to the beam direction with a coaxial Ge(Li) detector of 2.2 keV FWHM and 15% efficiency. Gamma-ray lines from radiactive sources of ²⁰⁷Bi and ^{56,60}Co were introduced into the spectra for calibration purposes and as a check on the gain stability of the system. The gamma spectra shown in figures 1a and 1c were taken at 90⁰ at which angle there is no Doppler shift. The spectra of figures 1b and 1d, taken at the same bombarding energy but with a detector angle of 0⁰, give examples of both large Doppler shifts (short lifetimes) and small Doppler shifts (long lifetimes which permit stopping of the nucleus before emission). The transitions with intermediate Doppler shifts with considerable line broadening are especially suitable for peak shape analysis.



Fig.1 - Typical gamma ray spectrum for the reaction ⁸⁸(Sr($\alpha, n\gamma$) at $E_{\alpha} = 10.8$ MeV. a) Ge(Li) detector at 90⁰ with respect to beam direction. b) Ge(Li) detector at 0⁰ with respect to beam direction.

3. Analysis of data

The lifetimes of the excited states were extracted both by the centroid-shift method and by gamma-ray line-shape analysis. The Doppler shifts AE7 were determined from a least-squares fit to the experimental γ -ray energies as a function of the cosine of the laboratory angle. The centroids of the peaks were obtained from first-moment calculations. The slope of the straight line ($\Delta E\gamma$) vs. $\cos\theta$ yields the value of the product $E\gamma(90^0) F(\tau) \beta(0)$. We use the approximation¹⁰

$$\beta(0) \simeq \beta_{\rm CM} [1 \pm 0.33R]$$



Fig.1 - Typical gamma ray spectrum for the reaction ⁸⁸ (Sr($\alpha, n\gamma$) at $E_{\alpha} = 10.8$ MeV. c) Ge(Li) detector at 90[°] with respect to beam direction. d) Ge(Li) detector at 0[°] with respect to beam direction.

where

$$B_{CM} = \sqrt{2M_1}E_1/(M_1 + M_2)$$

and R is the ratio of the recoil velocity in the center of mass system to the velocity of the center of mass in the laboratory system, i.e.

$$R = \Big\{rac{M_2M_3}{M_1M_4}\Big[1+rac{M_1M_2}{M_2}\Big]\Big\}^{1/2}$$

This approximation has been discussed in detail¹⁰ and holds rather well for the present incident energies (E, $\sim 1.4E_{\rm th}$). Also, at these incident energies no corrections for upper-level feeding were necessary, except in the case of the 1.205 MeV level which is fed from the 2.356 MeV level.

The theoretical $F(\tau)$ values were calculated with the computer code FTAU¹¹, which utilizes the stopping power theory of Lindhard-Blaugrund^{12,13,14.} The total errors in the mean lives were determined by adding, in quadrature, the experimental errors and the error derived from an assumed uncertainty of 15% in the electronic and nuclear stopping powers of the target material. The $F(\tau)$ values and lifetimes obtained are given in figure 2.

The energy of a gamma ray emitted by a recoiling nucleus depends on the velocity of the nucleus and the relative position of the detector. The basic shape of the resultant gamma line is a convolution of the detector resolution with the velocity distribution. The velocities of the recoiling nucleus are a function of the stopping process weighted by the exponential decay of the state in the recoiling nucleus. The gamma line-shape analysis was performed using the code SHAPE¹⁰ and also with a MONTECARLO program¹⁵ developed by us. While the code SHAPE¹⁰ is based on a phenomenological approach to the stopping power, the Monte Carlo code generates the gamma-line shapes as a function of r using the LSS formula for the stopping power;

$$-\frac{d\epsilon}{d\rho} = k\epsilon^{1/2} + \eta \frac{d\epsilon}{d\rho_{\rm nucl}}$$

The parameter k has a value¹⁶ of 0.16. The line shapes obtained are compared with experimental ones in figure 2. The errors on the mean lives were estimated from the χ^2 vs r curves.

4. Theoretical calculations of lifetimes

These calculations are based on a shell-model space of $p_{3/2}$ and $g_{9/2}$ proton orbitals and $d_{5/2}$, $d_{1/2}$, $d_{3/2}$ and $g_{7/2}$ neutron orbitals coupled to an inert ⁸⁸Sr core. The system is described by the Hamiltonian

$$H=H_{
m op}+H_{
m pp}+H_{
m on}+H_{
m pm}$$







Fig.2 - Results of the lineshape fitting programs SHAPE (dashed line) and MONTE CARLO (solid line); the filled circles are experimental points. a) E_{γ} =1.151 MeV (Ex = 2.356 MeV), E_{α} = 9.75 MeV; b) E_{γ} = 1.205 MeV; E_{α} = 10.3 MeV; c) E_{γ} = 1.467 MeV; E_{α} = 10.3 MeV; d) E_{γ} = 1.467 MeV; E_{α} = 10.8 MeV; e) E_{γ} = 1.882 MeV; E_{α} = 10.8 MeV; f) E_{γ} = 2.041 MeV; E_{α} = 10.8 MeV; g) E_{γ} = 2.132 MeV; E_{α} = 10.8 MeV.

where H_{op} and H_{on} represent the single-particle energies for protons and neutrons, respectively. The results of Ball et al.¹⁷ were used for the proton-proton interaction. The effective two-body proton-neutron residual interaction, H_{pn} , as used by Chuu et al.⁶, includes explicitly the surface 6-interaction (SDI)

$$V_{\rm pn} = 4\pi (G_0 P_0 + G_1 P_1) \delta(\Omega_{\rm pn}) \tag{1}$$

where P_0 , P_1 are, respectively, the projection operators for T = 0, T = 1 states and G_0 , G_1 the corresponding interaction strength. The eigenvalue problem is solved in the **basis** $|[(j_1 \ j_2)Jp, \ j_3]J >$, where j = (nlj) stands for the quantum numbers of the proton states (j_1, j_2) and neutron states (j_3) , J p is the total angular momentum of two protons, and J is the total angular momentum.

The electric quadrupole and magnetic dipole operators are

$$\mathcal{M}(E2,\mu) = e_i^{\text{eff}} \sum_{i=1}^3 r_i^2 Y_{2\mu}(\theta_1,\varphi_i)$$
(2)

$$\mathcal{M}(M1,\mu) = \frac{3}{4\pi}^{1/2} [g_{\ell} L_{\mu} + g_s^{\text{eff}} S_{\mu}]$$
(3)

where e_i^{eff} , for i = 1, 2, is the effective proton change, e_p^{eff} , and for i = 3, the effective neutron charge, e_n^{eff} , and g_ℓ and g_s^{eff} are, respectively, orbital and effective spin gyromagnetic ratios.

The mixing ratio $\delta(E2/M1)$ is given by¹⁸

$$\delta\left(\frac{E2}{M1}\right) = 0.835\left(\frac{E_{\gamma}}{\text{MeV}}\right)\left(\frac{D}{eb\mu_{\text{N}}^{-1}}\right)$$
(4)

with

$$D = \frac{\langle J_i \| \mathcal{M}(E2) \| J_f \rangle}{\langle J_i \| \mathcal{M}(M1) \| J_f \rangle}.$$
 (5)

The reduced transition probabilities are

$$B(\bar{\omega}; J_i \to J_f) = \frac{\langle J_i \parallel \mathcal{M}(\bar{\omega}L) \parallel J_f \rangle^2}{2J_i + 1}$$
(6)

with $\omega L = E2, M1$ for electric and magnetic cases, respectively. The matrix elements of the E2 and M1 operators are expressed as

$$\langle J_i \parallel \mathcal{M}(E2) \parallel J_f \rangle = (e_p^{\text{eff}}A + e_n^{\text{eff}}B)eb$$
 (7)

$$< J_i \parallel \mathcal{M}(M1) \parallel J_f >= (g_{\ell_p}C + g_{s_p}^{\text{eff}}D + g_{\ell_n}E + g_{s_n}^{\text{eff}}F)\mu_N$$
(8)

and the quantities A, B, C, D, E and F are calculated from the model wavefunctions.

The mean lifetime corresponding to the transition probability T of the decaying state J_i to J_f is

$$\tau(\bar{\omega}L; J_i \to J_f) = \frac{1}{T(\bar{\omega}L; J_i \to J_f)}$$
(9)

with

$$T(\bar{\omega}L; J_i \to J_f) = \frac{8\pi(L+1)}{L[(2L+1)!!]^2} \frac{3 \times 10^{23} s^{-1}}{137} \left(\frac{E_{\gamma}}{197 \text{ MeV}}\right)^{2L+1} \frac{B(\bar{\omega}L; J_i \to J_f)}{e^2 f m^{2L}}$$
(10)

For E2 transitions we have

$$T(E2) = 0.1236568 \ x \ 10^{-2} \left(\frac{E_{\gamma}}{\text{MeV}}\right)^{-5} \frac{B(E2)}{e^2 \ f \ m^4} p s^{-1} \tag{11}$$

The total mean life for more than one final state involved (E2 and M1 radiations) is expressed in terms of mixing ratios by

$$\tau = \frac{1}{\sum_{k} \left(1 + \frac{1}{\delta_{k}^{2}}\right) T(E2; J_{i} \to J_{f})}$$
(12)

where δ_k is given by eqs. (4) and (5) with J_{f_k} as final states.

The Hamiltonian was diagonalized with the following set of parameters⁶:

a - single proton energies $e_{p_{1/2}} = 0.0$, $e_{g_{9/2}} = 0.839$ MeV, and single neutron energies $e_{d_{5/2}} = 0.0$, $e_{s_{1/2}} = 1.08$ MeV, $e_{d_{3/2}} = 2.00$ MeV and $e_{g_{7/2}} = 2.34$ MeV;

b - SDI strength $G_0 = -0.45$ MeV and $G_1 = -0.09$ MeV.

The electromagnetic properties were evaluated with the following values of effective electric charges⁷ and the usual values of effective gyromagnetic ratios:

$$e_p^{\text{eff}} = 1.8e, \ e_n^{\text{eff}} = 1.5e,$$

 $S_{1p} = 1, \ g_{1n} = 0,$
 $g_{sp}^{\text{eff}} = g_{sp}^{\text{free}} = 5.58, \text{ and}$
 $g_{sn}^{\text{eff}} = 0.73 \ g_{sn}^{\text{free}} = -2.79.$

In the calculations of electric properties we use for the radial matrix elements $\langle J_a | r^2 | j_b \rangle$ the usual estimate $\langle r^2 \rangle = \frac{3}{5}R^2$ with a nuclear radius $R = 1.20 A^{1/3}$.

5. Results and Discussion

The mean lifetimes are a result of the analysis of spectra at three bombarding energies for the following methods of analysis: Centroid shift (FTAU), the analytical approach to peak shape reproduction (SHAPE), and Doppler peak shape simulation (MONTE CARLO). Greater weight has been given to the peak-shape methods for determining the Ionger mean lives, especially the mean life of the 1205 keV state. The mean life of the 2041 keV state is near the lower limit of lifetime measurements with the recoil velocities available; this is reflected in the rather large standard deviation. The recommended values for these measurements are given in the third column of table 1. Other measured values taken from the literature are given in the fourth column of table 1. It is difficult to reconcile the discrepant value of the 1205 keV state mean life measurements of the level at 1467 keV are inconsistent. The mean lives of the other states are in general agreement with other measurements.

The results of the theoretical calculations described in Section IV are given in column 5 of table 1. The calculated results were obtained by assuming that the experimentally observed levels at 0.000 MeV, 1.205 MeV, 1.467 MeV, 1.882 MeV, 2.041 MeV and 2.131 MeV correspond, respectively, to the theoretical levels 5/21, $1/2_1^+$, $5/2_2^+$, 7/21, 3/21 and $9/2_1^+$. The values of effective proton and neutron charges were taken from Gloeckner's work⁷. The usual values of the effective spin

gyromagnetic ratio were used. No other adjustments were made. There are a few other theoretical estimates of mean lives displayed in column 6 of table 1 for comparison. The calculated mean lives of column 5 are in agreement with the present measured values for the levels at 1.882 MeV and 2.13 MeV. Our calculated lifetimes for the levels at 1.205 MeV and 1.467 MeV are, in general, larger than any of the measured lifetimes. However, most of the measurements tend toward a longer mean life than given by the presently accepted mean lives²¹. The indication for the 2.042 MeV level is of a short lifetime which is supported qualitatively by the available measurements.

The weak coupling model, although suggested by earlier studies^{9,19}, does not give as complete a description as that afforded by the shell model calculation described in Section 4. These calculations furnished excitation energies for the lower lying energy levels and, more important, values of the transition probabilities for the decay of the first five excited states. The transition probabilities are a more sensitive probe of the wave functions. The components of the wave functions of the calculated states which contributed more than 4% are listed in table 2. It shows that the inclusion of the $g_{7/2}$ and especially the $d_{3/2}$ components is justified as mentioned in qualitative terms by Chuu et al.⁶. Although the predominant components of the low lying states are due to the $d_{5/2}$ neutron component and to the $p_{1/2}$ and $g_{9/2}$ proton components there is, as also indicated earlier, a strong component of the $s_{1/2}$ orbital in at least two levels. Earlier evidence from the ⁸⁸Sr($\alpha, n\gamma$) reaction²⁰ had indicated that the weak coupling picture breaks down in the description of the states at 1.88, 2.04, and 2.13 MeV. However, as seen in table 1, Metzger's³ interpretation of the weak coupling scheme agrees, within a factor of three, with this simple characterization for the states at 1.88 and 2.13 while the states at 1.47 and 2.04 MeV differ from the interpretation given by DuBard and Sheline¹⁹.

We present, in table 3, experimental values for magnetic dipole and electric quadrupole moments of the ground state and $B(E2\uparrow)$ of the first excited state taken from the literature, and mixing ratios have been indicated for the transitions from the second, third and fourth excited states and compared with our theoretical

	TABLE	1. Results EXP	and comparis ERIMENT	ons; mean liv THI	res EORY
Ex (MeV)	Jn	present results τ(ps)	others $\tau(ps)$	present results ~(ps)	others $\tau(ps)$
			1.20 ± .5	5 [°]	3.14 ^e 9.8 ^f
1.205	1/2+	.9 ± .2	0.0 .25 ± 0.0	9 ^b 1.51 7	
·			3.2 ± 5.0	c .	1.85 ⁹
			.62± .0	9 ^d	12.9 ⁱ
			.28 ± .1	6 ^{,6} 1	95 ^h
1.467	5/2*	.46 ± .15	.51 ± .1 1.10 ± .1	6 [°] 1.22 8 ^d	4.84 ⁱ , .010 ⁱ ,
1.882	7/2+	.24 ± .10	.11 + 0.2 .21 ± 0.2	.21	. 29 ^h 1. 39 ⁱ . 005 ⁱ ,
2.042	3/2*	.10 ± .05	0.030 ^b .016± .0	. 006	4.6 ^f .003 ^f .24 ⁹
			.14 ± .0	2 ^d	. 92 ¹ . 004 ¹
2.13	9/2*	.35 ± .10	.175± .0 .16 ± .0	30 [°] 43 1 [°]	. 15 ^h . 75 ⁱ . 003 ⁱ
a Ref. 1	f Ref.	2 for E2 t	ransition; f'	idem for M1	transition
o Ref. 2	g Ref.	8 for E2 t	ransition; g'	idem for $M1$	transition
: Ref. 3	h Ref.	3 for E2 t	ransition; h'	idem for M1	transition
1 Ref. 4	i Weiss	kopf estim	ate for E2 tr	ansition; i'	idem for M1
e Ref. 6	trans	ition			

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TABLE 2. Amplitudes of wave functions calculated for states of $^{91}{\rm Zr}.$ Only those amplitudes which are larger than 4% are listed.

1/2+1		3/21	
$ (g_{g/2})^2 0, s_{1/2}^>$	0.724	(g _{g/2}) ² 0, d _{3/2} >	0.651
(p _{1/2}) ² 0, s _{1/2} >	-0.524	$ (p_{1/2})^2 0, d_{3/2}^>$	-0.583
$ (g_{9/2})^2 2, d_{5/2}^>$	-0.398	$ (g_{9/2})^2 $ 4, $d_{5/2}$	0.275
		((g _{9/2}) ² 2, d _{5/2} >	0.252
5/21		5/21	
$ (p_{1/2})^2 0, d_{5/2}^>$	0.740	(p _{1/2}) ² 0, d _{5/2} >	0.625
$ (g_{9/2})^2 0, d_{5/2}^>$	-0.650	$ (g_{g/2})^2 2, d_{5/2} >$	0.593
		((g _{9/2}) ² 2, d _{5/2} >	0.462
7/2		9/2+1	
$ (g_{g/2})^2 2, d_{5/2}^>$	0.913	$ (g_{g/2})^2 2, d_{5/2}\rangle$	0.864
$ (g_{9/2})^2 4, d_{5/2}^>$	0.303	((g _{9/2}) ² 4, d _{5/2} >	0.320
(g _{9/2}) ² 0, g _{7/2} >	-0.200	(g _{9/2}) ² 6, d _{5/2} >	0.244
		(g _{9/2}) ² 4, s _{1/2} >	-0.211

			EXPERIMENT	THEORY
Ex	Jπ	_		present resul ts
0.0	5/2 ⁺	Q(eb) µ(nm)	21 ± .02 ^a -1.303± .001 ^a	-0.17 -1.301
1.205	1/2	B(E2 [†]) (e ² fm ⁴)	90 ± 40^{b} 34 ± 50^{b} $+115^{d}$ 412_{-150} 120 ± 30^{e}	70
1.467	5/2+	$\delta\left[\frac{E2}{M1}\right]$		+0.35
1.882	7/2+	$\delta\left(\frac{E2}{M1}\right)$	$4 \pm .1^{c}$ or $-1.25 \pm .15^{c}$	-0.68
			+2.7 ^f +1.0 4	
2.042	3/2*	$\delta\left(\frac{E2}{M1}\right)$	-10<δ<0.1 ^f	+0.096
a Ref. 21		d Ref.	2	
b Ref. 1		e prese	ent work	
c Ref. 3		f Ref.	22	

TABLE 3. Electromagnetic properties of ⁹¹Zr

calculations. The shell model results are closely consistent with the known lifetimes and other electromagnetic properties of low-lying energy levels of ⁹¹Zr.

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Resumo

As vidas médias de cinco níveis nucleares em ⁹¹Zr, até uma energia da excitação de ≈ 2.5 MeV, foram medidas pelo método de atenuação do deslocamento Doppler. Os resultados experimentais foram comparados com previsões teóricas baseadas em um modelo de camada que toma o ⁸⁸Sr como caroço inerte.