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Coherent states for certain time-dependent systems

I.A. Pedrosa

Departamento de Física, Universidade Federal da Paraíba, 58000, João Pessoa, PB, Brasil

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Abstract Hartley and Ray have constructed and studied coherent states for the time-dependent oscillator. Here we show how to construct coherent states for more general time-dependent systems. We also show that these states are equivalent to the well-known squeezed states.

1. Introduction

In a very interesting paper, Hartley and Ray[1] constructed exact coherent states for a special case of the time-dependent oscillator where only the frequency is allowed to change with time, by making use of the Lewis-Riesenfeld quantum theory for the time-dependent harmonic oscillator[2]. According to Hartley and Ray, these new coherent states have most, but not all, of the properties of the ordinary coherent states for the time-independent oscillator[3-5]. For example, these coherent states give the exact classical motion, but they are not minimun-uncertainty states.

In this paper, we wish to show how one can construct Hartley-Ray coherent states for more general time-dependent oscillators. To this end, we use a time-dependent canonical transformation, which reduces the more general time-dependent oscillator to the special time-dependent oscillator with a modified frequency, and an auxiliary time-dependent transformation. Thus, we can perform the Hartley-Ray construction for the transformed system and then map the states so obtained back to the original system via inverse transformations. We also show that these coherent states are equivalent to the well-known squeezed states.

This paper is organized as follows. In sec. 2 we briefly review some of the properties of the ordinary coherent states. In sec. 3 we construct coherent states for more general time-dependent oscillators. In sec. 4 we show that these states are equivalent to the squeezed states. Finally, some concluding remarks are added in sec. 5.

2. Review of coherent states for the time-independent oscillator

For the time-independent harmonic oscillator of mass equal to unity and frequency ω_0

$$H = \frac{p^2}{2} + \frac{1}{2}\omega_0 q^2 \ , \tag{2.1}$$

where p is the momentum conjugate to q with $[q,p] = i\hbar$. Now, writing the usual annihilation and creation operators

$$a = \left(\frac{1}{2\hbar\omega_0}\right)^{1/2}(\omega_0 q + ip) \ , \tag{2.2a}$$

$$a^+ = \left(\frac{1}{2\hbar\omega_0}\right)^{1/2}(\omega_0 q - ip) \ , \tag{2.2b}$$

which satisfy the commutation relation

$$[a, a^+] = 1 \ . \tag{2.3}$$

we can rewrite eq. **(2.1)** as

$$H = \hbar\omega_0\left(a^+ a + \frac{1}{2}\right) \ . \tag{2.4}$$

The operators a and a+ have the properties

$$a|n> = n^{1/2}|n-1> \ , \tag{2.5a}$$

$$a^+|n> = (n+1)^{1/2}|n+1> \ , \tag{2.5b}$$

$$|n> = \frac{(a^+)^n}{(n!)^{1/2}}|0> \tag{2.5c}$$

where $|n>$ are the number states and $|0>$ is the oscillator ground state.

The coherent states $|\alpha>$ where a = u + *iv* is a complex number may be defined [3,5,21] as (a) minimum uncertainty states (see eq.(2.12) below) that have the additional property Ap $= mw_0 \Delta q$ (in our case the mass m is equal to unity); (b) eigenstates of the annihilation operator, i.e., $a|\alpha>=\alpha|\alpha>$, and (c) as states displaced from the ground state via the operator $D(\alpha)=\exp[\alpha a^+ - \alpha^* a]$. However, for the time-independent harmonic oscillator all three definitions yield the same result[4], namely,

$$|\alpha>=\exp\left(-\frac{1}{2}|\alpha|^2\right)\sum_{n=0}^{\infty}\frac{\alpha^n}{(n!)^{1/2}}|n> \quad . \tag{2.6}$$

Using eq. **(2.6)** and the fact that $H|n>=\hbar\omega_0(n+1/2)$, we obtain the time dependence of the coherent states as

$$\begin{aligned}|\alpha,t> &= \exp(-iHt/\hbar)|\alpha> \\ &= \exp(-i\omega_0 t/2)|\alpha(t)> ,\end{aligned} \tag{2.7}$$

where

$$\alpha(t) = a\exp(-i\omega_0 t) \tag{2.8}$$

Then, trivially, from eq. (2.4) and eq. (2.7) we obtain the expectation value of H

$$<\alpha,t|H|\alpha,t>=\hbar\omega_0\left(|\alpha|^2+\frac{1}{2}\right)=<\alpha|H|\alpha> \tag{2.9}$$

Now, by calculating the uncertainties in q and p in the state $|\alpha,t>$, one finds

$$(\Delta q)^2 =<q^2>-<q>^2=\frac{\hbar}{2\omega_0} \tag{2.10}$$

and

$$(\Delta p)^2 =<p^2>-<p>^2=\frac{\hbar\omega_0}{2} \quad . \tag{2.11}$$

Thus, from eq. (2.10) and eq. (2.11) we see that the coherent states are minimum-uncertainty states, i.e.,

$$(\Delta q)(\Delta p) = \hbar/2 \ . \tag{2.12}$$

Also, using eq. **(2.7)** we calculate the expectation value of q for the coherent state $|\alpha, t>$ and find

$$< \alpha, t|q|\alpha, t >= \left(\frac{2\hbar|\alpha|^2}{\omega_0}\right)^{1/2} \sin(\omega_0 t + \delta) \ , \tag{2.13}$$

where 6 is the argument of the complex number a. As pointed out by Glauber[3], the classical limit is obtained from taking $h \longrightarrow 0$, $|\alpha| \longrightarrow \infty$, such that $\hbar|\alpha|^2 \longrightarrow$ finite. Now from eq. **(2.9)** we see that the classical Hamiitonian is

$$\begin{aligned} H_{cl} &=< \alpha|H|\alpha > -\frac{1}{2}\hbar\omega_0 \\ &= \hbar\omega_0|\alpha|^2 \ . \end{aligned} \tag{2.14}$$

Thus, the expectation value of q in eq. **(2.13)** follows the classical motion. It satisfies the classical equation of motion of a time-independent oscillator with energy given by eq. **(2.14).**

3. Time-dependent coherent states

We consider the time-dependent harmonic-oscillator Hamiltonian

$$H(t) = f(t)\frac{p^2}{2} + f^{-1}(t)\frac{\omega^2(t)}{2}q^2 \ , \tag{3.1}$$

where q is a canonical coordinate, **p** is its conjugate momentum and $\omega(t)$ and f *(t)* are arbitrary real functions of time. The variables q and **p** satisfy the canonical commutation relation

$$[q,p] = i\hbar \ . \tag{3.2}$$

The canonical equations of motion are

$$\dot{q} = \frac{1}{i\hbar}[q,H] = f(t)p \ , \tag{3.3}$$

$$\dot{p} = \frac{1}{i\hbar}[p, H] = -f^{-1}(t)\omega^2(t)q \ , \tag{3.4}$$

which, when combined, yield the equation

$$\ddot{q} + \gamma(t)\dot{q} + \omega^2(t)q = 0 \ , \tag{3.5}$$

where

$$\gamma(t) = -\frac{d}{dt}(\ln f(t)) \ . \tag{3.6}$$

Note that the so-called Kanai-Caldirola Hamiltonian[6] is recovered when f(t) = $\exp(-\gamma t)$ with constant 7.

To construct time-dependent coherent states for the Hamiltonian eq. (3.1) we proceed as follows. Consider the time-dependent canonical transformation given by the generating function'

$$F(q, P, t) = \frac{1}{2}(qP + Pq)f^{-1/2}(t) - \frac{\gamma(t)}{4}q^2 f^{-1}(t) \ . \tag{3.7}$$

The transformation equation are Q $= \partial F/\partial P$, p $= \partial F/\partial q$, from which we obtain the new canonical variables

$$Q = qf^{-1/2}(t) \ , \tag{3.8a}$$

$$P = pf^{1/2}(t) + \frac{\gamma(t)}{2}qf^{-1/2}(t) \ . \tag{3.8b}$$

This is a generalization of the canonical transformation proposed by Gzy1[8]. Also, note that [Q,$P] = [q, p]$ which implies that the commutation relations remain the same in both coordinates. Then, under this transformation the Hamiltonian eq. (3.1) is transformed into a new Hamiltonian H, (t) = $H(t) + \partial F/\partial t$ which, in terms of the new variables, is expressed as

$$H_1(t) = \frac{P^2}{2} + \frac{\Omega^2(t)}{2}Q^2 \ , \tag{3.9}$$

where

$$\Omega^2(t) = \omega^2(t) - \left(\frac{\gamma^2(t)}{4} + \frac{\dot{\gamma}(t)}{2}\right) \tag{3.10}$$

is the modified frequency. Here we observe that the Hamiltonian eq. (3.9) is of the form of that considered by Lewis and Riesenfeld[2] and Hartley and Ray[1]. Hence, an exact invariant for (3.9) is given by[1,2]

$$I(t) = \frac{1}{2}\left[(\rho P - \dot{\rho} Q)^2 + (Q/\rho)^2\right] , \tag{3.11}$$

where $Q(t)$ satisfies the equation of motion

$$\ddot{Q} + \Omega^2(t)Q = 0 \tag{3.12}$$

and $\rho(t)$ is a c-number quantity satisfying the auxiliary equation

$$\ddot{\rho} + \Omega^2(t)\rho = 1/\rho^3 \ . \tag{3.13}$$

In order to make $I(t)$ Hermitian, we choose only the real solutions of eq. (3.13).

Let us now introduce the time-dependent transformation[7]

$$\rho(t) = x(t) f^{-1/2}(t) \ , \tag{3.14}$$

where $x(t)$ is a real function of time which is to be determined. Then using eqs. (3.8), (3.10) and (3.14), the equation of motion (3.12) is converted into eq. (3.5) and the auxiliary eq. (3.13) into the equation

$$2 + \gamma(t)\dot{x} + \omega^2(t)x = f^2(t)/x^3 \ . \tag{3.15}$$

The invariant eq. (3.11) is converted into the form

$$I(t) = \frac{1}{2}\left[(px - f^{-1}\dot{x}q)^2 + (q/x)^2\right] . \tag{3.16}$$

Thus, eq. (3.16) is an exact invariant for the Hamiltonian eq. (3.1) with p given by eq. (3.3) and $q(t)$ and $x(t)$ satisfying, respectively, eqs. (3.5) and (3.15). For $f(t) = 1$ we recover the invariant for the time-dependent Hamiltonian where only the frequency is allowed to change with time[1,2]. Note that in this case the function (3.7) generates the identity transformation.

Next we consider the time-dependent operators[1,2]

$$b(t) = \left(\frac{1}{2\hbar}\right)^{1/2} \left[(Q/\rho + i(\rho P - \dot{\rho}Q)\right] , \tag{3.17a}$$

$$b^+(t) = \left(\frac{1}{2\hbar}\right)^{1/2} \left[(Q/\rho - i(\rho P - \dot{\rho}Q)\right] , \tag{3.17b}$$

where

$$[b(t), b^+(t)] = 1 . \tag{3.18}$$

From eq. **(3.17)** we can deduce

$$Q = \left(\frac{\hbar}{2}\right)^{1/2} \rho(b^+(t) + b(t)) , \tag{3.19a}$$

$$P = i\left(\frac{\hbar}{2}\right)^{1/2} \left(\left(\frac{1}{\rho} - i\dot{\rho}\right)b^+(t) - \left(\frac{1}{\rho} + i\dot{\rho}\right)b(t)\right) . \tag{3.19b}$$

The invariant operator' given by eq. *(3.11)* can be written in terms of $b(t)$ and $b^+(t)$ as [1,2]

$$I(t) = h\left(b^+(t)b(t) + \frac{1}{2}\right) . \tag{3.20}$$

Using eqs. *(3.18)* and *(3.20)* the eigenvalue problem for $I(t)$ can be exactly solved just as for the Hamiltonian in the time-independent case. Thus, we have[1,2]

$$I(t)|n,t> = \hbar(n + 1/2)|n,t> , \tag{3.21}$$

$$b(t)|n,t> = n^{1/2}|n-1,t> , \tag{3.22}$$

$$b^+(t)|n,t> = (n+1)^{1/2}|n+1,t> . \tag{3.23}$$

The general solutiori of the time-dependent Schrõdinger equation for $H_1(t)$ in eq. *(3.9)* is given by

$$|\psi, t>_S = \sum_n c_n \exp\left(i\alpha_n(t)\right)|n,t> , \tag{3.24}$$

where the c_n are constant, the subscript S indicates that the states evolve in time according to the Schrödinger equation and the phase functions $\alpha_n(t)$ are given by

$$\alpha_n(t) = -\left(n+\frac{1}{2}\right)\int_0^t \frac{d\eta}{\rho^2(\eta)} , \qquad (3.25)$$

The coherent states for the Hamiltonian eq. *(3.9)* are given by [1,9]

$$|\alpha,t>_S = \exp(-|\alpha|^2/2)\sum_{n=0} \frac{\alpha^n}{(n!)^{1/2}} \exp\left(i\alpha_n(t)\right)|n,t> \ . \qquad (3.26)$$

On the other hand, by using eq. *(3.14)* the eq. *(3.25)* is converted into the equation

$$\alpha_n(t) = -\left(n+\frac{1}{2}\right)\int_0^t \frac{f(\eta)}{x^2(\eta)}d\eta \ . \qquad (3.27)$$

Also, using eqs. *(3.8)* and *(3.14)* the operator in eq. *(3.17)* are converted into the form

$$b(t) = \left(\frac{1}{2\hbar}\right)^{1/2}\left(q/x + i(xp - f^{-1}\dot{x}q)\right) \qquad (3.28a)$$

$$b^+(t) = \left(\frac{1}{2\hbar}\right)^{1/2}\left(q/x - i(xp - f^{-1}\dot{x}q)\right) \qquad (3.28b)$$

These operators are a generalization of the operators originally introduced by Lewis[10] to factor the invariant eq. *(3.11)* as eq. *(3.20)* in the first exact quantum treatment of the harmonic oscillator with a time-dependent frequency. Thus, the states $|\alpha, t>_S$ are coherent states for the time-dependent Hamiltonian eq. *(3.1)* with an exact invariant given by eq. *(3.16)*. Notice that for $f(t) = 1$ the states eq. *(3.26)* become the coherent states constructed by Hartley and Ray.

The coherent states $|\alpha,t>_S$ are eigenstates of the operator $b(t)$ the eigenvalue $\alpha(t)$:

$$b(t)|\alpha,t>_S = \alpha(t)|\alpha,t>_S \ , \qquad (3.29)$$

where

$$\alpha(t) = a\exp(2i\alpha_0(t)), \qquad (3.30)$$

$$\alpha_0(t) = -\frac{1}{2}\int_0^t \frac{f(\eta)}{x^2(\eta)}d\eta \qquad (3.31)$$

Also, these states can be created from then oscillator ground states by the unitary displacement operator $D(\alpha) = \exp[\alpha a^+ - a^* a]$ using the same procedure of ref. 1.

From eqs.(3.20) and (3.29) we obtain the expectation value of $I(t)$

$$_S < \alpha, t|I(t)|\alpha, t >_S = \hbar(|\alpha|^2 + 1/2) \ . \tag{3.32}$$

After some calculation, using eqs. (3.8), (3.14), (3.19), (3.29) and the well-known tricks, we find that the uncertainties in q and p in the state $|\alpha, t >_S$ are

$$(\Delta q)^2 = \frac{\hbar}{2} x^2 \ , \tag{3.33a}$$

$$(\Delta p)^2 = \frac{\hbar}{2}(\dot{x}^2 f^{-2} + 1/x^2) \ . \tag{3.33b}$$

Thus, the uncertainty product is expressed as

$$(\Delta q)(\Delta p) = \frac{\hbar}{2}(x^2 \dot{x}^2 f^{-2} + 1)^{1/2} \tag{3.34}$$

and, in general, does not attain its minimum value.

Next, using eqs. (38a), (3.14), (3.19a) and (3.29), we find that the expectation value of q in the state $|\alpha, t >_S$ is given by

$$_S < \alpha, t|q|\alpha, t >_S = (2\hbar|\alpha|^2 x^2)^{1/2} \sin[\phi(t) + \delta] \ , \tag{3.36}$$

where

$$\phi(t) = -2\alpha_0(t) = \int_0^t \frac{f(\eta)}{x^2(\eta)} d\eta \tag{3.37}$$

and 6 is the argument of the complex number a. Here we observe that the results eqs. (3.33) and (3.36) can also be obtained from eqs. (3.28) and (3.29). On the other hand, it is known[11] that the solution to the equation of motion for the classical time-dependent harrnonic oscillator

$$\ddot{q}_{c1} + \gamma(t)\dot{q}_{c1} + \omega^2(t) q_{c1} = 0 \tag{3.38}$$

can be expressed as

$$q_{cl} = A_0 x(t) \sin[\phi(t) + \delta] \ , \tag{3.39}$$

where A_0 is a constant and $x(t)$ satisfies the auxiliary eq.(3.15). In this case, the invariant $I(t)$ is defined in the same way as eq. (3.16) but using classical variables. Notice that eq. (3.39) provides a physical interpretation for $x(t)$ and $\phi(t)$ as quantities related to the amplitude and the phase of the time-dependent oscillator eq.(3.38). Now, as we have already mentioned in the last paragraph os sec. 2, the classical limit is obtained from taking $\hbar \longrightarrow 0$, $|\alpha| \longrightarrow \infty$, such that $\hbar|\alpha|^2 \longrightarrow$ finite. Thus, the expectation value of q in eq. (3.36) is exactly the solution for a classical time-dependent oscillator with invariant (see eq. (3.32)) $\hbar|\alpha|^2 = {}_S\!< \alpha, t|I(t)|\alpha, t >_S - \frac{1}{2}\hbar$.

In finishing this section, we remark that for $f(t) = 1$ all our results reduce to those of Hartley and Ray[1]. Clearly, all of the properties of the coherent states constructed in ref.1, are possessed also by the coherent states $|\alpha, t>_S$ given in eq. (3.26).

4. Squeezed states

In this section, we wish to show that the time-dependent coherent states $|\alpha, t>_S$ constructed in the previous section are equivalent to the so-called squeezed states[12-17]. To show this let us consider the operators $b(t)$ and $b^+(t)$ given by eq. (3.28). It is easy to verify that these operators can be expressed in terms of the operators a and a+ (see eq.(2.2)). In fact, using eqs. (2.2) and (3.28) we can write $b(t)$ and $b^+(t)$ as

$$b(t) = \mu(t)a + \nu(t)a^+ \ , \tag{4.1a}$$

$$b^+(t) = \mu^*(t)a + \nu^*(t)a^+ \ , \tag{4.1b}$$

where

$$\mu(t) = \left(\frac{1}{4\omega_0}\right)^{1/2}\left(\frac{1}{x} - if^{-1}\dot{x} - x\omega_0\right) \ , \tag{4.2a}$$

$$\nu(t) = \left(\frac{1}{4\omega_0}\right)^{1/2}\left(\frac{1}{x} - if^{-1}\dot{x} + x\omega_0\right) , \tag{4.2b}$$

Also, a straightforward calculation shows that the complex *c*-numbers $\mu(t)$ and $\nu(t)$ satisfy the relation

$$|\mu|^2 - |\nu|^2 = 1 . \tag{4.3}$$

Thus, from eqs.(3.29), (4.1a) and (4.3) we see that the coherent states $|\alpha, t>_S$ are, by definition, equal to the well-known squeezed states[12-17]. The properties of the states have been studied in detail by some authors[12,16].

On the other hand, it is known that the uncertainties in q and p for a squeezed state are given by[12,17,18]

$$(\Delta q)^2 = \frac{\hbar}{2\omega_0}|\mu - \nu|^2 , \tag{4.4a}$$

$$(\Delta p)^2 = \frac{\hbar}{2\omega_0}|\mu + \nu|^2 , \tag{4.4b}$$

whence

$$(\Delta q)(\Delta p) = \frac{\hbar}{2}|\mu + \nu||\mu - \nu| . \tag{4.5}$$

From eq. (4.5) we see that the squeezed states, in general, are not minimum-uncertainty states[12,17]. Now, the uncertainty produc eq. (4.5) is minimized if $\mu = r\nu$ for r real (see refs. 12, 13 and 18). Also, notice that the relations (4.4) and (4.5) are equivalent to eqs. (3.33) and (3.34).

Therefore, from the arguments presented above we see that the coherent states $|\alpha, t>_S$ for the time-dependent harmonic oscillator eq. (3.1) are equivalent to the well-known squeezed states. Here, we remark that these states are also known as two-photon coherent states in quantum-optics literature[12,15].

5. Concluding remarks

In this paper we have used a time-dependent canonical transformation, which can be implemented as a unitary change of representation[8], an auxiliary time-dependent transformation and the procedure developcd in ref. *1* to construct coherent states for the time-dependent system described by the Hamiltonian eq. *(3.1)*. These coherent states have been expressed in terms of the eigenstates of the invariant eq. *(3.16)* and are more general than those of ref. *1.* Furthermore, we have shown that these states are equivalent to the well-known squeezed states which have recently been the focus of considerable attention, specially owing to its prospective application in quantum optics. Also, we have obtained a natural generalization of the operators originally introduced by Lewis[10].

The technique developed here can also be applied to other time-dependent systems. As an example, we consider the system described by thc Hamiltonian

$$H(t) = f(t)\frac{p^2}{2} + f^{-1}(t)\frac{\omega^2(t)}{2}q^2 + \frac{f(t)}{x^2}g(q/x) \ , \qquad (5.1)$$

which possesses an invariant given by[7]

$$I(t) = \frac{1}{2}\left[(x - f^{-1}\dot{x}q)^2 + (q/x)^2 + 2g(q/x)\right] \ , \qquad (5.2)$$

where $x(t)$ satisfies the auxiliary eq. *(3.15)*. Then, following the same steps as those of sec. *3,* we convert the Hamiltonian eq. *(5.1)* into the form

$$H_1(t) = \frac{P^2}{2} + \frac{\Omega^2(t)}{2}Q^2 + \frac{1}{\rho^2}g(Q/\rho) \ , \qquad (5.3)$$

where $\Omega(t)$ is given by eq. *(3.10)* and $\rho(t)$ satisfies eq. *(3.13)*. The invariant eq. (5.2) is converted into the form

$$I(t) = \frac{1}{2}\left[(P\rho - \dot{\rho}Q)^2 + (Q/\rho)^2 + 2g(Q/\rho)\right] \ . \qquad (5.4)$$

Now, Ray[19] constructed coherent states for time-dependent systems described by Hamiltonians of the form eq. *(5.3)*. Thus, it seems that it would not be any problem to construct coherent states for the time-dependent systems associated with eq. (5.1) using the same technique presented in this paper and that of ref. 19. Work in this direction is in progress.

Finally, we mention that Dodonov and Man'ko[20] have constructed coherent states for a special case of the Hamiltonian eq. (3.1) with $f(t) = \exp[-2\Gamma(t)]$. However, the approach used by Dodonov and Man'ko is considerably different from the one presented in this paper. Moreover, they have considered linear invariants and have not expressed their states in terms of the eigenstates of the invariant. Also, those authors have not obtained the relationship to the classical motion.

As a concluding remark we wish to point out that Hamiltonians of the form eq.(3.9) are of physical relevance. In fact, the standard technique for generation of squeezed states, i.e., parametric amplification corresponds to the Hamiltonian eq. (3.9) with a frequency $\Omega(t) = \Omega_0^2[1+2\sin(2\Omega_0 t)]$. For a detailed discussion on parametric amplification (theoretical and experimental) see refs. 22 and 23.

The author wishes to thank the referee for supplying the contents of the last paragraph of sec. 5.

References

1. J.G. Hartley and J.R. Ray, Phys. Rev. D25, 382 (1982).
2. H.R. Lewis, Jr. and W.B. Riessenfeld, J. Math. Phys. 10, 1458 (1969).
3. R.J. Glauber, Phys. Rev. 131, 2766 (1963).
4. M.M. Nieto and L.M. Simmons, Jr., Phys. Rev. D20, 1321 (1979).
5. J.R. Klauder and B.-S. Skagerstam, ***Coherent States: applications in Physics and Mathematical Physics*** (World Scientific, Singapore, 1985).
6. E. Kanai, Prof. Theor. Phys. 3, 440 (1948); P. Caldirola, Nuovo Cimento 18, 393 (1941); see also H. Dekker, Phys. Rep. 80, 1 (1981).
7. I.A. Pedrosa, J. Math. Phys. 28, 2662 (1987).
8. H. Gzyl, Phys. Rev. A27, 2297 (1983).
9. J.M. Cervero and J. Villarroel, J. Phys. A17, 2963 (1984).
10. H.R. Lewis, Jr., Phys. Rev. Lett. 18, 510 (1967).
11. D.C. Khandekar and S.V. Lawande, J. Math.Phys. 20, 1870 (1979).

12. H.P. Yuen, Phys. Rev. A13, 2226 (1976).
13. A.K. Rajagopal and J.T. Marshall, Phys. Rev. A26, 2977 (1982); see also M. Schubert and W. Vogel, Phys. Rev. A28, 3668 (1983).
14. D.F. Walls, Nature 306, 141 (1983).
15. R.A. Fisher, M.M. Nieto and V.D. Sandberg, Phys. Rev. D29, 1107 (1984).
16. M.M. Nieto, Los Alamos National Laboratory Report No. LA-UR-84-2773, 1984, presented at the NATO Advanced Study Institute: Frontiers of Nonequilibrium Statistical Mechanics (unpublished).
17. H.A. Haus and Y. Yamamoto, Phys. Rev. A34, 270 (1986).
18. I.A Pedrosa, Phys. Rev. D36, 1279 (1987).
19. J.R. Ray, Phys. Rev. D25, 3417 (1982).
20. V.V. Dodonov and V.I. Man'ko, Phys. Rev. A20, 550 (1979).
21. S. Howard and S.K. Roy, Am. J. Phys. 55, 1109 (1987).
22. B.R. Mollow and R.J. Glauber, Phys. Rev. 160, 1076 (1967); 160, 1097 (1967).
23. Squeezed States of the Electromagnetic Field, Journal of the Optical Society of America 4, (10), 1987.

Resumo

Hartley and Ray têm construído e estudado estados coerentes para o oscilador dependente do tempo. Aqui mostramos como construir estados coerentes para sistemas dependentes do tempo mais gerais. Também mostramos que estes estados são equivalentes aos bem conhecidos estados comprimidos.