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Qualitative description of the electron's anomalous magnetic moment

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Abstract It is shown that the first order radiative correction to the electronic magnetic rnoment rnay be explained by vacuurn fluctuating electromagnetic fields using a modification of the Bargmann-Michel-Telegdi equation which allows for the instantaneous velocity.

The most direct and extremely successful radiative corrections provided by QED perturbative scheme are the Lamb shift and the anomalous magnetic moment[1]. Whereas for the former a simple intuitive explanation is given in terms of the smearing of the Coulomb potential caused by the vacuum fluctuating electromagnetic fields[2], for the latter a similar qualitative description seems to be Iacking, apart from an attempt based on stochastic ideas[3].

The purpose of the present not is to show that the electron's anomalous magnetic moment can also be qualitatively explained by the vacuum electromagnetic fluctuations if use is made of a modification of the Bargmann-Michel-Telegdi equation[4] (BMT) which allows for the instantaneous velocity necessary to justify the spin gyromagnetic ratio g = 2.

For this purpose we write the BMT equation for a spin 1/2 interaction with an external field as

$$\frac{da^\mu}{d\tau} = c_1 F^{\mu\nu} a_\nu + c_2 u^\mu F^{\nu\lambda} u_\nu a_\lambda \qquad (1)$$

where τ is the proper time iind a'' is a vector which in the rest frame defined by $p^\mu = (m,\vec{o})$ takes the form $a^\mu = (o,\vec{\varsigma})$ with $\vec{\varsigma} = < \vec{\sigma} >$. The difference with the usual presentation[5] of the BMT equations is that the instantaneous velocity 4-vector u^μ is not equal to p^μ/m. Because of the antisymmetry of the field tensor $F^{\mu\nu}$ and the condition a.u $= 0 = \vec{\varsigma}\cdot\vec{v}$ in the rest frame necessary for the instantaneous velocity to build the intrinsic spin, the second term of eq.(1) is the only allowed one.

If in the rest frame we have only magnetic externa1 field, from eq.(1)

$$\frac{d\varsigma^i}{dt} = c_1 F^{ij}\varsigma_j + c_2 u^i F^{jk} u_j \varsigma_k \tag{2a}$$

$$o = c_2 u^0 F^{ij} u_j \varsigma_j \tag{2b}$$

The instantaneous velocity terms must be considered as time averaged. Disregarding at first the effect of the fluctuating fields, in terms of the zitterbewegung velocity

$$< u^i u_j > = -\frac{1}{3}\frac{v^2}{1-v^2}\delta^i_j \tag{3a}$$

$$< u^0 u_i > = 0 \tag{3b}$$

Assuming $v^2/(1-V^2) \simeq 1$ and that the second term of eq.(2a) is responsible, through the zitterbewegung, for the increase of the gyromagnetic ratio from 1 to 2 we need

$$c_1 = -\frac{c_2}{3} = \mu = \frac{e}{2m} \tag{4}$$

We now include the effect of the electric field of the radiation zero-point by means of a non-relativistic fluctuation[2]

$$\delta\ddot{\vec{r}} = \frac{e}{m}\vec{E} \tag{5}$$

which in Fourier components means

$$\delta\Gamma_\omega = -\frac{eE_\omega}{m\omega^2}, \quad \delta v_\omega = -i\frac{eE_\omega}{m_\omega} \tag{6}$$

Being $|E_\omega|^2 = \omega^2/(2\pi^2)$ to reproduce the radiation zero-point energy, the change in the averaged instantaneous velocity is

$$< |\delta v|^2 >= \frac{e^2}{27r^2 m^2} \int d\omega\ \omega \tag{7}$$

Taking as ultraviolet cutoff $\omega_c \simeq \lambda_c^{-1} = m$

$$< |\delta v|^2 >= \frac{\alpha}{\pi} \quad , \quad \alpha = \frac{e^2}{4\pi} \tag{8}$$

Therefore, considering that the radiation zero-point and zitterbewegung effects do not overlap, the anomalous term is

$$-\frac{1}{3}c_2 < |\delta v|^2 > \delta^i{}_j\ F^{jk}\ \varsigma_k \approx \mu\ \frac{\alpha}{\pi}\ F^{ij}\ \varsigma_j \tag{9}$$

and the complete rest frame spin equation eq.(2a) is

$$\frac{d\varsigma^i}{dt} \cong 2\mu\left(1 + \frac{\alpha}{2\pi}\right)F^{ij}\ \varsigma_j \tag{10}$$

which coincides with the first order radiative correction coming from the perturbative QED theory.

One must remark that the Lamb-shift estimation is more strictly determined since its calculation is independent on the spreading of Coulomb law due to zitterbewegung which gives way to the Darwin term. Here, on the contrary, the anomalous magnetic moment is estimated from the BMT equation once the zitterbewegung contribution is fixed to be responsible for the increase of the Landé factor from the standard value 1 for orbital momentum to that coming from Dirac equation g $= 2$.

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Resumo

Mostra-se que a correção radiativa de primeira ordem para o momento magnético do eletron pode ser explicada por campos eletromagnéticos com vácuos flutuantes usando uma modificação da equação de Bargmann-Michel-Telegdi que leva em conta velocidades instantâneas.