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Quantum gravity, classical geometry: a coherent treatment

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Abstract After the work of many physicists - synthesized in a recent paper by Grishchuk, Petrov and Popova[1] - there is no more doubt that Einstein's exact General Relativity admits a complete formal description in terms of a field theory in an (auxiliary) Minkowski background manifold. We explore here this property in order to propose a model in which gravity is to be quantized, although the observable metrical properties of space-time remain a classical structure. Thus, quantum fluctuations of the gravitational field can produce microscopic excitations without recurring to the metrical concept. In the macroscopic world - that is, in the observed domain of General Relativity. e.g. $\ell >> \ell_{\text{Planck}}$ - only the geometrical quantities constructed from the classical (non-quantum) metric $g^{\mu\nu}$ produce observable gravitational effects.

1. Although there is not a single evidence that the gravitational field should have a quantum version, there is a general belief that in order to allow a future unified treatment of all physical interactions, the fields of physics should be quantized

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Einstein's geometrical approach to the description of gravitational forces makes such quantization a very hard job. One is led to suppose the existence of many exotic situations like, for instance, that geometry fluctuates (where ?); that for dimensions comparable to Planck's length ($L_{Pl} \sim 10^{-33} cm$) the metric undergoes "quantum instabilities"; that during the elementary Planck time ($\Delta t \sim 10^{-43} sec$) one cannot define but average values for the geometry in a hypothetical super space beyond the ordinary arena of physical events; that the causal structure of the world should be dramatically modified once the null cone fluctuates and the distinction between past and future "might become blurried"; and so on.

A series of alternative schemes for this quantization process have been presented. Nevertheless, each one has serious drawbacks, which is precisely the reason for not having obtained a general acceptance.

We think that the main reason for this situation is due to the very fundamental principle which selects the geometry of space-time as the true variable to describe gravity. If we deal with Einstein's geometric variables, there seems to be no way out for such a difficulty - that is a quantum version is to be associated to unobservable "geometric fluctuations."

We are thus led to argue that one should try another set of variables to describe gravity. This set should be somehow conciliated to Einstein's geometric scheme since General Relativity is, for the time being, the best theory to describe gravitational processes.

This dilemma can be circumvented if we make use of a flat space-time field description of Einstein's theory as it has been presented recently by Grishchuk and co-workers. Although the idea to describe General Relativity in terms of an equivalent field theory in flat space, time is not a new one, it seems to us that Grishchuk et al gave a very convincing and extremely simple model to deal with the complete, exact Einstein's theory of gravity in flat space-time.

This new characterization opens a natural way to a quantization procedure in the standard scheme of field theory which could provide the solution of some of the main difficulties associated up to now to Einstein's geometrical view.

Let us thus present here the initial steps of such a program that **can** conduct us to the quantum gravitational road.

2. The Grishchuk-Petrov-Popova (GPP) scheme

Let $y_{(X)}^{muv}$ be the metric of the flat Minkowski space-time, written in an **arbi-**trary system of coordinate. The associated Christoffel symbol $\gamma^{\alpha}_{\mu\nu}$ **is** defined in the usual way

$$\gamma^{\alpha}_{\mu\nu} = \frac{1}{2}(\gamma_{\lambda\mu,\nu} + \gamma_{\lambda\nu,\mu} - \gamma_{\mu\nu,\lambda}) \ . \tag{1}$$

The curvature tensor associatèd to such a connection vanishes

$$R^{\alpha}_{\beta\mu\nu}[\gamma^{\rho\sigma}] = 0$$

Let $\varphi^{\mu\nu}$ be the gravitational field, defined on this flat space, the dynamics of which is given by the Lagrangian

$$\mathcal{L}_{(g)} = -\frac{1}{2K_E}\sqrt{-\gamma}(\gamma^{\mu\nu} + \varphi^{\mu\nu})[K^{\alpha}_{\mu\nu;\alpha} - K_{\mu;\nu} + (KK)_{\mu\nu}] \tag{2}$$

in which

$$(KK)_{\mu\nu} \equiv K_{\alpha}K^{\alpha}{}_{\mu\nu} - K^{\alpha}{}_{\mu\beta}K^{\beta}{}_{\nu\alpha}$$

$$K_{\alpha} \equiv K^{\epsilon}{}_{\alpha\epsilon}$$

and $K^{\alpha}{}_{\mu\nu}$ is a functional of $\varphi^{\mu\nu}$ containing up to first order derivativa of $\varphi^{\mu\nu}$ and K_E is **Einstein's** constant. The symbol ; stands for the covariant derivative in the flat space that is, for **instance,**

$$K_{\mu;\nu} = K_{\mu,\nu} - \gamma^{\epsilon}{}_{\mu\nu}K_{\epsilon}$$

We consider independent variations of $\varphi^{\mu\nu}$ and $K^{\alpha}{}_{\mu\nu}$ thus obtaining **correspond-**ingly the dynamics of $\varphi^{\mu\nu}$ and the functional **dependence** of $K^{\alpha}{}_{\mu\nu}$ in **terms** $\varphi^{\mu\nu}$.

The next step is to define an associated geometrical tensor $g^{\mu\nu}$ through the definition

$$\sqrt{-g}g^{\mu\nu} = \sqrt{-\gamma}(\gamma^{\mu\nu} + \varphi^{\mu\nu}) \tag{3}$$

in which $g \equiv \det\ g_{\mu\nu}$ and $7 = \det \gamma_{\mu\nu}$.

The associated Christoffel symbol $\Gamma^{\alpha}{}_{\mu\nu}$ induced by the new metric $g^{\mu\nu}$ can be separated in a very convenient way under the form

$$\Gamma^{\alpha}_{\mu\nu} = \gamma^{\alpha}_{\mu\nu} + K^{\alpha}_{\mu\nu} \tag{4}$$

Although all tensorial indices are to be lowered and raised by means of the metric of the flat space-time $\gamma^{\mu\nu}$, the inverse of $g^{\mu\nu}$ that is $g_{\mu\nu}$ r $(g_{\mu\nu})^{-1}$, is defined by

$$g_{\mu\nu}\ g^{\nu\lambda} = \delta^{\lambda}{}_{\mu}$$

(note that $g_{\mu\nu}$ is **not** given by $\gamma_{\mu\alpha}\ \gamma_{\nu\lambda}\ g^{\alpha\lambda}$).

The associated contracted riemannian curvature tensor can be written

$$R_{\mu\nu}(g) = K_{\mu;\nu} - K^{\alpha}{}_{\mu\nu,\alpha} - (KK)_{\mu\nu} \tag{5}$$

We can thus re-write the gravitational Lagrangian L_g given by eq. (2) in terms of the associated metric variables (up to an unimportant divergence term)

$$L_{(g)} = \frac{-1}{2K_E}\sqrt{-g}R_{\mu\nu}\ g^{\mu\nu} \tag{6}$$

which yields precisely Einstein's equation of motion. Then the description of the gravitational field in terms of a modification of the geometry of space-time (variable $g^{\mu\nu}$) or as a field $(\varphi^{\mu\nu})$ in the usual Minkowski space-time becomes a matter of choice. The generalizatiori to the case in which we consider the sources of gravity is straightforward: we have only to substitute in the matter Lagrangian the auxiliary metric $\gamma^{\mu\nu}$ and its corresponding connection $\gamma^{\alpha}{}_{\mu\nu}$ by $g^{\mu\nu}$ and $\Gamma^{\alpha}{}_{\mu\nu}$ given by eqs. (3) and (4). This is nothing but a consequece of the universal coupling of gravity to all existing matter (see GPP for more details).

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3. Geometry, like temperature, is a macroscopic concept. One can generalize both to deal with microscopic quantities - but in a very artificial way

For instance, one can consider like de Broglie, that it makes sense to examine the thermodynamical properties of an isolated particle. However, this should be made after the introduction of an extrinsic unobservable thermostate. In the micro-world one could also deal with geometric quantities but one should face analogous conceptual difficulties.

There is a simple way to deal with such a situation: to accept that classical gravity should have a quantum version although leaving the geometry as a classical macroscopic quantity.

From what we have said previously this can be achieved by the correspondence between the (quantum) gravitational field $\varphi^{\mu\nu}$ to the (c-number) geometry $g^{\mu\nu}$ through the modified formula

$$\sqrt{-g}g^{\mu\nu} = \sqrt{-\gamma}(\gamma^{\mu\nu} + <\varphi^{\mu\nu}>) \tag{7}$$

in which $<\varphi^{\mu\nu}>$ is nothing but the expectation value of the gravitational field in a given (quantum) state.

This simple formula has far-reaching consequences. It implies, for instance, that at the quantum level the universality of the gravitational field is broken.

Indeed, this can be seen by an examination of GPP's proof of the classical equivalence quoted above in section 2 (Cf. eq.(3)).

This could be thought of as an heresy. However, it does not conflict with any actual observation. Besides, if we are led by the guidance of the behavior of the electromagnetic field in these quantum regions (say, at the microscopic level), then new short range gravitational effects may appear.

We know, for instance, that in the leptonic world the short range counterpart of electromagnetic forces is represented by weak processes (the ancient Fermi interaction, responsible for the radioactive decay). This property admits a unified treatment of the Electro-Weak forces, the so-called $SU(2)_L \times U(1)$ gauge theory.

However, electrons and neutrinos interact not only with the photon and thevector bosons that mediate weak processes, but also with gravity. The description of gravity into the $SU(2)$ x $U(1)$ gauge structure of the leptons induces the existence of a new short range force mediated by massive spin-two particles[2].

As has been done in the electro-weak case, this new force can be interpreted as the short range counterpart of the (long range) gravitational field.

If this structure does indeed exist and gravity does not break the $SU(2)\times U(1)$ symmetry of the leptonic world, it follows naturally that universality of gravity is broken at this level.

How can we prove this?

If the electron coupling to gravity is given by Einstein's constant $K_E = 1$ in natural geometric units then, recent observations from the Supernova 1987 tell us that the coupling of the neutrino to gravity must be given by $\xi = K_E(1+\epsilon) = 1+\epsilon$ with $\epsilon \leq 10^{-3}$. Even this very small value of a possible violation of the distinction of gravity coupling in the leptonic world is enough to prove the existence of the new short range gravitation-like force.

Indeed, the leptons (say, the electron and its neutrino) are described in terms of an $SU(2)$ doublet L $= \frac{1-\gamma_5}{2}\begin{pmatrix}\nu\\ \epsilon\end{pmatrix}$ and a singlet R $= \frac{1+\gamma_5}{2}\epsilon$ in which v represents the neutrino field and ϵ the electron field. A simple algebraic manipulation shows that the tensorial leptonic current that couples to gravity is to be written as

$$T_{\mu\nu}(\epsilon) + \xi T_{\mu\nu}(\nu) = \frac{\xi+1}{2}\bar{L}\gamma_{(\mu}D_{\nu)}L + \bar{R}\gamma_{(\mu}D_{\nu)}R + \frac{\xi-1}{2}\bar{L}\gamma_{(\mu}D_{\nu)}\tau_3\ L + h.c. \tag{8}$$

Thus, unless there is a very extreme fine tuning Ç $\neq 1$; then, besides the identity of the $SU(2)$ algebra $[\bar{L}\gamma_{(\mu}D_{\nu)}L]$ there appear also a τ_3 component. In order to close the algebra (in case gravity does not spoil the $SU(2)\times U(1)$ symmetry of the leptonic world) we must deal with charged tensorial currents, e.g. $\bar{L}\gamma_{(\mu}D_{\nu)}\tau^{\pm}$L. These currents cannot couple directly to gravity, but only to charged (massive) spin-two fields, which then become the local counterpart of gravity, in analogy to the case of the electro-weak interaction.

This leads naturally to the idea that, at least under the condition stated above for the leptonic world, gravity is not **universally** coupled to **all** matter, at the microscopic level. This in turn **is** a strong support to our program to treat geometry only in the classical domain, even if gravity becomes quantized through **eq.(7).**

Moreover it **seems** almost a necessary requirement, if the conditions in the **leptonic** world described above are to be **fulfilled. In** any case, this scheme makes the question of quantization of gravity to **become** again to be decided by observation and not by theoretical prejudgements.

References

1. L.P. Grishchuk, A.N. Petrov and A.D. Popova, Comm. Math. Phys. 94, 379 (1984).
2. M. Novello, E. Elbaz, The Aussois paper (1989), to be published.

Resumo

Após o trabalho de muitos pesquisadores - sintetizado em artigo recente de Grishchuk, Petrov e Popova[1] - não há mais dúvidas de que a Relatividade Geral exata de Einstein admite uma descrição formal completa em têrmos de uma teoria de campos em uma variedade Minkovskiana (auxiliar) de fundo. Aqui, exploramos esta propriedade para propor um modelo no qual a gravidade deve ser quantizada, apesar de as propriedades métricas observáveis do espaço-tempo permanecerem uma estrutura clássica. Assim, Autuações **quânticas** do campo gravitacional podem produzir excitações microscópicas sem recurso ao conceito métrico. No mundo macroscópico - ou seja, no domínio observado da Relatividade Geral p.ex. $\ell >> \ell_{Planck}$ - apenas as quantidades geométricas construídas a partir da **métrica** clássica (não quântica) $g^{\mu\nu}$ produzem efeitos gravitacionais observáveis.