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A semi-topological Lagrangian

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Abstract Local Abelian symmetry yields, through an extended gauge model, a Lagrangian whose gauge invariance is achieved by supplemeting a total derivative term by a contribution that contains only dynamical effects. From such a combination, the asymptotic behavior plays a significant role in the vector-boson self-interactions. The corresponding Chern-Simons term is derived, Bianchi identities with sources are presented. The analysis, also including the kinetic contribution, is carried out in the framework of differential forms.

Symmetry principles have appeared as the present main source to generate interactions and dynamics in physics. Historically, different proposals have been developed for the understanding of the motion of a given body. For Aristotles, a motion should be accompanied by a cause. A surprising fact in our days is that the dynamics of the interactions among physical entities may be controlled by a non-measurable real parameter $\alpha(x)$, which guides gauge invariance. The machinery of gauge theories is not switched on by physical entities such as mass, electric charge or the equivalence principle. Perhaps, it is an interesting aspect to note that, although the gauge parameter $\alpha(x)$ does not belong to the class of natural phenomena, it stipulates rules.

The gauge parameter works as a source for generating quanta. Through the local gauge invariance principle, it sets up field theories such as QED and QCD.

Nevertheless, a non-rigid interpretation of $\alpha(x)$ can also be developed. Geometric entities, like the vielbein and the spin-connection of a certain higher-dimensional manifold, may provide arguments, through a Kaluza-Klein mechanism, for extending the consequences from this arbitrary function of the space-time variables. It also contains the property of generating N distinct quanta in the same group[1]. Another support to substitute such a spontaneous generation of differences from a same symmetry can be obtained from supersymmetry. Studying the cases $N = \frac{1}{2} - D = 2$, $N = 1 - D = 3$, and a (2,0) supersymmetric model, the presence of a second potential field transforming under the same $U(1)$ - gauge group is established[2].

In this note, we intend to study some properties of an extended Abelian gauge theory[3]. Mainly, the consequences from a dual field-strenght tensor $\tilde{Z}_{\mu\nu}$ shall be discussed. For simplicity, tbe calculations will be performed in the framework of the differential forms[4]. As any tensor field, a p-form field is a C^{∞}-mapping that associates to each point of a manifold M a completely skew-symmetric covariant tensor of ranke p. It provides a coordinate-frame description that offers a synthesis to express, for example, elements as the Euler-Lagrange equations, Noether currents and energy-momentum tensor of a Lagrangian field theory. Therefore, the following operations will be useful: the exterior product A, the exterior derivative d, the hodge star, the co-differential c, and the Lie derivative[4].

The mechanism that exhibits N potential fields transforming in the same Abelian context can be controlled by different sets of fields $\{D_\mu, X^i_\mu\}$ which represent the so-called set of constructor fields. There, the quanta associated to each field is not isolated, but the gauge field transformation is isolated by acting only on the D_μ-field. However, it is more physical to work with a set of fields that correspond to each physical mass: these are referred to as the physical fields, $\{G^I\}$, whose corresponding gauge transformations assume the form

$$G^I_\mu(x) \rightarrow G^I_\mu(z) + u^{IO}\partial_\mu a(x) \qquad (1)$$

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The coefficients u^{IO} are elements of an orthogonal matrix, U, that connects the two different sets of fields:

$$\begin{pmatrix} G_1 \\ \vdots \\ G_N \end{pmatrix} = U \begin{pmatrix} D \\ X_1 \\ \vdots \\ X_{N-1} \end{pmatrix} \tag{2}$$

Here, we shall be concerned with the physical fields, $G^I_\mu(x)$, where the index I labels the N potential fields that transform under the same group. This set yields the following gauge-invariant Lagrangian

$$L = L + k + L_I + L_m + L_{\text{total derivative}} \tag{3}$$

where

$$L_K = a_{IJ}\partial_\mu G^I_\nu \partial^\mu G^{\nu J} + b_{IJ}\partial_\mu G^I_\nu \partial^\nu G^{\mu J} + \\ c_{IJ}\partial_\mu G^{\mu I}\partial^{\nu J} \tag{4}$$

$$L_I = a_{IJK}\partial_\mu G^I_\nu G^{J\mu} G^{K\nu} + b_{IJK}\ \partial^\mu\ G^{\nu J}\ G^J_\nu\ G^{\nu K} + \\ + c_{IJKL}\ G^I_\mu\ G^{\mu J}\ G^K_\nu\ G^{L\nu} \tag{5}$$

$$L_m = d_{IJ} G^I_\mu G^{\mu J}\ , \tag{6}$$

where the coefficients a_{IJ}, b_{IJ} and so on depend on more primitive coefficients related to the symmetry. The importance of these coefficients is that they provide relations for organizing the gauge invariance of the mode1[3].

Adopting the language of differential forms for investigating the properties of the Lagrangian eq.(3), the "physical" gauge potentials are taken as 1-forms given by

$$G^I = G^I_\mu\ dx^\mu \tag{7}$$

Upon use of the operators mentioned above[4], one derives:

$$dG^I = \frac{1}{2!}(\partial_\mu G^I_\nu - \partial_\nu G^I_\mu)dx^\mu \wedge dx^\nu \tag{8}$$

$$*dG^I = \frac{1}{4}\epsilon^{\mu\nu}\rho\sigma(\partial_\mu G^I_\nu - \partial_\nu G^J_\mu)dx^\rho \wedge dx^\sigma \tag{9}$$

However, in order that eq. *(3)* be described in terms of differential forms, one has to impose some constraints on the coefficients. We consider the conditions,

$$\begin{aligned} a_{IJ} &= -b_{IJ} \\ a_{IJK} &= a_{I[JK]} \end{aligned} \tag{10}$$

where *[JK]* stands for anti-symmetrization in J,K. Under eq. *(10)* it is possible to rewrite eq. *(3)* as the following 4-form:

$$\begin{aligned} L = & -a_{IJ}(dG^I) \wedge^* \ DG^J + c_{IJ}(d^*G^I) \wedge^* d^*G^J + \\ & - d_{IJ}G^I \ A* \, G^J - \frac{1}{2}a_{IJK}(dG^I) \ A* \, (G^J \ A \ G^K) + \\ & b_{IJK}(d^*G^I) \ A* \, (G^J \ A* \, G^K) \\ & + c_{IJKL}(G^I \ A* \, G^J) \ A* \, (G^K \ A* \, G^L) + \\ & + \text{total derivatives terms} . \end{aligned} \tag{11}$$

From eq. (11), some properties of the involved coefficients can be automatically read off:

$$\begin{aligned} a_{IJ} = a_{(IJ)} \quad &; \quad c_{IJ} = c_{(IJ)} \\ d_{IJ} = d_{(IJ)} \quad &; \quad b_{IJK} = b_{I(JK)} \\ c_{IJKL} = c_{(IJ)(KL)} &= c_{(KL)(IJ)} \end{aligned} \tag{12}$$

The Lagrangian eq. **(3)** generalizes the usual photon and Proca Abelian fields. Here, besides their mixing through kinetic and interaction terms, they can also be associated to other vector fields. Another feature of eq. **(3)** is the appearance of gauge-invariant mass terms, given in eq. (6). Observe that this result is perfeclty compatible with the presence of a massless photon. Indeed, one can prove that such kind of extended models organizes, at most, the presence of (N – **1)** massive fields[5] along with a massless vector that we interpret as the photon.

Geometrical objects, defined independently of any coordinate system, yield equations with no explicit tensor indices. Then, concise and elegant equations can be written down. Adopting the variational principle to derive the classical equations of motion, one gets the following formal structure:

$$\delta L \equiv [\text{Noether current}] + \delta G^I \wedge \ [\text{equations of motion}] \tag{13}$$

Thus, from the first term, one arrives at N equations of motion, each one corresponding to a physical field. They read:

$$\begin{aligned}
&a_{IJ} d^* dG^J + c_{IJ}^{*} d^* d^* G^J + d_{IJ}^{*} G^J + \\
&+ \frac{1}{4} a_{IJK} d^*(G^J \wedge G^K) + \frac{1}{2} a_{IJK}(^*dG^J) \wedge G^K + \\
&\frac{1}{2} b_{IJ\overset{*}{K}} d^*(G^J \wedge^* G^K) - b_{JKI}(^*d^*G^J) \wedge^* G^K + \\
&2\ c_{IJKL}(^*G^J) \wedge^* (G^K \wedge^* G^L) = 0\ .
\end{aligned} \tag{14}$$

Using the nilpotency of the exterior derivative operator, N conserved currents show up:

$$d^* J_I = 0$$

with

$$\begin{aligned}
J_I = -^*\{&2c_{IJ} c^* c G^J + 2d_{IJ}^{*} G^J + a_{JIK}(^*dG^J) \wedge G^K + \\
&b_{IJK} c(G^J \wedge^* G^K) - 2b_{JKI}(cG^J) \wedge^* G^K = \\
&- 4c_{IJKL}(^*G^J) \wedge^* (G^K \wedge^* G^L)\}\ ,
\end{aligned} \tag{15}$$

where c $\equiv (-1)^{n(p-1)+1}$ *d*denotes the co-differential operator (p is the degree of the form and n the space-time dimension). Although we are working with an

Abelian model not coupled to matter, eq. (15) shows the existence of N non-zero charges. They are probably originates from the local invariance determined by the inclusion of N potential fields in the same $U(1)$-group. Note that eq. (15) shows a contribution for the currents from the symmetric part of the kinetic term, from the mass terms and from the trilinear and quadriliriear self-interactions.

Information on the Noether theorem is obtained from the first term on the right-hand side of eq. (13). Considering that

$$\delta G^I = u^I d\alpha \tag{16}$$

one gets the following relations

$$\begin{aligned} &d\alpha \wedge u^I[2a_{IJ}d^* dG^J + \frac{1}{2}a_{I[JK]}d^*(G^J \wedge G^K)] + \\ &+^* d\alpha \wedge u^I[-2c_{IJ}d^* d^* G^J + b_{IJK}d^*(G^J \wedge G^K)] = 0 \ . \end{aligned} \tag{17}$$

and

$$d^* d\alpha \wedge U^I[2c_{IJ}^* d^* G^J + b_{IJK}^*(G^J \wedge^* G^K)] = 0 \ , \tag{18}$$

Observe that eq.(17) appears divided in two terms, depending on da and $^*d\alpha$, which correspond, respectively, to fhe anti-symmetric and syrnmetric sectors of the theory. Thus, such a splitting of sectors appears as a nice by-product of adopting the differential form treatment.

Reading off (17) and (18) in components, one gets the two following local pieces of information:

$$\partial_\mu \alpha(x)\{u^{Io}\partial_\nu(t_I^{[\mu\nu]})(G) + t_I^{(\mu\nu)}(G))\} = 0$$

where

$$t_I^{[\mu\nu]}(G) = 2a_{IJ}(\partial^\mu G^{\nu J} - \partial^\nu G^{\mu J}) - \frac{1}{2}a_{I[JK]}(G^{\mu J}G^{\nu K} - G^{\nu J}G^{\mu K})$$

$$t_I^{(\mu\nu)}(x) = \eta^{\mu\nu}(2c_{IJ}\partial.G^J + b_{I(JK)}G^J.G^K) \ , \tag{19}$$

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and

$$\partial_\mu \partial_\nu \alpha(x) \ \{u^{Io} \eta_{\nu\mu} t_I^{(\mu\nu)}\} = 0 \ . \tag{20}$$

Thus, from the Noether analysis, a global current does not show up. This is so because there is no matter coupled to this $U(1)$-system. Eqs. (19) and (20) are only a response to the local gauge invariance. They represent symmetry constraints on the model.

Translational invariance also yields another conservation law. In terms of forms, the canonical energy-momentum tensor is given by

$$\begin{aligned}
\tau_\epsilon = & -{}^*\{(i_\epsilon G^I) \wedge [2a_{IJ} d^* dG^J + \\
& + \frac{1}{2} a_{IJK} d^*(G^J \wedge G^K) + d_{IJ}^* G^J - c_{IJKL}^* G^J \wedge^* (G^K \wedge G^L)] + \\
& + (i_\epsilon {}^*G^I) \wedge [-2c_{IJ} dcG^J + g_{IJK} {}^*c(G^J \wedge^* G^k) + \\
& - d_{IJ} G^J - c_{IJKL} G^J \wedge^* (G^K \wedge {}^*G^L)] + \\
& - a_{IJ}(i_\epsilon dG^I) \wedge^* dG^J + c_{IJ}(i_\epsilon d^* G^I) \wedge cG^J + \\
& + a_{IJ}(dG^I) \wedge i_\epsilon {}^*dG^J + \frac{1}{2} a_{IJK}(dG^I) \wedge i_\epsilon {}^*(G^J \wedge^* G^K)\}
\end{aligned} \tag{21}$$

where $i_\epsilon p$ means the operator that defines the scalar product between the infinitesimal 1-form ϵ and an arbitrary pform w.

In the first part of this article we have observed some basic properties of a general Abelian Lagrangian, such as the one in eq. (11). Now, the most interesting part of this program is its systematization in terms of different types of gauge scalars. The introduction of N physical fields in the same $U(1)$-group also defines generalized gauge-invariant antisymmetric field-strenghts. **A** gauge invariant 2-form associated to each physical field is

$$Z_{AI} = 2\alpha_I dG_I + \frac{1}{2} \rho_{[IJ]} G_I \wedge G^J \tag{22}$$

where the index **I** labels a fixed field, while the index **J** is meant to be summed over. The parameters α_I and $\sigma_{[IJ]}$ are calculated through the gauge symmetry[3]. For instance, in the case of three potential fields, the field strength associated to the physical field G_1 is

$$Z_{A1} = 2\alpha_1 dG_1 + \frac{1}{2}\rho_{[12]}G_1 \wedge G^2 + \frac{1}{2}\rho_{[13]}G_1 \wedge G^3 \tag{23}$$

As a check, notice that structurally, eq. *(22)* contains the usual Abelian and non-Abelian cases when just one field is considered. This means that for $\sigma_{[IJ]} \to 0$ and $\sigma_{[IJ]} \to \delta_{IJ}$, QED and QCD can be respectively reproduced.

$$L = L_Q + L_{S-T} + \text{symmetric contributions} \tag{24}$$

where

$$L_Q = \sum_{I,J=1}^{N} Z_{AI} \wedge^* Z_{AJ} - d_{IJ}G^I \wedge^* G^J \tag{25}$$

and

$$L_{S-T} = \sum_{I=1}^{N}\sum_{1>I}^{N} Z_{AI} \wedge Z_{AJ} \tag{26}$$

L_Q stands for the contribution which generators propagation and interactions for the N associated quanta. Most of the physical aspects, at least in perturbation theory, are found in L_Q. However, there is another possible achievement, L_{S-T}, which is also dictated by gauge invariance. It contains the totally antisymmetric $\epsilon_{\mu\nu\rho\sigma}$ tensor.

Now, two new features can be derived from eq. *(22)*. They regard the organization of the Bianchi identities with sources and the development of an extended Chern-Simons term. As both do not depend on the symmetric sector of the model, the restrictions from eq.(10) will not be relevant.

Chern-Simons terms are directly related to a topological invariant, the Pontryagín index, that may be used to characterize the principal fiber bundle associated to the gauge group. Although our model contains a set of different potential fields, the basic structures it is built upon are the (1+3) space-time and the local $U(1)$ symmetry group. Therefore, it displays only one Chern-Simons term. In order to better discuss it, one rewrites eq. (11) as

$$L_{S-T} = \frac{1}{2}[Z_A \wedge Z_A + Z_{AI} \wedge Z_A^I] \ , \tag{27}$$

where

$$Z_A = \sum_{I=1}^{N} Z_{AI} \ . \tag{28}$$

Working out eq. (26), one gets two contributions

$$L_{S-T} = L_{\text{top}} + L_{\text{ACT.}} \ , \tag{29}$$

where L_{top} depends only upon the boundary conditions. It is an exact form

$$L_{\text{top}} = 2\alpha_I d(\alpha^I G_I \wedge dG^I + \alpha_J G^I \wedge dG^J) \ . \tag{30}$$

Thus, unlike minimal gauge theories, the Chern-Simons-like term corresponding to **(30)** consists of a polynomial involving the N fields, but which $N(N+1)/2$ terrns. As can be seen from eq. (27), the topological Lagrangian eq. **(30)** must be complemented by another term in order to achieve gauge symmetry. **Moreover,** this complement contains only interactions terms. It is given by

$$\begin{aligned} L_{\text{act.}} = & \, 2\alpha_I \rho_{[IJ]}(dG^I) \wedge G^I \wedge G^J + \\ & + 2\alpha_I \rho_{[JK]}(dG^I) \wedge G^J \wedge G^K + \\ & \frac{1}{4}\rho_{[IJ]}\rho_{[KL]} G^I \wedge G^J \wedge G^K \wedge G^L \end{aligned} \tag{31}$$

The emphasis is that, although eq. (31) does not generate quanta in the canonical way, it is sensitive to the physical reality. The symmetry reveals, through eq. (31), physical entities such as an associated Noether current, equations of motion, energy-momentum tensor and the corresponding Lorentz force. They are respectively

$$\alpha_I d[\rho_{[IJ]} \delta G^J \wedge G^I \wedge G^J + \rho_{[JK]} \delta G^I \wedge G^J \wedge G^K] = 0 \tag{32}$$

as the expression that characterizes $\tilde{J}_{\text{NOETHER}}$, and the corresponding equations of motion for this active piece

$$\alpha_I\rho_{[IJ]}d(G^I\wedge G^J)+\alpha_I\rho_{[JK]}d(G^J\wedge G^K)=$$
$$=-2\alpha_I\rho_{[IJ]}(dG^I)\wedge G^J-2\alpha_J\rho_{[IJ]}(dG^J)\wedge G^J$$
$$-2\alpha_K\rho_{[IJ]}G^J\wedge dG^K-\frac{1}{2}\rho_{[IJ]}\rho_{[KL]}G^J\wedge G^K\wedge G^L \tag{33}$$

Neglecting total derivative contributions, one gets the following energy-momentum tensor

$$\tau^{\epsilon}_{\text{ACT}}=-^*\{(i_\epsilon G^I)\wedge[\alpha_I\rho_{[IJ]}(dG^I)\wedge G^J+$$
$$+\alpha_I\rho_{[IJ]}(dG^J)\wedge G^J+\alpha_K\rho_{[IJ]}(dG^K)\wedge G^J]\} \tag{34}$$

Finally, by coupling external sources to the physical fields, one gets the following generalized Lorentz force

$$d^*\tau^{\epsilon}_{\text{act}}=\tilde{F}_{\text{LORENTZ}}\ ,$$

where

$$\tilde{F}_{\text{LORENTZ}}=-\frac{1}{2}(dG^I)\wedge i_\epsilon\ {}^*J_I$$

for

$$\tilde{L}=L_{S-T}+G^J\wedge\ {}^*J_I\quad . \tag{35}$$

The conclusion we could draw is that, physically, this semi-topological Lagrangian has two insights. A first part, S_{top}, can be put in the form of a total derivative, which is not relevant for the classical dynamics. It depends only on the asymptotic behavior of the potentials and may be related to the topological excitations. So, it may affect the excitation spectrum of the extended gauge model. In QCD, paths connecting vacua which are topologically different, but physically identical, have been active line of research[6]. Thus for a further investigation, we have to study the physical consequences from adding the term S_{top}. Similarly to QCD[7], we should infer about the value of the corresponding **θ-parameter** of this extended model. The difference here is that such proposed question is not to be answered so isolatedely as in QCD. This is because gauge invariance requires that the causality for S_{top} must be complemented with the causality of S_{act}.

The second insight regards S_{act}. It does not carry any quanta but contains self-interaction terms. Eqs. (32), (33), (34) and (35) stress consequences of its dynamics. Thus, unlike the usual case[S] where topological means the presence of observable but with no corresponding action, eq. (29) shows a semi-topological Lagrangian: a no-quanta with dynamics. Probably a non-perturbative effect is to be studied. It seems not to be abusive to compare it with the case of symmetry breakdown. In this sense a dynamics with massive vector fields is connected with a choice of a determined vacuum. Whereas here the vacuum can also be discussed as an observational fact which is suplemented by an additionai dynamics.

Our suggestion is that this semi-topological Lagrangian will have two kinds of phenomenological consequences: on the physical spectrum through the vacuum dependence on some theta parameter[9] and on testing vector-boson self interaction in vector-boson scattering at e^+e^- and $p\bar{p}$ colliders. For instance, coupling of photons with massive vector mesons as p, $\omega, J/\psi$ [10].

The second feature of the field strenght Z_{AI} concerns an identity which is not consequence from symmetry, but is just an algebraic conclusion. Taking the derivative in eq. (22) one gets the following inhomogeneous Bianchi identity:

$$dZ_{AI} = \rho_{[IJ]} d(G_I \wedge G^J) \tag{36}$$

The nature of such algebraic identities is based on the property $d^2 = 0$, dimensional analysis and gauge invariarice. Eq. (36) result satisfies all these requirements but adds up a new aspect to such identities: It is the presence of a source. In order to verify such results, one can rewrite eq.(36) as

$$\begin{aligned} dZ_{AI} &+ \frac{\rho_{[IJ]}}{2\alpha_I} G_I \wedge Z_A^J - \frac{\rho_{[IJ]}}{2\alpha_I} Z_{AI} \wedge G^J = \\ &+ \frac{1}{4}\Big(\frac{\rho_{[IJ]}\rho_{[JK]}}{\alpha_I} + \frac{\rho_{[IJ]}\rho^{[J}{}_{K]}}{\alpha_I}\Big) G_I \wedge G^J \wedge G^K \end{aligned} \tag{37}$$

Then, considering the limit case for just one field, and taking the QED and QCD values for $\rho_{[IJ]}$, the usual Abelian and non-Abelian Bianchi identities can be reproduced.

We would like to conclude this note with two conjectures. The first one is based on possible implications for this extended Abelian field strenght two-form, Z_{AI}. It could be associated to the biochemistry of the brain''. Another observation is about a justification for the appearance of terms without total derivatives in eq. (29).

The study of a gauge theory with N potential fields shows that, to each considered field, a corresponding Bianchi identity is associated. Then, by writing these equations in terms of Z_{AI}, one gets no sourceless identities. Our conjecture here is not to associate such a non-zero divergence to monopoles. Our effort intends to is to stimulate the idea that the corresponding electric and magnetic extended Abelian fields associated to Z_{AI} are useful for studying the electromagnetic properties that influence brain mechanisms. Briefly, we assume that the mechanism of the memory would be based on a dynamics that requires a self-feeding mechanism. So, in mathematical terms, the brain waves should be derived from self-interacting terms and with a weak correlation to external-matter sources. These two features would mean, respectively that memory would generate memory and that it does not depend strongly on matter cells. What is the simplest gauge theory which develops such structures?

One expects a field-theoretic model for the brain biochemistry based on electromagnetic reactions, but with non-linear coupling. In this model, the information would be guaranteed through self-interaction, and so, be kept preserved from assistance of external sources (J_ψ and J_ϕ). An Yang-Mills field theory fulfills such a requirement. However, it does not have the same nature as QED. Thus, eq. (35) represents the simplest case of equations that refeed themselves. Our suggestions is that mechanisms containing electric and magnetic fields with self interacting termo should be adequate for organizing the electrodynamics of the mernory in the neurons.

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Another consequence of this possibility based upon more potential fields is that the contribution $Z_A \wedge Z_A$ appears with terms which are not a total derivative. This result, perhaps, can have support in the fact that the presence of more potential fields introduce mass terms in the theory. The total derivative term in $Z_A\, h\, Z_A$ is originated from a massless gauge field, with long range oscillations, that correspond to the surface effects. Thus, the second conjecture, is that the term $L_{act.}$ stems from massive fields - short range oscillations, which do not have topological consequences on the surface but influences the theory perturvatively when L_Q is considered.

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Resumo

Através de um modelo de gauge estendido para uma simetria abeliana local, obtém-se uma Lagrangeana invariante de gauge constituída por uma parte contendo apenas derivada total mais outra que contém apenas efeitos dinâmicos. Para tal combinação, o comportamento assintótico ganha significado para compreender-se as auto-interações dos bosons vetoriais. O correspondente termo de Chern-Simons é derivado, e identidades de Bianchi com sortes são apresentadas. Para a análise, que também inclui a contribuição cinética, usa-se como sistema de trabalho as formas diferenciais.