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The ρ ω mass difference in a relativistic potential model with pion corrections

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Abstract The problem of the $\rho-\omega$ mass difference is studied in the framework of the relativistic, harmonic, S+V independent quark model implemented by center-of-mass, one-gluon exchange and pion-cloud corrections stemming from the requirement of chiral symmetry in the (u, $d)SU(2)$ flavour sector of the model. The pionic self-energy corrections with different intermediate energy states are instrumental to the analysis of the problem, which requires an appropriate parametrization of the mesonic sector different from that previously used to calculate the mass spectrum of the S-wave baryons. The correct $\rho-\omega$ mass splitting is found, together with a satisfactory value for the mass of the pion, calculated as a bound-state of a quark-antiquark pair. An analogous discussion based on the cloudy-bag model is briefly presented.

This work is devoted to a discussion of the problem of the $\rho-\omega$ mass difference in the framework of the chiral version[1] of a relativistic, harmonic, $S+V$ independent quark model[2], implemented by center-of-mass and one-gluon exchange corrections[3].

We note that in the $S+V$ chiral model, the quark axial current is not conserved due to the presence of the Lorentz scalar (S) component of the confining potential. In order to restore chiral symmetry in the massless (u, $d)$ $SU(2)$ flavour sector of the model, an elementary Goldstone-like pion field is introduced, interacting with the quarks of the valence "core" of the hadron in a linear way. As a consequence,

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pion exchange and self-energy effects give rise to additional contributions to the physical mass spectrum of the hadrons - the so-called pion-cloud corrections.

It is well-known that *non-chiral* models lead to $M_\rho = M_\omega$. On the other hand, early attempts to improve this result by using chiral models led to the wrong result $M_\rho > M_\omega$ [4]. However, as will be shown, by treating the pionic self-energy corrections in a sufficiently detailed way, namely, with the inclusion of the available self-energy diagrams with one Goldstone pion, it is possible to obtain the $\rho-\omega$ mass difference, both in sign and magnitude, provided appropriate corrections due to the non-degenerate intermediate states are introduced. We note that our approach parallels that of Myhrer, Brown and Xu[5], although the present method for calculating the pionic mass shifts is much improved by taking into account the model form-factors in a more satisfactory way.

Firstly, we briefly recall that in the framework of the $S+V$ harmonic potential model, each constituent quark in a hadron obeys the Dirac equation

$$\left[\vec{\alpha}\cdot\vec{p}+\beta m_i+\frac{1}{2}(1+\beta)V(r)\right]\Psi_i(\vec{r})=E_i\Psi_i(\vec{r}) ,$$

where i is a quark (flavour) index and

$$V(r)=V_0+\frac{1}{2}Kr^2 .$$

The S-wave solution $\psi_i(r)$ is given by[3]

$$\Psi_i(\vec{r})=N_i\left[\frac{1}{\pi R_i^2}\right]^{3/4}\begin{pmatrix}\exp(-r^2/2R_i^2)\chi\\ \frac{1}{X_i}\vec{\sigma}\cdot\vec{p}\exp(-r^2/2R_i^2)\chi\end{pmatrix} \tag{1}$$

where χ is a Pauli spinor and

$$R_i=\left[\frac{2}{X_iK}\right]^{1/4} , \quad X_i=E_i+m_i . \tag{1'}$$

The term $\frac{1}{2}(1+\beta)V(r)$ in the Dirac equation represents, in a phenomenological way, the non-perturbative multi-gluon interactions which give rise to quark confinement. The residual interactions arising out of one-gluon and one-pion exchange are treated perturbatively in lowest order. As **shown** in ref. 3, the hadron masses M can be written in the form

$$M = \left[(E_0 + E_1)^2 - <\vec{P}^2>\right]^{1/2} + \Delta E_\pi \qquad (2)$$

where $E_0 = \sum_i E_i$. The term $E_1 = \Delta E_M + \Delta E_E$ is the sum of the magnetic and electric **parts** of the one-gluon exchange contribution **whereas** the term $<\vec{P}^2>$ in eq. (2) is the **center-of-mass** energy correction, given by

$$<\vec{P}^2> = \sum_i \frac{1}{R_i^2}\left(\frac{5}{2} - N_i^2\right)$$

(See ref. 3 also for the explicit relations of the one-gluon exchange corrections in **eq.(2))**. For the **present** purposes, we **explicitly need** the "pion-cloud" self-energy of the hadron AE,, given by[1]

$$\Delta E_\pi = -\frac{1}{3} f_{NN\pi}^2 \frac{9}{25} C(H) I_\pi \qquad (3)$$

where

$$C(H) = < H| \sum_{i,j} \vec{\sigma}_i \cdot \vec{\sigma}_j \; \vec{\tau}_i \cdot \vec{\tau}_j |H > \qquad (4)$$

and

$$I_\pi = \frac{1}{\pi m_\pi^2} \int_0^\infty \frac{k^4}{w_\pi^2} u^2(k) dk \quad , \quad w_\pi^2 = k^2 + m_\pi^2 \quad .$$

In eq. (7) k is the pion momentum and $u(k)$ **is** the form-factor characteristic of the harmonic S + V model, given by

$$u(k) = (1 - A R_0^2 k^2)\exp(-k^2 R_0^2/4) \quad , \quad A = \frac{E_0 - m_0}{2(5E_0 + 7m_0)} \qquad (5)$$

where the subscript zero indicates ordinary (u or d) quarks.

We note that **eq.(3)** is valid in the **PCAC** limit, with an **elementary** pion field of small but finite mass (m, = 134 **MeV)**. Furthermore, the expression for the pion correction ΔE_π, eq. (3), is also valid is the cloudy bag model **(CBM),** provided we take for the form-factor the well-known **expression**[6]

$$u(k) = \frac{3j_1(kR_B)}{kR_B} \tag{6}$$

where R_B is the bag radius.

When calculating the pionic self-energy corrections to the mass of a given hadron H, by means of eq.(3), the possibility of contributions from hadronic intermediate states H' belonging to the same multiplet as H must be taken into account. These intermediate energy states contributiona to the hadron self-energy can be obtained by using, instead of the spin-isospin matrix element $C(H)$ in eq. (3), the matrix element $C'(H)$ defined by

$$C'(H) = \sum_{H'} C(HH') = \sum_{H'}\sum_{i,j} < H|\vec{\sigma}_i\vec{\tau}_i|H' >< H'|\vec{\sigma}_j\vec{\tau}_j|H > \tag{7}$$

where $\sum_{H'}$ indicates a sum over all the available intermediate states. Each term $C(HH')$ is a transition matrix element, associated to the vertex $HH'\pi$. The available diagrams for tha hadrons considered in the present work are shown in fig. 1. As we shall see, an important role is played by the diagram of the ρ-meson containing two pions.

Nevertheless, due to the (dominant) effect of the one-gluon interactions, the states H and H' are no longer degenerate. Thus, we have[5]

$$C'(H) = \sum_{H'} \delta_H(H')C(HH') \quad , \tag{7'}$$

where

$$\delta_H(H') = \int_0^\infty \frac{k^4 u^2(k) dk}{w_\pi(w_\pi + M_{H'} - M_H)} \Big/ \int_0^\infty \frac{k^4 u^2(k) dk}{w_\pi^2} \tag{8}$$

with $\delta_H(H) = 1$.

To perform the integrais in $\delta_H(H')$, eq. (8), it is convenient to make the change of variables

$$x = k^2/m_\pi^2 \quad \rightarrow \quad w_\pi = m_\pi(x+1)^{1/2}$$

leading to

$$\delta_H(H') = \frac{m_\pi}{2\pi I_\pi} \int_0^\infty \frac{x^{3/2} u^2(x) dx}{(x+1)[1+d(x+1)^{-1/2}]} \qquad (9)$$

where

$$d = \frac{\delta_M}{m_\pi} = \frac{M_{H'} - M_H}{m_\pi}$$

On performing the numerical integration in eq. (9), the principal value is taken whenever a pole at $x = d^2 - 1$ appears. We evaluated the intermediate mass corrections given by eqs. (8) and (9), as a function of the mass difference $\delta_M = M_{H'} - M_H$ for several values of tha parameter R_0, defined by eq. (1'), as shown by the curves of fig. 2. We note that the presence of a form-factor in eq. (8) makes δ_H (H') model dependent and is, on the other hand, essential for the convergence of the integrais. As can be seen from eq. (8), for $\delta_M + m_\pi < 0$ we have a pole at $x = d^2 - 1 \Longleftrightarrow k^2 = \delta_M^2 - m_\pi^2$ which is responsible for the change of sign in $\delta_H(H')$ (see fig. 2b). The negative slope of the curves for $\delta_M < 0$ is governed, in our model, by the exponential present in the form-factor eq. (5), or by the function $j_1(kR_B)$ in eq. (6), in the CBN case. Of course, the presence of $j_1(kR_B)$ introduces oscillatory contributions in the region of large k values, giving rise to a slower convergence of the integrals in the CBM.

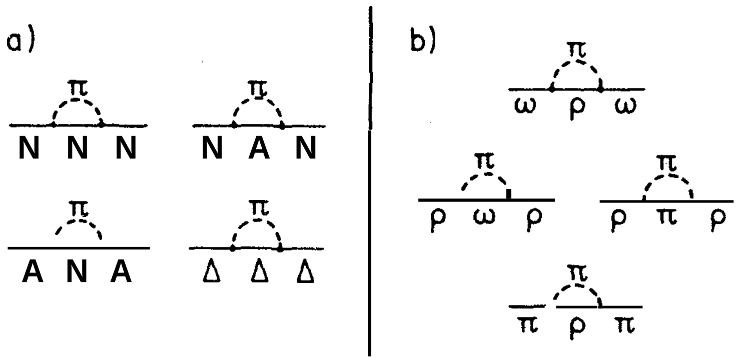
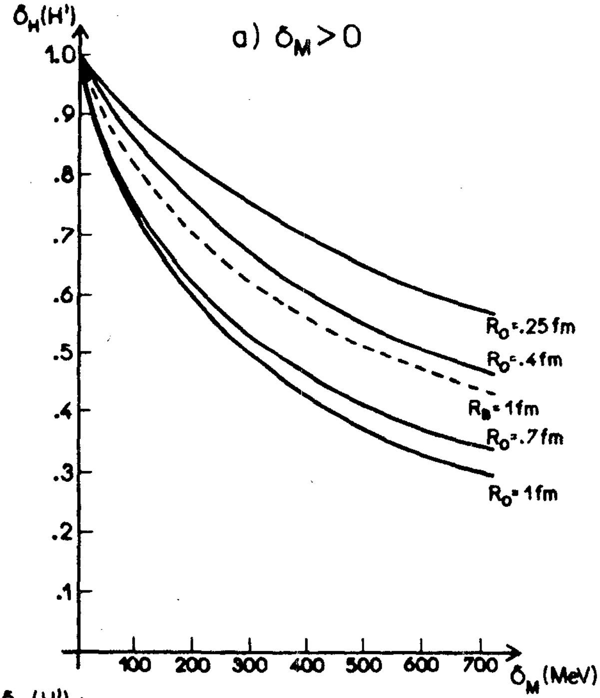


Fig.1 - Available self-energy diagrams for: a) N and h baryons and b) ω, ρ ad π mesona.



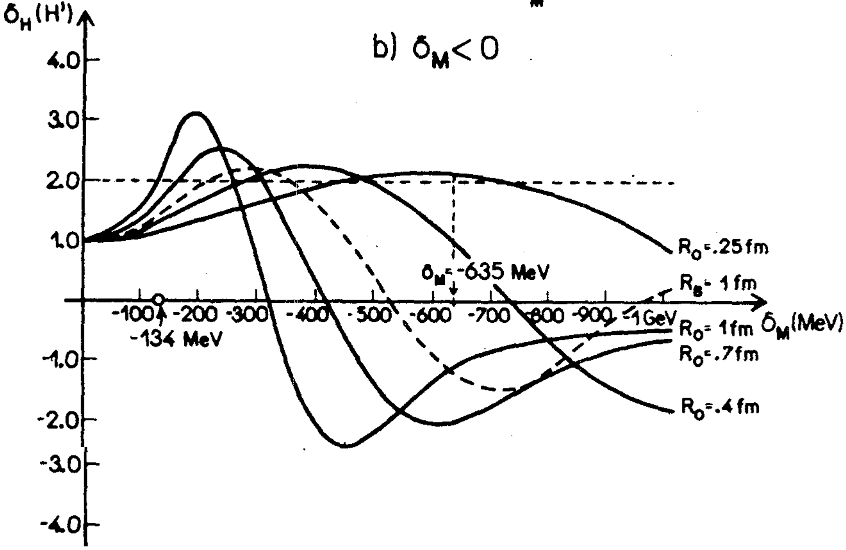


Fig.2 - Intermediate mass carrection δ_H (H') as a function of the mass difference $\delta_M = M_{H'} - M_H$ for several values of R_0. The dashed curve refers to the bag model with R_B = I fm.

In table 1, the matrix elements $C(H)$, $C(HH')$ and the corrections δ_H (H') due to the non-degenerate intermediate states, are given. As can be seen from table 1, when $C(\rho) = 16$ is broken to $C(\rho\omega) = 8$ plus $C(\rho\pi) = 8$ and the corrections $\delta_\rho(H')$ are applied, we get $\delta_\rho(\pi) > 2.00$. Consequently, $C'(\rho) > 24$ and thus the correct splitting, with the lightest p, is found. This can be achieved only if we adjust the radius R_0 for mesons to a convenient small value corresponding to a curve passing through the upper region of fig. 2b, with δ_H (H') > 2 for $\delta_M = M_\pi - M_\rho = -635$ MeV.

The several mass corrections for the N, A baryons and for the ρ, w and π mesons are given in table 2, where ΔE_M denotes the magnetic part of one-gluon exchange correction, ΔE_E the corresponding electric part and $< P^2 >$, the center-of-mass correction[1]. As indicated in table 2, our fitting corresponds to taking $R_0^{(B)} = 0.58$ fm for the baryons and $R_0^{(M)} = 0.27$ fm for the mesons. The corresponding values for the CBM case are $R_B^{(B)} = 1$ fm and $R_B^{(M)} = 0.38$ fm, respectively. Notice that in the fitting of the N and A baryons use was made of the colour relation[7]

$$V_{qq} = \frac{1}{2}V_{q\bar{q}} \quad , \tag{10}$$

so that we have

$$< V(r) >_{\text{Baryon}} \simeq \frac{1}{2} < V(r) >_{\text{Meson}} \quad .$$

This leads to a value of the spring constant K circa ten times larger for the mesons than for the baryons. A similar effect is also expected to appear in the bag model case, with respect to the bag pressure. As can be seen from table 2, the energy corrections found for the mesons are larger than those for the baryons; this is also a consequence of the smaller radius of the mesons. A similar effect is expected in the bag model case.

Several points are worth of comment. Firstly, as discussed before, the diagram containing two pions in fig. 1 plays a decisive role in the mechanism of the ρ – w mass difference. We wish to emphasize that one of the pions in that diagram is a quantum of the elementary (Goldstone) field, whereas the other appears as a quark-antiquark bound-state belonging to the pseudoscalar **flavour** nonet. This dual role of the pion is a common feature of the **chiral** models such as ours. It **is** gratifying that, in a simple **model** as ours, the $q\bar{q}$ pion appears consistently with a mass ***M,*** (qq) very nearly equal to the mass value m_π = **134 MeV** taken for the Goldstone pion in the PCAC limit.

Table **1** - Corrections to the matrix elements $C(H)$ due to the non-degenerate intermedinte states, eqs. **(7)-(7').**

H	H'	$C(H)$	$C(HH')$	$\delta_M = M_{H'} - M_H$	$\delta_H(H')$	$\delta_H(H') \cdot C(HH')$	$C'(H)$
ρ	ω	16	8	13	0.97	7.76	24.32
ρ	π	16	8	-635	2.07	16.56	24.32
ω	ρ	24	24	-13	1.00	24.00	24.00
π	ρ	24	24	635	0.572	13.73	13.73
N	N	57	25	0	1	25	44.26
N	Δ	57	32	294	0.602	19.26	44.26
Δ	Δ	33	25	0	1	25	44.44
Δ	N	33	8	-294	2.43	19.44	44.44

Table 2 - Energy corrections and particle **masses** (in MeV). The parameters of the **fitting** are: m_u = 7 MeV, α_c = **0.626,** $V_0^{(B)} = \frac{1}{2}V_0^{(M)}$ = 71 MeV, $K^{(B)} = \frac{1}{10}K^{(M)}$ = **42.3.10^6** MeV^3. With these **values** for V_0 and K, ***we*** get R_0 = **0.58 fm** (R_B = **1** fm) for the baryons and R, = **0.27** fm (R_B = **0.38** fm) for the mesons.

Particle	E_0	ΔE_M	ΔE_E	$< P^2 >^{1/2}$	*AE,*	*M*	M_{EXP}
N	**1878.8**	-509.5	0	**791.0**	**-179.0**	**938.7**	**938**
A	**1878.8**	**-254.7**	0	**791.0**	**-179.7**	**1238.6**	**1232**
p	**2678.6**	**-367.9**	0	**1389.3**	**-1077.3**	**769.2**	**770**
w	**2678.6**	**-367.9**	0	**1389.3**	**-1063.1**	**783.3**	**783**
R	**2678.6**	**-1103.7**	0	**1380.3**	**-608.2**	**133.7**	**134**

Second, as we have shown, if one **wishes** to obtain the correct $\rho - w$ **mass** splitting it is necessary to take sufficiently small R_0 values. This also implies a

separate parametrization of the meson sector with respect to the baryon sector, in accord with the colour relation eq.(10).

Finally, it is interesting to remark that the η and η' pseudo-scalar mesons are devoid of pion cloud. This statement is a consequence of the fact that, independent of the mixing angle, all the matrix elements $C(HH')$ with $H = \eta$ or η' vanish, as can be easily verified by direct calculation.

An extension of this work treating the strange and also the heavy meson sectors is scheduled for a later date.

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Resumo

Estuda-se o problema da diferença de massa $\rho - w$, utilizando-se um modelo relativista de quarks independentes com um potencial harmônico do tipo escalar-vetorial, em que correções de centro de massa, correções devidas à troca de um glúon e correções devidas à troca de um píon de Goldstone são introduzidas. O último tipo de correção origina-se da imposição de simetria quiral no setor (u,d) de $SU(2)$ sabor do modelo. Correções de auto-energia dos píons com diferentes estados hadrônicos intermediários são importantes na análise do problema, que requer uma parametrização apropriada do setor mesônico diferente daquela previamente utilizada para o cálculo do espectro de massa dos bárions de onda-S. Encontra-se o valor correto da diferença de massas p $- \omega$, que também leva a um valor satisfatório da miisa do píon, calculado como um estado ligado de um par quark-antiquark. Apresenta-se também uma breve discussão do caso anáiogo envolvendo o modelo de sacola nublada.