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Structure of ^{88}Kr nuclei within the cluster phonon coupling model

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Abstract The description of the low energy spectra of ^{88}Kr is performed in terms of the model in which two neutrons are coupled to the quadrupole vibrational field. The residual interaction among the valence neutrons is approximated by the pairing force. Excitation energies and $B(E2)$ values are calculated and compared with the corresponding experimental data. Our results are also compared with those obtained using the interacting boson approximation *(IBA-2* MODEL).

1. Introduction

The level schemes of even-even krypton nuclei with N $<$ 50 have been investigated earlier and calculations have been performed[1-7]. In the case of ^{84}Kr nuclei the experimental information[4-6] was compared with the model calculation[7] in which two neutron holes are coupled to the vibration field. Based on the satisfactory results of this calculations it is therefore of great interest to extend this work to see how well this model can predict collective and particle excitations above the $N = 50$ neutron shell.

Recently[8] a detailed investigation of the level structure of ^{88}Kr was performed by measurements on chemically separated bromine samples. A total of 146 γ-rays have been assigned to the decay of ^{88}Br and a level scheme with 58 levels was proposed. The collectivity of ^{88}Kr has been investigated within the framework of the interacting boson approximation **(IBA-2** MODEL).

In view of the above mentioned situation we propose in the present work a theoretical interpretation of the structure of ^{88}Kr nucleus within the two-neutron cluster phonon model.

2. The cluster vibrator model and parameters

Since a detailed description of the model can be found in the literature[9,10], only the main formulas are presented here. The model hamiltonian is

$$H = H_0 + H_{\mathrm{res}} + H_{\mathrm{int}} \tag{1}$$

where H_0 represents the energy of the unperturbed system consisting of quadrupole vibrational field and valence neutrons in a central field. The residual interaction energy among the neutrons in the shell model cluster, H_{res}, only includes the pairing forces explicity. The particle vibration interaction is given by the expression

$$H_{\mathrm{int}} = -\frac{\beta_2}{\sqrt{5}} \sum_{\mu=-2}^{2} \left[b_2^{+\mu} + (-)^{\mu} b_2^{-\mu}\right] \sum_{i=1}^{2} K(r_i) Y^{*}_{2\mu}(\theta_i, \phi_i) \tag{2}$$

where $K(r_i)$ is the interaction intensity and β_2 is the quadrupole deformation parameter.

The coupling constant of the matrix element of H_{int} is defined as

$$a = \frac{< K > \beta_2}{\sqrt{20\pi}} \tag{3}$$

where (K) is the mean value of the radial matrix element of the interaction.

The hamiltonian is diagonalized in the basis $|\{[j_1 j_2], \mathrm{J}, NR\} I >$, where j = (nlj) stands for the quantum numbers of the particle state, **J** is the total angular momentum of the two neutrons, N and R represent the phonon number and the angular momentum, respectively, and **I** is the total angular momentum.

The electric quadrupole operators consist of a particle and a collective part

$$M(E2,\mu) = e_p^{\text{eff}} \sum_{i=1}^{2} r_i^2 Y_{2\mu}(\theta_i,\phi_i) + \\ + \frac{3R_0^2}{4\pi} e_v^{\text{eff}} [b_2^{\mu+} + (-)^\mu b_2^{-\mu}] \tag{4}$$

where e_p^{eff} is the effective particle charge and $e_v^{\text{eff}} = Ze\beta_2/\sqrt{5}$ is the effective vibrator charge.

The reduced $B(E2)$ value is given by

$$B(E2, I_i \to I_f) = \frac{| < I_f \| M(E2) \| I_i > |^2}{(2I_i + 1)} \tag{5}$$

The reduced matrix elements of the E2 operator are expressed in the form

$$< I_f \| M(E2) \| I_i > = (e_p^{\text{eff}} A + e_v^{\text{eff}} B) eb \tag{6}$$

and the quantities A, B are calculated from the model wave functions.,

The hamiltonian was diagonalized with the following set of parameters:

(a) Single particle energies $\epsilon d_{5/2} = 0$, $\epsilon s_{1/2} = 1.08$ MeV, $\epsilon d_{3/2} = 2.00$ MeV and $\epsilon g_{7/2} = 2.34$ MeV, were taken from the work of Chuu et al.[11];

(b) pairing strength G = 0.26 MeV, which follows from the estimate of Kisslinger and Sorensen[12] (G = $23/A$ MeV);

(c) phonon energy $\hbar w_2 = 1.56$ MeV, is the experimental energy of the 2^+ state in the single-closed-shell ^{86}Kr nucleus[13];

(d) particle-vibration coupling constant a = 0.74 MeV, which results from $\beta_2 = 0.13$ (as measured in the Coulomb excitation process on ^{86}Kr (ref. 13) and $< K > = 45$ MeV (as estimated numerically using wave functions obtained from the Woods-Saxon potential[14]).

In this parametrization, without any adjustable parameter, we diagonalize the Hamiltonian by including all vibrational states up to two phonons.

The electric properties were evaluated with the usual values of the effective electric charge

$$e_p^{\text{eff}} = 0.5e, \quad e_v^{\text{eff}} = 2.09e ,$$

3. Results and Discussion

In order to test the parametrization quoted in the previous section we first briefly discuss the available experimental data for the N = 51 nuclei[6]. The energy spectra are compared in fig. 1. It should be noted that the agreement between the calculated and the measured energy spectra for ^{87}Kr nuclei can be improved by lowering the particle-phonon coupling constant. As an example, in fig. 1 is also exhibited the calculated spectrum for a = 0.50 MeV

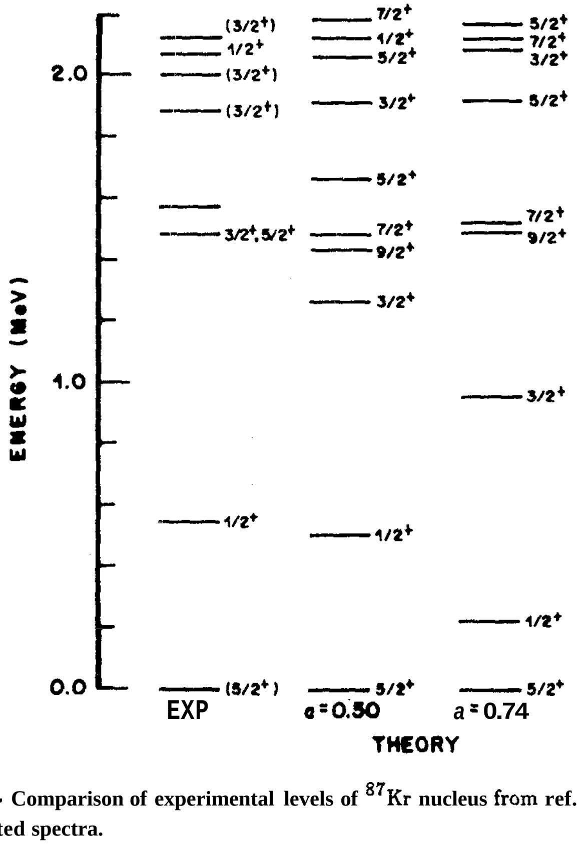

Fig. 1 - Comparison of experimental levels of ^{87}Kr nucleus from ref.6 with the calculated spectra.

The calculated energy levels of ^{88}Kr are shown in fig. 2 and compared with experiment and theoretical calculations of ref. 8. There is a satisfactory agreement between our calculation with a = *0.74* MeV and the experimental results. In particular the occurrence of a vibrational-like doublet of 2^+ and 4^+ states at about twice the energy of the first 2^+ is reproduced by the calculations, although the order of these levels is invertsd. The levels experimentally known at an excitation energy of *2.6* MeV are well described and we reinforce the spin and parity assigment $I = 3^+$ to the level at *2.342* MeV and indicate $I = 1^+$ to the level at *2.420* MeV. For completeness we also present in fig. *2* the calculated spectrum for a = *0.50* MeV. It is worth noting that by lowering the particle-phonon coupling constant the calculated energy spectrum is very weakly influence.

The components of the wave functions of the 0_1^+, 0_2^+, 0_3^+, 2_1^+, 2_2^+, *23*, 4_1^+, 1_1^+, 3_1^+ levels in ^{88}Kr which contributed more than *4%* are listed in table 1. It appears that all states have mixed characteristics with two-particle and two-particle-one-phonon configurations, only the 1_1^+ and 3_1^+ also have mixed two-particle-two phonon configurations.

Experimental information on the $B(E2)$ values are displayed in table 2. We also show our theoretical (for a = 0.74 MeV) results and those obtained with IBA-2 in ref.8. By inspecting the experimental and theoretical results one sees that the magnitudes of the measured $B(E2)$ values are reasonably well reproduced by our calculations except for the $0_3^+ \to 2_1^+$ transition. The values calculated with IBA-2 disagree for the transitions $0_2^+ \to 2_1^+$, $0_3^+ \to 2_2^+$, $0_3^+ \to 2_1^+$ and the value obtained for the $2_2^+ \to 2_1^+$ are the upper limit.

4. Conclusions

We have demonstrated that the low-lying level properties of the ^{88}Kr nucleus arise from a two-neutron cluster with the quadrupole vibrations fields. Within this picture the experimental energy spectrum and the $B(E2)$ values for $0_2^+ \to 2_2^+$, $0_2^+ \to 2_1^+$, $0_3^+ \to 2_3^+$, $0_3^+ \to 2_2^+$, $2_2^+ \to 0_1^+$, $2_2^+ \to 2_1^+$ and $4_1^+ \to 2_1^+$ are well reproduced without any parameter adjustment.

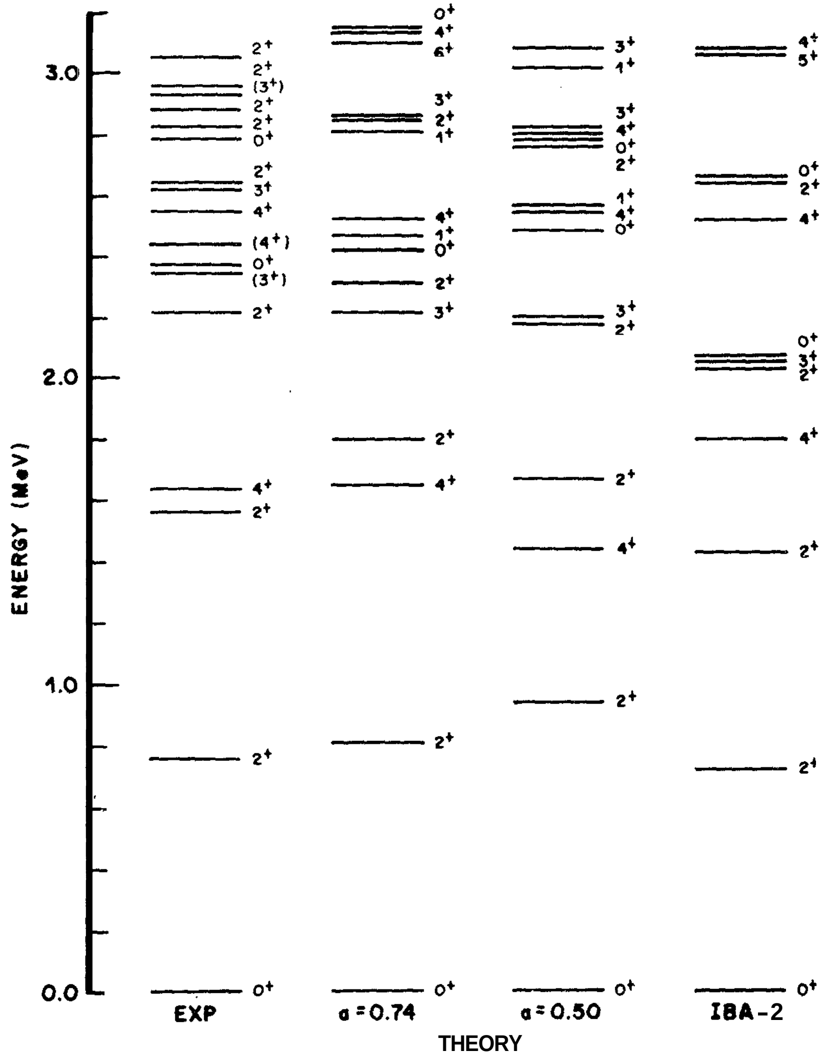

Fig. 2 - Comparison of experimental levels (EXP)[8] with our calculation (THEORY) and using the IBA-2 model[8] for the ^{88}Kr spectra.

Table 1 - Wave functions of a few low-lying states in ^{88}Kr. Only amplitudes larger than 4% are listed.

0_1^+		0_2^+		0_3^+	
$\vert(d_{5/2})^2 0,00 >$	.698	$\vert(s_{1/2})^2 0,00 >$	.485	$\vert(d_{3/2})^2 0,00 >$	.407
$\vert(d_{5/2})^2 2,12 >$	-.362	$\vert(d_{5/2},s_{1/2})2,12 >$	.397	$\vert(d_{5/2},s_{1/2}) >$	-.362
$\vert(d_{5/2},s_{1/2})2,12 >$	.345	$\vert(d_{5/2})^2 2,12 >$	.396	$\vert(d_{5/2})^2,2,12 >$	.344
$\vert(s_{1/2})^2 0,00 >$	.275	$\vert(d_{1/2})^2 0,00 >$	$-.388$	$\vert(g_{7/2})^2 0,00 >$	.309
		$\vert(s_{1/2},d_{3/2})2,12 >$	.324	$\vert(d_{3/2},g_{7/2}) >$	.267
		$\vert(d_{5/2})^2 2,22 >$	.238	$\vert(d_{5/2})^2 0,00 >$	.256
				$\vert(d_{5/2})^2 0,20 >$	.241

2_1^+		2_2^+		2_3^+	
$\vert(d_{5/2})^2 0,12 >$	.522	$\vert(d_{5/2})^2 2,00 >$	.556	$\vert(d_{5/2})^2 0,12 >$	.475
$\vert(d_{5/2})^2 2,00 >$	$-.422$	$\vert(d_{5/2})^2 2,12 >$	.475	$\vert(d_{5/2})^2 4,12 >$	.369
$\vert(d_{5/2}s_{1/2})2,00 >$	.346	$\vert(d_{5/2}s_{1/2})2,00 >$	.352	$\vert(d_{5/2}s_{1/2})2,00 >$	.294
$\vert(d_{5/2}s_{1/2})2,12 >$	.259	$\vert(d_{5/2}s_{1/2})3,12 >$	.226	$\vert(d_{5/2})^2 2,12 >$	.244
$\vert(d_{5/2})^2 4,12 >$	.245	$\vert(d_{5/2}d_{3/2})2,00 >$	.224	$\vert(d_{5/2}d_{3/2})2,00 >$	.243
$\vert(s_{1/2})^2 0,12 >$	.230			$\vert(d_{5/2})^2 2,00 >$	.223
				$\vert(d_{5/2}s_{1/2})2,12 >$	.211

4_1^+		1_1^+		3_1^+	
$\vert(d_{5/2})^2 4,00 >$	.688	$\vert(d_{5/2}d_{3/2}1,00 >$	.587	$\vert(d_{5/2}s_{1/2}3,00 >$	.659
$\vert(d_{5/2})^2 2,12 >$	$-.343$	$\vert(d_{5/2})^2 2,12 >$	.361	$\vert(d_{5/2})^2 4,12 >$	.472
$\vert(d_{5/2}s_{1/2})2,12 >$	.296	$\vert(d_{5/2}d_{3/2})2,12 >$	$-.329$	$\vert(d_{5/2})^2 2,12 >$	.273
$\vert(d_{5/2})^2 4,12 >$	$-.244$	$\vert(d_{5/2}s_{1/2})3,12 >$	$-.263$	$\vert(d_{5/2}s_{1/2})3,12 >$	$-.230$
		$\vert(d_{5/2})^2 4,24 >$	$-.251$		
		$\vert(d_{5/2},g_{7/2})2,12 >$	.214		
		$\vert(d_{5/2}d_{3/2}3,12 >$	$-.202$		

Table 2 - Comparison of relative experimental transition rates to $B(E2)$ values (EXP)[8] with our calculation (THEORY) and IBA-2 model[8] for ^{88}Kr.

Transition	Exp.	Theory**	IBA-2
$0_2^+ \rightarrow 2_3^+$	-	0.14	0.10
$0_2^+ \rightarrow 2_2^+$	1.0	1.2	1.0
$0_2^+ \rightarrow 2_1^+$	0.05	0.04	0.53
$0_3^+ \rightarrow 2_3^+$	1.0	0.7	1.0
$0_3^+ \rightarrow 2_2^+$	0.078	0.027	0.13
$0_3^+ \rightarrow 2_1^+$	0.0033	0.27	0.013
$2_2^+ \rightarrow 0_1^+$	0.009	0.004	0.013
$2_2^+ \rightarrow 2_1^+$	< 1.0*	0.5	1.0
$4_1^+ \rightarrow 2_1^+$	1.0	1.0	1.0
$4_1^+ \rightarrow 2_2^+$	-	0.015	0.076
$2_1^+ \rightarrow 0_1^+$	-	1.1	-

(*) - upper limit based on a pure E2 transition.

(**) - for absolute values in (Wu) unit to multiply by **7.916.**

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Resumo

A descrição do espectro de baixa energia do ^{88}Kr e efetuada através de um modelo no qual dois neutrons são acoplados a um campo vibracional quadrupolar. A inferação residual entre os neutrons de valéncia é aproximada pela força de emparelhamento. Energias de excitação, spin, paridade e probabilidades de transição $B(E2)$ são calculadas e comparadas com os dados experimentais correspondentes. Nossos resultados são também comparados com os valores obtidos usando a aproximação de bosons interagentes (IBA-2).