Revista Brasileira de Física, Vol. 19, nº 2, 1989

Model calculations in the three-body approach to deuteron stripping reactions

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Received on Dezember 26, 1988

Abstract Deuteron stripping reactions are treated as a three-body problem involving a neutron, a proton and an inert core. The neutron-core interaction is described by a separable potential which acts on both s and d waves making it a suitable model for describing the (d,p) reactions on ^{16}O. The Coulomb force is not taken into account and the proton-core interaction is either assumed to be identical to the neutron-core interaction (Mitra model) or neglected (Amado model). The neutron-proton interaction is assumed to have the Yamaguchi form. The transition amplitudes are determined by numerically solving the Alt, Grassberger and Sandhas (AGS) equations.

1. Introduction

The three-body model for deuteron stripping reactions assumes that the collision of a deuteron with a target nucleus is a process involving only three particles: the neutron and the proton in the deuteron and an inert (structureless) core which is introduced as an approximation for the target. In spite of its crudeness, the three-body model has enough degress of freedom to permit the occurrence of (d,p) and (d,n) stripping reactions, deuteron break-up and deuteron elastic scattering.

One speaks of a three-body approach if the assumed three-body model is treated in the framework of the Faddeev[1] formalism for the three-body problem. The three-body approach to deuteron stripping reactions was considered by many authors[2] in the past. The calculations have always been performed using a nucleon-core potential which acts only on the s wave. This s-wave model can describe only the stripping leading to an $s_{1/2}$ single-particle state of the residual nucleus. Thus, in the specific case of (d,p) reactions on ^{16}O (an example used very often to illustrate the calculations[3-6]), the model can account only for the stripping leading to the first excited state of ''O, which is a $2s_{1/2}$ single- particle state.

To improve the model, one must use a nucleon-core interaction which acts on more partia1 waves and/or includes the Coulomb interaction. The long range character of the Coulomb interaction is troublesome for calculations of the Faddeev type. Even though there exist several ways[7,8] to formally incorporate this interaction, the numerical treatment of the three-body problem with charged particles is well known to be very difficult and approximations must be made. In the specific case of deuteron stripping, the approximations made in references 5 and 6 are rather questionable and one cannot tell how conclusive the results are. In the absence then of a reliable way to solve the Coulomb-Faddeev equations in the case of deuteron stripping, the present calculation will no include the proton-core Coulomb interaction and the model will be improved only by the inclusion of more partial waves in the nucleon-core interaction.

Having in mind the specific example of (d,p) reactions on ^{16}O, we consider a neutron-core interaction which acts not only on the s wave but also on the d wave. With the inclusion of a d-wave term, the model will also describe the transition leading to the ground state of ^{17}O which is a $d_{5/2}$ single-particle state, thus the usefulness of the present model-calculation. The terms acting on partial waves other than the $s_{1/2}$ and the $d_{5/2}$ should not be very important since, for the low deuteron energies considered in our calculation, the kernel of the AGS-Faddeev equations would be dominated by the bound state poles occurring in the $d_{5/2}$ and $s_{1/2}$ components of the t-matrix.

For the nuclear part of the proton-core interaction, we make the assumption that it is equal to the neutron-core interaction (this symmetric model is sometimes called Mitra model[g]). We also perform a calculation neglecting the proton-core interaction completely. This simple three-body model for (d,p) reactions was proposed by Amado[10] a long time ago. Although the Amado model is apparently too rough, it may provide a reasonable description of the (d,p) process if, in the real case, the Coulomb repulsion and the nuclear attraction produce a small net effect on the proton.

This paper is organized as follows. In Sec. 2, we present the description of the model. We specify the form that we choose for the potentials and give the expressions for the corresponding t-matrices.

In Sec. 3, we apply the AGS-Faddeev formalism to our problem. We write the transition amplitudes in terms of the AGS X amplitudes. We then perform the angular momentum decomposition of the X amplitudes and write the AGS integral equations for the partia1 wave components.

In Sec.4, we indicate how the singular integral equations obtained in Sec.3 are solved. The results are compared with experiment and discussed.

2. Description of the model

As we said before, we assume that stripping reactions can be considered as rearrangement processes in a system consisting of only three particles: the in coming neutron and proton in the incident deuteron and a core (C) which remains inert. For simplicity, we consider C to be infinitely heavy. The proton and the neutron (numbered respectively by 1 and 2) have mass m.

We shall consider in detail the symmetric model (the AGS equations for the Amado model will follow as a special case). We denote by V_{12} the nucleon-nucleon interaction and by V_1 and V_2 the nucleon-core interaction. Since (d, p) reactions on ^{16}O will be used to illustrate the calculations, we consider, from the very beginning, a nucleon-core interaction which is appropriate for the nucleon-^{16}O system. We thus require that the nucleon-core potential produces two bound states: the $1d_{5/2}$ ground state and the $2s_{1/2}$ excited state. We use a separable potential of the form

$$< \vec{P}_i|V_i|\vec{P}_i' > = \sum_{\ell j} -\frac{\Lambda_{\ell j}}{2m} v_{\ell j}(P_i) v_{\ell j}(P_i') \times \sum_{\mu} < \hat{P}_i|\mathcal{Y}_{\ell j \mu} >< \mathcal{Y}_{\ell j \mu}|\hat{P}_i' > \quad (i = 1, 2), \tag{1}$$

where $\vec{P}_i$ is the momentum of nucleon i,

$$< \hat{P}_i|\mathcal{Y}_{\ell j\mu} >= \mathcal{Y}_{\ell j\mu}(\hat{P}_i) = \sum_{m_\ell m_s} (\ell m_\ell \frac{1}{2} m_s|j\mu) Y_\ell^{m_\ell}(\hat{P}_i)|\frac{1}{2}, m_s >$$

and (ℓj) take the values (0 $\frac{1}{2}$) and (2 $\frac{5}{2}$).

The energy $\epsilon_{\ell j}$ of a bound state is determined by the equation $1 = I_{\ell j}(\epsilon_{\ell j})$, where

$$I_{\ell j}(s) = \Lambda_{\ell j} \int_0^\infty dQ \frac{Q^2 [v_\ell j(Q)]^2}{Q^2 - 2ms}, \tag{2}$$

and the corresponding wave function is given by

$$\phi_{\ell j\mu}(\vec{Q}) = N_{\ell j} \frac{v_{\ell j}(Q)}{Q^2 - 2m\epsilon_{\ell j}} \mathcal{Y}_{\ell j\mu}(\hat{Q}), \tag{3}$$

$N_{\ell j}$ being a normalization factor. The t-matrix associated with the potential given by eq.(1) is written as

$$< \vec{P}_i|t_i(s)|\vec{P}_i' >= \sum_{\ell j\mu} v_{\ell j}(P_i)\tau_{\ell j}(s)v_{\ell j}(P_i') < \hat{P}_i|\mathcal{Y}_{\ell j\mu} > . < \mathcal{Y}_{\ell j\mu}|\hat{P}_i' >, \tag{4}$$

where

$$\tau_{\ell j}(s) = -\frac{\Lambda_{\ell j}}{2m} \frac{1}{1 - I_{\ell j}(s)} \tag{5}$$

The form factors are chosen in the following way

$$v_{d_{5/2}}(Q) = (Q^2 + \alpha^2_{d_{5/2}})Q^2 \exp\left(-\frac{1}{2}\beta^2 Q^2\right), \tag{6}$$

$$v_{s_{1/2}}(Q) = (Q^2 + \alpha^2_{s_{1/2}})(\frac{3}{2} - \beta^2 Q^2) \exp(-\frac{1}{2}\beta^2 Q^2). \tag{7}$$

This kind of potential was used by the author and collaborators in previous works[11,12]. The experimental values of the single-particle energies, $\epsilon_d = -4.146$ MeV and $\epsilon_s = -3.275$ MeV, are reproduced if we use

$$\beta = 1.613 \text{ fm},$$

$$\Lambda_{d_{5/2}} = 3.470 \text{ fm}^9, \qquad \alpha_{d_{5/2}} = 1.163 \text{ fm}^{-1},$$

$$\Lambda_{s_{1/2}} = 1.398 \text{ fm}^5, \qquad \alpha_{s_{1/2}} = 1.033 \text{ fm}^{-1}.$$

The corresponding values of the root mean square radius are 3.48 fm for the $1d_{5/2}$ state and 4.12 fm for the $2s_{1/2}$ state. The $\delta_{d_{5/2}}$ and the $\delta_{s_{1/2}}$ phase shifts agree with the experimental data (for details, see ref.11).

For the neutron-proton interaction, we use a separable potential which acts only on the s wave:

$$< \vec{p}|V_{12}|\vec{p}' >= -\frac{\Lambda}{m}v(p)v(p')\sum_{M_S}|1,M_S >< 1,M_S| \ , \tag{8}$$

where $\vec{p}$ is the relative momentum $\frac{1}{2}(\vec{P}_1 - \vec{P}_2)$ and

$$|1,M_S >= \sum_{m_{s_1},m_{s_2}} (\frac{1}{2}m_{s_1}\frac{1}{2}m_{s_2}|1M_S)|\frac{1}{2},m_{s_1} > |\frac{1}{2},m_{s_2} >$$

is the spin wave function of the triplet state. The corresponding bound state wave function is

$$\phi_{M_S}(\vec{p}) = N\frac{v(p)}{p^2 - m\epsilon}|1,M_S > ,$$

where $-\epsilon$ is the binding energy of the deuteron and N is a normalization factor. The t-matrix is given by

$$< \vec{p}|t_{12}(s)|\vec{p}' >= \sum_{M_S} v(p)\tau(s)v(p')|1,M_S >< 1,M_S| \ , \tag{9}$$

where

$$\tau(s) = -\frac{\Lambda}{m}\frac{1}{1 - I(s)}, \quad I(s) = 4\pi\int_0^\infty dp\frac{p^2[v(p)]^2}{p^2 - ms} \ . \tag{10}$$

For $v(p)$, we choose the 'Yamaguchi form

$$v(p) = \frac{1}{p^2 + \alpha^2} \cdot \tag{11}$$

The values of the triplet parameters α and A are

$$\alpha = 1.406 \ \ \mathrm{fm}^{-1} \ , \ \ \Lambda = 0.382 \ \ \mathrm{fm}^{-3} \ .$$

They are determined from the experimental values a = 5.42 fm and r_0 = 1.76 fm for the scattering length and the effective range of the triplet neutron-proton scattering. For the binding energy of the deuteron they give $\epsilon = -2.223$ MeV. In the symmetric model, the total isospin T and the total spin S are constants of the motion. Since the reactions are initiated by deuterons, we must have $T = 0$ and $S = 1$. Therefore it is not necessary to consider the singlet neutron-proton interaction.

3. The AGS equations

The Hamiltonian of the three-body system is given by

$$H = H_0 + V_1 + V_2 + V_{12} \ , \tag{12}$$

where H_0 is the kinetic enexgy. The channel Hamiltonians are

$$H_i = H_0 + V_i \qquad (i = 1, 2) \ , \tag{13}$$

for the stripping channels, and

$$H_{12} = H_0 + V_{12} \ , \tag{14}$$

for the deuteron channel.

The entrance channel is described by the wave function $|\chi_{d(\vec{P}^0, M_s^0} >$ which describes an in corning deuteron with momentum $\vec{P}^0$ (kinetic energy $E_d = P^{0^2}/4m$) and spin projection M_s^0. It is an eigenstate of H_{12}

$$H_{12}|\chi_{d(\vec{P}^0, M_s^0)}> = (E_d + \epsilon)|\chi_{d(\vec{P}^0, M_s^0)}> = E|\chi_{d(\vec{P}^0, M_s^0)}> \quad . \tag{15}$$

A similar channel wave function

$$|\chi_{d(\vec{P}, M_s)}> \quad (P^2 = P^{0^2}) \quad ,$$

describes the scattered deuterons.

The stripping channel in which nucleon **2** is captured in a single-particle state $\phi_{\ell j \mu}$ and nucleon *1* moves away with momentum $\vec{P}_1$ and spin projection m_s' is described by the wave function

$$|\chi_{1(\vec{P}_1, m_s'), 2(\ell j \mu)}> \quad ,$$

which is an eigenstate of H_2

$$H_2|\chi_{1(\vec{P}_1, m_s'), 2(\ell j \mu)}> = \left(\frac{P_1^2}{2m} + \epsilon_{\ell j}\right)|\chi_{1(\vec{P}_1, m_s'), 2(\ell j \mu)}> \quad , \tag{16}$$

with $P_1^2/2m + \epsilon_{\ell j} = E$. Similarly,

$$|\chi_{2(\vec{P}_2, m_s'), 1(\ell j \mu)}>$$

describes the stripping channel in which particle 1 is captured and particle *2* is outgoing.

The Alt, Grassberger and Sandhas (AGS)[13] operators appropriate for the description of the (d,d), $(d,1)$ and $(d,2)$ reactions are

$$U_{dd}(z) = (V_1 + V_2) + (V_1 + V_2)\; G(z)\; (V_1 + V_2) \quad , \tag{17}$$

$$U_{1d}(z) = (z - H_2) + (V_1 + V_2) + (V_{12} + V_1)\; G(z)\; (V_1 + V_2) \quad , \tag{18}$$

$$U_{2d}(z) = (z - H_1) + (V_1 + V_2) + (V_{12} + V_2)\; G(z)\; (V_1 + V_2) \quad , \tag{19}$$

where $G(z) = (z - H)^{-1}$. In terms of these operators, the transition amplitudes are written as

$$T_{d(\vec{P},M_S);d(\vec{P}^0,M_s^0)} = \langle \chi_{d(\vec{P},M_S)} | U_{dd}(E+i0) | \chi_{d(\vec{P}^0,M_s^0)} \rangle \quad , \tag{20}$$

$$T_{1(\vec{P}_1,m_s'),2(\ell j\mu);d(\vec{P}^0,M_S^0)} = \langle \chi_{1(\vec{P}_1,m_s'),2(\ell j\mu)} | U_{1d}(E+i0) | \chi_{d(\vec{P}^0,M_S^0)} \rangle \tag{21}$$

and a similar expression with **1** and **2** interchanged.

The X amplitudes are defined by

$$X_{d(M_S);d(M_S^0)}(\vec{P},\vec{P}^0;z) = \langle \vec{P} | \langle v_{M_S} | G_0(z) U_{dd}(z) G_0(z) | v_{M_S^0} \rangle | \vec{P}^0 \rangle \quad , \tag{22}$$

$$\begin{aligned} &X_{1(m_s'),2(\ell j\mu);d(M_S^0)}(\vec{P}_1,\vec{P}^0;z) = \\ &\langle \vec{P}_1 | \langle \frac{1}{2}, m_s' | \langle v_{\ell j\mu} | G_0(z) U_{1d}(z) G_0(z) | v_{M_S^0} \rangle \vec{P}^0 \rangle, \end{aligned} \tag{23}$$

$$\begin{aligned} &X_{2(m_s'),1(\ell j\mu);d(M_S^0)}(\vec{P}_2,\vec{P}^0;z) = \\ &\langle \vec{P}_2 | \langle \frac{1}{2}, m_s' | \langle v_{\ell j\mu} | G_0(z) U_{2d}(z) G_0(z) | v_{M_S^0} \rangle \vec{P}^0 \rangle, \end{aligned} \tag{24}$$

where $G_0(z) = (z - H_0)^{-1}$, and $|v_{M_S} >$ and $|v_{\ell j\mu} >$ are such that

$$< \vec{p} | v_{M_S} >= v(p) | 1, M_S > \quad , \tag{25}$$

$$< \vec{P}_i | v_{\ell j\mu} >= v_{\ell j}(P_i) < \hat{P}_i | \mathcal{Y}_{\ell j\mu} > \quad . \tag{26}$$

The transition amplitudes can be written in terms of the on-shell X amplitudes:

$$T_{d(\vec{P},M_S);d(\vec{P}^0,M_S^0)} = \left(\frac{N}{m}\right)^2 X_{d(M_S);d(M_S^0)}(\vec{P},\vec{P}^0;E+i0) \quad , \tag{27}$$

$$T_{1(\vec{P}_1, m_s'), 2(\ell j \mu); d(\vec{P}^0, M_S^0)} = \frac{N_{\ell j} N}{(2m)m} X_{1(m_s'), 2(\ell j\mu); d(M_S^0)}(\vec{P}_1, \vec{P}^0; E + i0) \tag{28}$$

and a similar expression with 1 and *2* interchanged. The momenta P^0, P, P_1 are **all** taken on-shell.

The U operators are determined by the AGS equations

$$\begin{aligned} U_{dd} &= T_2 \ G_0 \ U_{1d} + T_1 \ G_0 \ U_{2d} \quad , \\ U_{1d} &= z - H_0 + T_{12} \ G_0 \ U_{dd} + T_1 \ G_0 \ U_{2d} \quad , \\ U_{2d} &= z - H_0 + T_{12} \ G_0 \ U_{dd} + T_2 \ G_0 \ U_{1d} \quad , \end{aligned} \tag{29}$$

where $T_i(z)$ $(i = 1, 2)$ and $T_{12}(z)$ are the T-operators for the two-body sybsystems:

$$\begin{aligned} T_i &= V_i + V_i \ G_o \ T_i \quad (i = 1, 2) \quad , \\ T_{12} &= V_{12} + V_{12} \ G_0 \ T_{12} \quad . \end{aligned}$$

After multiplying eqs. (29) by appropriate bras and kets, one gets the integral equations for the X amplitudes. We then perform the angular momentum decompositions

$$X_{d(M_S); d(M_S^0)}(\vec{P}, \vec{P}^0; z) = \sum_{JM_J} \sum_{LM_L} (LM_L 1 M_S | JM_J) \cdot \frac{X^{(JM_J)}_{d(L1); d(M_S^0)}(P, P^0; z)}{PP^0} Y_L^{M_L}(\hat{P}) \quad , \tag{30}$$

$$X_{1(m_S'), 2(\ell j \mu); d(M_S^0)}(\vec{P}_1, \vec{P}^0; z) = \sum_{JM_J} \sum_{j'\mu'} \sum_{\ell' m_\ell'} \left(\ell' m_\ell' \frac{1}{2} m_s' | j'\mu'\right) \cdot (j'\mu' j\mu | JM_J) \frac{X^{(JM_J)}_{1(\ell' j'), 2(\ell j); d(M_s^0)}(P_1, P^0; z)}{P_1 P^0} Y_{\ell'}^{m_\ell'}(\hat{P}_1), \tag{31}$$

and an analogous expression with 1 and 2 interchanged. In these expansions, we have taken the polar axis in the direction of the momentum $\vec{P}^0$. The integral equations for the partial wave amplitudes are

$$X^{(JM_J)}_{d(L1);d(M_S^0)}(q,P^0;z) = \sum_{\ell''j''}\sum_{\ell'''j'''} 2 \cdot \int_0^\infty dq' A^J_{L1;\ell''j'',\ell'''j'''}(q,q';z)\tau_{\ell''j''}\left(z-\frac{q'^2}{2m}\right) \cdot X^{(JM_J)}_{1(\ell'''j'''),2(\ell''j'');d(M_S^0)}(q',P^0;z) \quad , \tag{32}$$

$$\begin{aligned} X^{(JM_J)}_{1(\ell'j'),2(\ell j);d(M_S^0)}(q,P^0;z) = \sum_{L'} \frac{\sqrt{2L'+1}}{4\pi}(L'01M_S^0|JM_J) \\ \cdot C^J_{\ell j,\ell'j';L'1}(q,P^0;z) \\ + \int_0^\infty dq' B^J_{\ell j,\ell'j'}(q,q';z)\tau_{\ell'j'}\left(z-\frac{q'^2}{2m}\right) \\ \cdot X^{(JM_J)}_{1(\ell j),2(\ell'j');d(M_S^0)}(q',P^0;z) \\ + \sum_{L'}\int_0^\infty dq' C^J_{\ell j,\ell'j';L'1}(q,q';z)\tau\left(z-\frac{q'^2}{4m}\right) \\ \cdot X^{(JM_J)}_{d(L'1);d(M_S^0)}(q',P^0;z) \quad , \end{aligned} \tag{33}$$

where we have used the symmetry property

$$X^{(JM_J)}_{1(\ell'j'),2(\ell j);d(M_S^0)}(q,P^0;z) = X^{(JM_J)}_{2(\ell'j'),1(\ell j);d(M_S^0)}(q,P^0;z) \quad . \tag{34}$$

The **functions** A, **B and** C **are given by**

$$A^{J}_{L1;\ell j,\ell' j'}(q,q';z) = (-1)^{j+j'-J}\left[(2j+1)(2j'+1)3(2L+1)\right]^{1/2} \cdot \begin{Bmatrix} \ell & \ell' & L \\ 1/2 & 1/2 & 1 \\ j & j' & J \end{Bmatrix} A^{\ell j}_{L\ell'}(q,q';z) \quad , \tag{35}$$

$$B^{J}_{\ell j,\ell' j'}(q,q';z) = (-1)^{j+j'-J} 2mq' v_{\ell j}(q')(2mz - q^2 - q'^2)^{-1}\, q v_{\ell' j'}(q) \quad , \tag{36}$$

$$C^{J}_{\ell j,\ell' j';L1}(q,q';z) = A^{J}_{L1;\ell j,\ell' j'}(q',q;z) \quad , \tag{37}$$

where

$$A^{0\frac{1}{2}}_{LL}(q,q';z) = 2m\sqrt{\pi}\; V^{0\frac{1}{2}}_{L}(q,q';z) \quad , \tag{38}$$

$$A^{2\frac{5}{2}}_{L\ell'}(q,q';z) = 2m\sqrt{\pi}\left[\frac{5(2\ell'+1)}{2L+1}\right]^{1/2} \cdot \Big\{ (20\ell'0|LO)\left[V^{2\frac{5}{2}}_{\ell'}(q,q';z)q^2 + V^{2\frac{5}{2}}_{L}(q,q';z)q'^2\right] - \sqrt{30}\sum_{\mathcal{L}}\sqrt{2\mathcal{L}+1}(10\mathcal{L}0|L0)(10\ell'0)|\mathcal{L}0)W(11L\ell';2\mathcal{L}) \cdot V^{2\frac{5}{2}}_{\mathcal{L}}(q,q';z)qq' \Big\} \tag{39}$$

and

$$V^{\ell j}_{\lambda}(q,q';z) = qq'\int_{-1}^{1} \frac{v(|\frac{1}{2}\vec{q} - \vec{q}'|)v_{\ell j}(|\vec{q} - \vec{q}'|)}{(2mz - q^2 - 2q'^2 + 2\vec{q}\cdot\vec{q}')|\vec{q} - \vec{q}'|^{\ell}} P_{\lambda}(x)dx \quad , \tag{40}$$

x being the cosine of the angle between $\vec{q}$ and $\vec{q}'$.

The equations corresponding to the Amado model are similar to eqs.(32) and (33), the only differences being the absence the factor 2 in eq. (32) and the absence of the term containing B in eq.(33).

4. Results and Discussions

The singular integral equations (32)-(33) were solved using the method of contour rotation[14,15]. The analysis of the singularities shows that it is possible to continue analytically the integral equations to a half-line of the fourth quadrant of the complex momentum plane. The angle ϕ between this half-line and the positive half axis must satisfy the condition

$$\phi < \mathrm{Min}\left\{\tan^{-1}\frac{2\alpha}{P^0},\quad \tan^{-1}\frac{\sqrt{4m|\epsilon|}}{P^0},\quad \tan^{-1}\frac{\alpha}{P_1}\right.$$

$$\left.\tan^{-1}\frac{\sqrt{P_1^2-2mE}}{P_1},\quad \tan^{-1}\frac{\sqrt{P_1^{*2}-2mE}}{P_1^*}\right\}\ , \qquad (41)$$

where P_1 and P_1^* refer to the stripping to the ground and excited states respectively. Also, because of the exponential in the form factor *(6)* and (7), ϕ must be taken smaller than $\pi/4$. In the calculation, we took a value which was about one half of the maximum allowed value.

The equations on the rotated contour are free from singularities and can be treated by standard numerical methods. We approximate each integral by a finite sum (using the Gauss quadrature formula) and solve the resulting system of algebraic equations. The number N_G of Gauss points were varied until an acceptable stability of the calculated amplitudes was reached. For $N_G = 24$, the results were stable within less than a few percent. Below the threshold of the deuteron break-up (E < 0), the correctness of the calculation was checked using the optical theorem. So, we compared the amplitude for the forward elastic scattering with the sum of the total (angle-integrated) cross sections for all processes. Since we do not calculate the break-up, this kind of test cannot be used above the threshold of this process (namely, at positive energies).

The differential cross sections for stripping and elastic scattering are given by

$$\left(\frac{d\sigma}{d\Omega}\right)_{str.} = \frac{1}{3}\sum_{M_S^0}\sum_{m'_s}\sum_{\mu}(2m)m\frac{P_1}{P^0}(2\pi)^4\left|T_{1(\vec{P}_1,m'_s),2(\ell j\mu);d(\vec{P}^0,M_S^0)}\right|^2 , \quad (42)$$

$$\left(\frac{d\sigma}{d\Omega}\right)_{el.} = \frac{1}{3}\sum_{M_S^0}\sum_{M_S}(2m)^2(2\pi)^4\left|T_{d(\vec{P},M_S);d(\vec{P}^0,M_S^0)}\right|^2 \quad (43)$$

These cross sections were calculated for three values of E_d : 1.964 MeV, 2.425 MeV and 3.0 **Mev.** We restricted the calculation to low energies because, otherwise, we would have to handle too many partial waves. Even for such low energies, we have to include about thirteen partial waves (those corresponding to $J^\pi = 0^-, 1^+, 1^-, ..., 6^+, 6^-$).

In figs. **1-3** we give the results obtained for the stripping to the excited $2s_{1/2}$ state. The calculated angular distributions for both the symmetric model and the Amado model agree qualitatively with experiment, the frontal peak being clearly reproduced. It should be mentioned that the early calculations of refs.3-6 already reproduced this pattern. Also, it should be remarked that a peaked shape for the angular distribution is obtained even in calculations using the Born approximation (direct reaction model[16]); the position of the peak depends on the

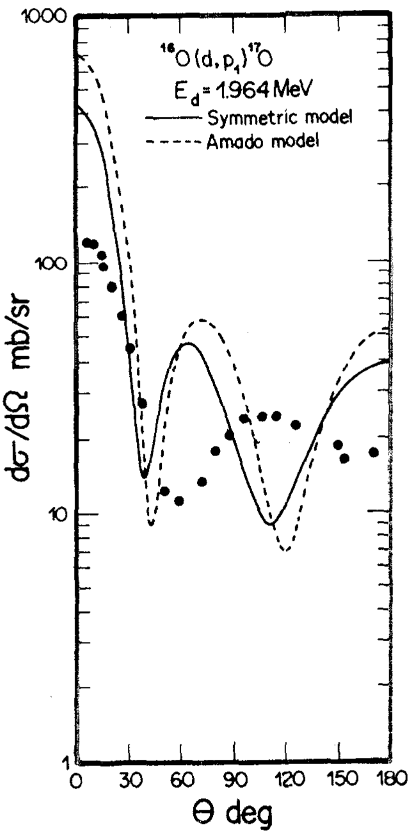

Fig.1 – Angular distributions for the stripping to the excited state of ^{17}O for E_d=1.964 MeV. The Experimental points in Figs. 1-9 (indicated by dots) are from ref. 17.

angular momentum ℓ of the captured neutron and, for the specific value $\ell = 0$ (transition to an s state), the peak occurs right in the forward direction. Our calculation indicates that the direct transition of the neutron to the $2s_{1/2}$ state is not appreciably disturbed by the three-body dynamics.

Although the calculated angular distribution is of the desired form, the absolute value of the cross section is about four times the experimental value, in the forward direction. At the same time, the theoretical curves oscillate more rapidly than one would desire. We expect that the inclusion of the Coulomb interaction

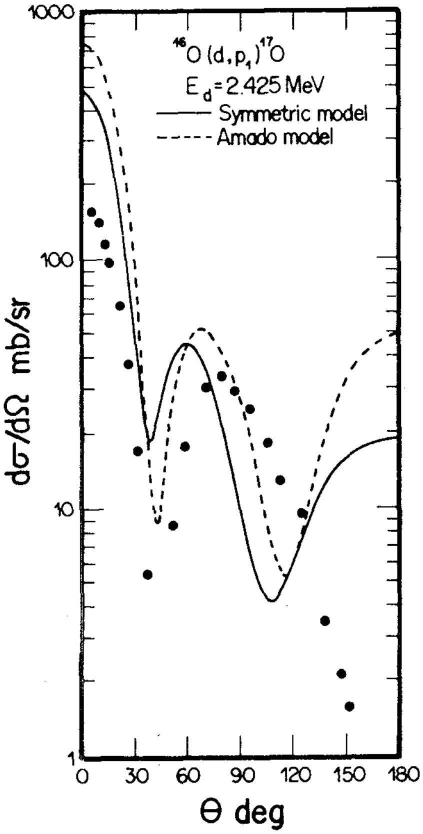

Fig.2 – The same as fig.1, but for E_d=2.425 MeV.

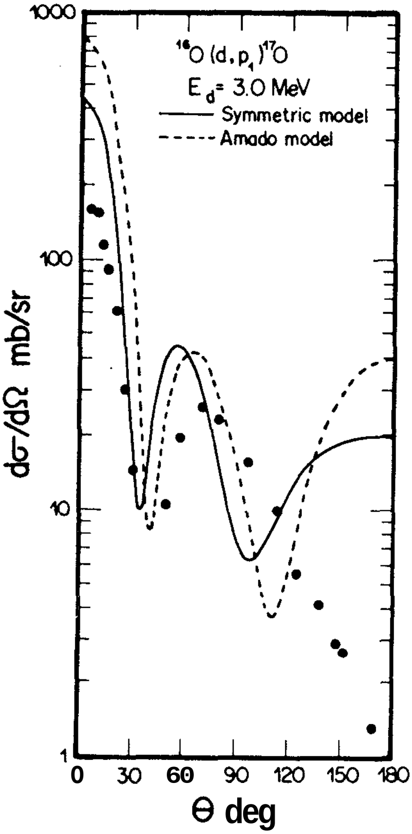

Fig.3 – The same as fig. 1, but for E_d=3.0 MeV.

will improve the results as the Coulomb barrier keeps the low energy deuteron farther away from the reaction region decreasing the probability for the neutron to be captured. Of course, the Coulomb repulsion would affect also the deflection of the outgoing proton. Intuitively, we expect that the bending of the proton orbit towards larger scattering angles would correspond, in figs. **1-3,** to an expansion of the theoretical curves to the right. Calculations performed within the Born approximation[16] are in agreement with such a prediction.

Figs.4–6 refer to the stripping to the ground state. As we mentioned before, this transition was not considered in the calculations performed by other authors. Again, the agreement with experiment is only qualitative, improving as the energy increases. The angular distributions have a better shape for the Amado model. These exhibit the peak which is characteristic of a direct stripping leading to a d state ($\ell = 2$). However, the symmetric model gives better absolute values for the cross sections in the region of the characteristic peak. In these figures, we exhibit also the result for the symmetric model when the $2s_{1/2}$ term in the single-particle potential is neglected. The curves become similar to those of the Amado model, no longer showing the unpleasant rise in the forward direction. Therefore, in the symmetric model the s term of the nucleon-core potential has too much effect on the motion of the outgoing particle. The s term may act at any point inside the core since there is no centrifugal barrier for the s wave. However, in a real situation, the effect of the s term has to be smaller because the Coulomb repulsion acts as a barrier even for the s wave.

Finally, in figs. 7-9, we show the angular distributions for the elastically scattered deuterons. Here also, the calculated curves are too compressed to the left. Of course, because we neglect the Coulomb interaction, the results for small angles are meaningless.

In conclusion, we would say that the result predicted by our model calculation shows a reasonable qualitative agreement with experiment, and this justifies further investigations. Although the quantitative agreement is still poor, we believe that the results will be much improved when the Coulomb force is properly taken into account. The investigation of this point is under way.

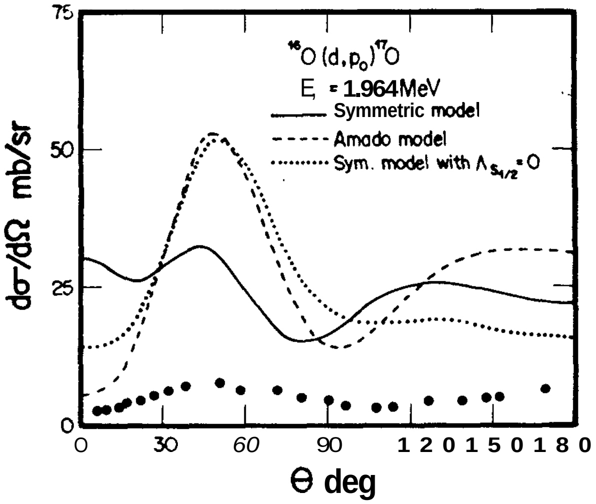

Fig.4 - Angular distributions for the stripping to the ground state of ^{17}O for E_d=1.964 MeV.

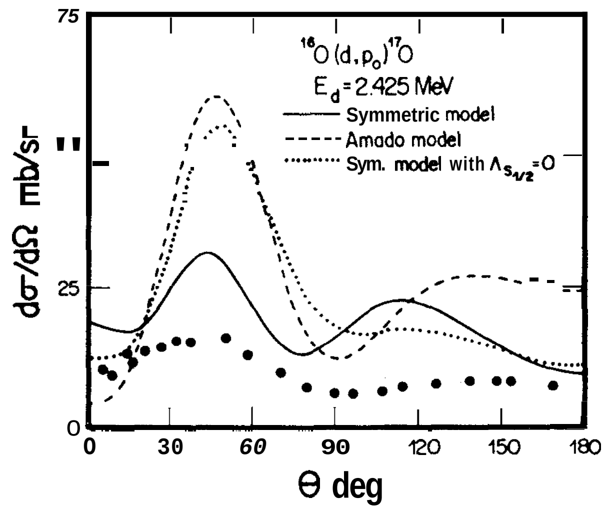

Fig.5 - The same as fig. 4, but for E_d=2.425 MeV.

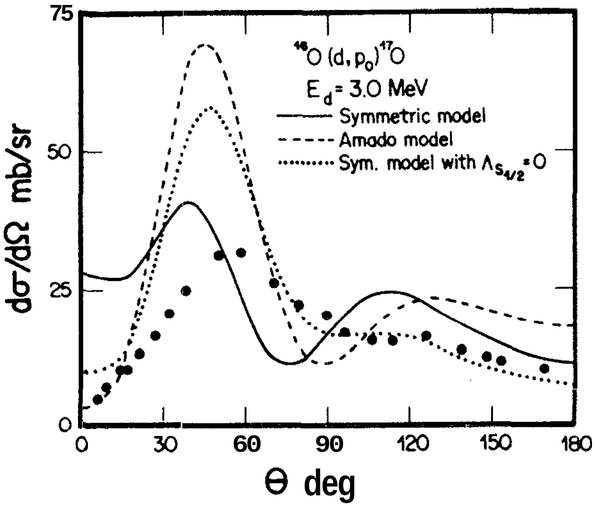

Fig.6 - The same as fig. 4, but for E_d=3.0 MeV.

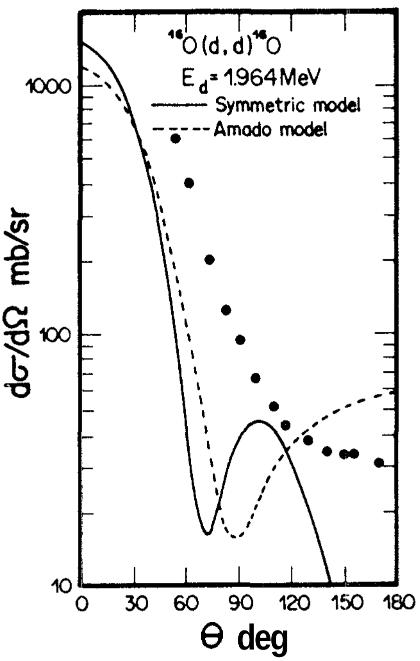

Fig.7 – Angular distribution for the elastic scattering of deuterons. E_d=1.964 MeV.

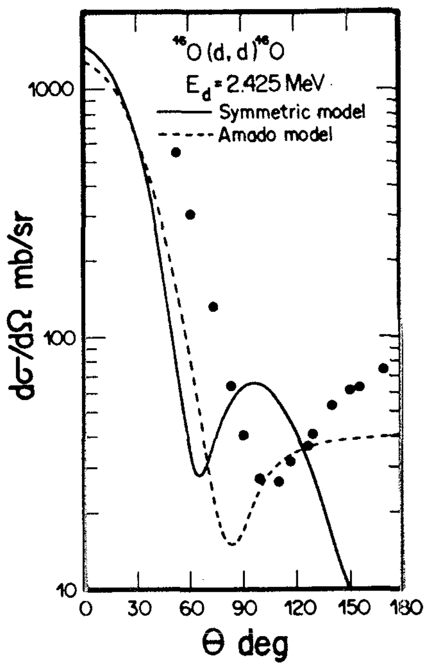

Fig.8 – The same as fig. 7, but for E_d=2.425 MeV.

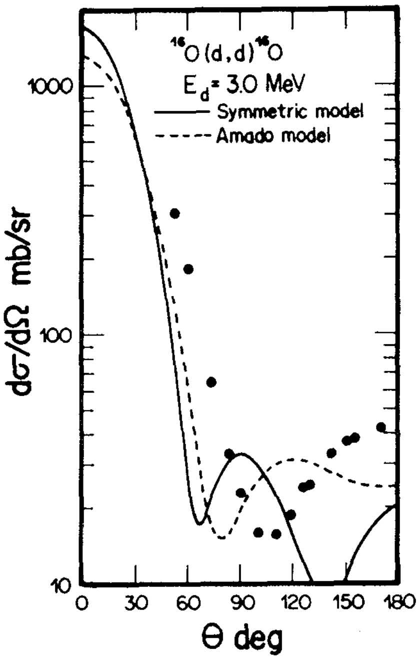

Fig.9 - The same as fig. 7, but for $E_d = 3.0$ MeV.

The numerical calculations were carried at the Centro de Computação Eletrônica (CCE) of the University of São Paulo.

I would like to thank Dr. C.P. Malta, Prof. L. C. Gomes and Prof. W. Gloeckle for the discussions.

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Resumo

As reações de stripping do dêuteron são tratadas como um problema de três corpos envolvendo um nêutron, um próton e um caroço inerte. A interação nêutron-caroço é descrita por um potencial separável que atua tanto na onda s como na onda d, o que a torna apropriada para descrever reações (d,p) no ^{16}O. *A* força Coulombiana não é levada em conta e a interação próton-caroço ou é considerada idêntica à interação nêutron-caroço (modelo de Mitra) ou é desprezada (modelo de Amado). Para a interação nêutron-próton admitimos a forma de Yamaguchi. As amplitudes de transição são determinadas resolvendo-se numericamente as eqcações de Alt, Grassberger e Sandhas (AGS).