Revista Brasileira de Física, Vol. 19, nº 3, 1989

Angular momentum quasiparticle approach to cluster states in nuclei and quasi-free reactions

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Received January 27, 1989

Abstract Quasi-free $(\vec{p}, 2p)$ reactions at medium energies (with polarized incident protons) are considered. In this context, a simple and rather direct method to extract informations about cluster correlations in nuclei is presented. It is shown that, if the initial nucleus has a non-vanishing spin, the quasi-free effective polarization for a knock-out leading to a given final state can be expressed as a linear combination of effective polarizations defined on the basis of an angular momentum quasiparticle approach to cluster states in nuclei. Analyzing the effects of the nuclear medium on the quasi-free effective polarization, some information about nuclear structure - particularly that associated with orbital angular momenta and/or cluster correlations in nuclei - may be obtained.

1. Introduction

Numerous experimental and theoretical investigations on quasi-free nucleon-nucleus reactions, performed in the last three decades, have resulted in a large body of information about binding energies and momentum distributions of nuclear nucleons, widths of one-hole states and spin-orbit splittings of nuclear single-particle shells'. This considerable amount of work has established, in a very consistent way, the basic conceptual framework of quasi-free processes, i.e., the understanding of the reaction mechanism, the limitations of the Distorted Wave Impulse Approximation (DWIA) and of the Factorization Approximation, the role of the residual

Supported in part by Conselho Nacional de Desenvolvimento Científico e Tecnológico (CNPq, Brasil) and by Fundação de Amparo à Pesquisa no Estado do Rio Grande do Sul (FAPERGS, Brasil).

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interactions in the initial and final states and the properties of the states of the final nucleus.

Quasi-free nucleon-nucleus scattering at medium energies emerged from these studies as one of the most important tools for investigating the single-particle properties of a nucleus (its shell structure), in special for the most strongly bound states, and effects of the nuclear environment on the short range structure of the bound nucleons.

With the advent of quasi-free processes with incident polarized particles a new dimension in the investigation of the nuclear structure was added. Theoretical predictions[2], which have been later on experimentally confirmed[3,4,5,6], have shown that the nuclear nucleons which is knocked-out in these processes was in general (effectively) polarized in the nucleus. In recent years, considerable progress has been made in this subject both on the theoretical and experimental domains. Experimentally, intense polarized medium-energy proton beams with a good duty cycle have become available and have been used at TRIUMF* [5] in $(\vec{p}, 2p)$ reactions and also precise (p, pn) scatterings have become possible[7]. The most extensive $(\vec{p}, 2p)$ measurements have been made for the nuclei ^{16}O and ^{40}Ca at 200 MeV[3,4,5] and quite recently for ^{40}Ca at 300 MeV[1]. In these experiments, the measured asymmetries have clearly confirmed a large (effective) polarization of the nucleons in the nucleus. The matrix element for the scattering of two protons is strongly spin dependent - by using a polarized proton beam this effective polarization may be measured in $(\vec{p}, 2p)$ reactions. Theoretically, the exploitation of the spin[2] and isospin[8,9] degrees of freedom has been shown to eliminate many uncertainties in the description of this kind of reactions. This allows one to detect, for instance, the influence of the nuclear medium on the short-range structure of a nucleon.

The calculations are most often based on the single-particle shell model and the Distorted Wave Impulse Approximation (DWIA)[11]. The Impulse

* Quasi-free experiments with polarized protons were also performed in Dubna[6] at 635 MeV on ^{5}Li and ^{40}Ca. In these experiments appreciable asymmetries, compatible with estimates in the DWIA, have been observed. The energy resolution of the experiments did not allow, however, for a clear separation of single- hole states. These states in ^{16}O and ^{40}Ca were resolved in the TRIUMF experiments for particular kinematical conditions.

Approximation[2] states that, if the energy of the incoming proton and the momentum transfer to the nuclear target-nucleon are sufficiently large, the dynamical influence of the other nucleons in the nucleus on the knock-out process itself can be neglected. This approximation then assumes that the rest of the nucleons play the role of "passive spectators" during the process of ejection of a nuclear nucleon. Of course, this assumption would be completely wrong if, for instance, the target-nucleon is strongly correlated to another nucleon in the nucleus which could, in this case, also be ejected. This latter process corresponds, however, to a two-hole final state and does not contribute to the single-hole quasi-free spectrum, only to its background. The same occurs, of course, for more complex long-range correlations between the target-nucleon and the environment. And, in fact, investigations[13] have shown that the short-range effects are dominant in quasi-free reactions. The rest of the nucleus has, however, a significant influence on the dynamics of the knock-out process due to multiple scatterings of the incident and emerging particles by the nuclear nucleons. These effects are taken into account in the DWIA. In this approximation, the wave functions of the incoming and outgoing protons are distorted by complex optical potentials[14,15,16]. As was pointed out in ref. 13, the distortion smears out the shapes of the angular correlations, naïvely expected from the momentum distributions in the various shells, and also reduces the intensity. The smearing out of the angular correlations (provided by the real part of the optical potentials) corresponds to diffraction processes of the wave functions of the colliding particles in the mean field of the spectador nucleons. The reduction in the intensity of the angular correlations (from the imaginary or absorptive part of the optical potentials) reflects inelastic multiple scattering out of the quasi-free channel. The intensity reduction caused by absorption can be one order of magnitude. Fortunately, the large number of inelastically scattered particles do not upset the "quasi-free" spectrum because their energies and momenta are spread over a large energy range originating, in special for the low momentum components, a smooth background in the summed energy spectra[13]. For the ^{16}O and ^{40}Ca nuclei, in experiments in geometries for which the on-shell DWIA is expected to be a good approximation, the measured effective polarizations are in reasonable agreement with the theoretical expectations.

In this note we consider quasi-free reactions and the cluster model description of nuclear states. We shall see that if the inifial nucleus has a nonvanishing spin, we can express quasi-free the effective polarization for a knock-out leading to a given final state as a linear combination of effective polarizations defined on the basis of an angular momentum quasiparticle approach to cluster states in nuclei, weighted by distorted momentum distributions of the nuclear nucleons. By determining the quasi-free effective yolarization, which strongly depends on spin correlations, details of the nuclear struciure may be investigated. An interesting example of the nuclear structure information contained in the effective polarization is given by the quasi-free reaction on a nucleus that is modeled as consisting of an inert core surrounded by a deuteron cluster; such models have often been used for ^6Li, ^{14}N and other nuclei. The triplet spin correlation in the deuteron results in an effective polarization that has the opposite sign to the one expected for the single-particle shell-model description of the same nucleus[20]. Therefore the observation of the asymmetry in such a case might help to shed light on the two models.

2. Review of the theory

In the following we assume an experimental situation where the DWIA is applicable. In this case, the cross section is given by*[13]

$$\frac{d^6\sigma_{fi}}{dE_1\,d\Omega_1\,dE_2\,d\Omega_2} = \frac{4}{(\hbar c)^2}\,\frac{k_1 k_2 \bar{E}_0}{k_0 E_3}\cdot|g'(\vec{k}_3)|^2\cdot\frac{d\sigma^{fr}}{d\bar{\Omega}}(T,\bar{\Theta},P_0P_3)$$
$$\cdot\,\delta(E_1+E_2+E_{A-1}-E_0-E_A)\;. \qquad (1)$$

* **The DWIA produces a drastic reduction of the unmanageable number of the nuclear many-body degrees of freedoni, but we still have to make an additional approximation to obtain the factored form of eq.(1). In the DWIA, we arrive at a two-body scattering-matrix element taken between the (distorted) wave functions of the initial and final states. The distortion introduces new momentum cornponents into the scattering states in addition to the asymptotic mornenta**

Above and further on, we use the following notation: 0, 1, 2, and 3 refer, respectively, to the incoming particle (0), to the two outgoing particles (1 and 2), and to the particle in the nucleus that is subsequently knocked-out (3). The symbols i and f (or A and $A-1$) indicate quantities of the initial nucleus (in its ground state) and the final nuclear system of $A-1$ nucleons. E denotes the full energy, T the kinetic energy, and $\vec{k}$ is a momentum. The barred quantities are taken in the centre of mass system of the nucleons 1 and 2. The selection of the final nuclear state is experimentally achieved (via the energy conserving 6 function) by fixing E, $+ E_2$. The centre of mass cross section for the free scattering of the nucleons 0 and 3, with their actual momenta and polarizations in the laboratory system, is dexioted by $d\sigma^{fr}/d\bar{\Omega}(T,\bar{\Theta},P_0,P_3)$.*

The momentum of particle 3 is taken to be $\vec{k}_3 = \vec{k}_1 + \vec{k}_2 - \vec{k}_0 = -\vec{k}_{A-1}$ (see fig. 1). The momentum distribution, $|g'(\vec{k}_3)|^2$, is given by

$$|g'(\vec{k}_3)|^2 = \frac{1}{2j_i+1} \sum_{m_f m_i} \sum_{\mu} |g'^{\mu}_{m_f m_i}(\vec{k}_3)|^2 \ , \tag{2}$$

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and spins of the particles. If the value of the two-body matrix element is smooth compared to the nuclear parts of the full matrix element, it is reasonable to consider this asymptotic value, as a good average and take it out of the overlap integral. This gives the factored form of eq.(1) which comprises the Factorization Approximation. This approximation then assumes that the two-body knock-out collision occurs with the finally observed momenta and spins. The factorization becomes questionable for outgoing nucleons with low kinetic energy, for pions under and above the A-resonance and in the regions where the undistorted momentum distributions are small (in a minimum or in a high-momentum tail). It would also become questionable if charge exchange and spin-flip were important effects in this kind of reactions. It seems, however, that these last effects produce only small corrections in quasi-free reactions. (See ref. 1 for a discussion of all these points.)

* Compared to the free-scattering of a nuclear nucleon we have in the energy-conserving delta function of eq.(1), instead of the free kinetic energy (nonrelativistically $k_3^2/2m$), the term $\delta' = -|E_{\text{Binding}}| + k_{A-1}^2/2M_{A-1}$. Since in the extreme single-particle model we take $\vec{k}_{A-1} = -\vec{k}_3$, the energy is, therefore, off-shell by an amount A,, $=$ 6' $- k_3^2/2m \propto k_3^2$, bringing a certain arbitrariness in the choice of the free cross section $d\sigma^{fr}/d\bar{\Omega}$. Studies[1] have shown that for not too large momentum components, off-shell effects are usually of no great importance for nucleon-nucleon scattering. For quasi-free $(e,$e'p) scattering the strong energy and momentum dependence of the matrix element makes an unfactored calculation desiderable.

where the $g'^{\mu}_{m_f m_i}(\vec{k})$ represent distorted momentum amplitudes and μ the spin projections $(\mu = +or-)$. In the language of quantum field theory the distorted momentum amplitude may be written in the form

$$g'^{\alpha}_{m_f m_i}(\vec{k}) = (2\pi)^{-3/2} \int \exp(-i\vec{k}\cdot\vec{r}) D_{fi}(\vec{r}) < A-1; j_f m_f | a^{\alpha}(\vec{r}) | j_i m_i; A > \mathrm{d}^3 r. \tag{3}$$

In this expression $|j_i m_i; A >$ and $|j_f m_f; A-1>$ denote, respectively, the wave functions of the initial and final nuclear states characterized, respectively, by total angular momenta j_i and j_f and their projections m_i and m,; the distortion function $D_{fi}(\vec{r})$ represents the effects of multiple scattering of the incident and emerging particles from the rest of the nucleus (the symbols f and i indicate that the distortion depends on the selected geometry); the quantity $a^{\alpha}(\vec{r})$ is the annihilation operator annihilating a nucleon at position $\vec{r}$ and relevant quantum numbers denoted by a. (All angular momenta are quantized orthogonally to the scattering plane.)

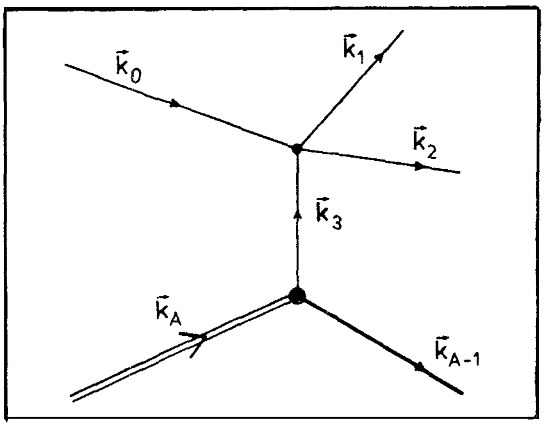

Fig.1 - First order diagram for a quasi-free process.

We introduce the function $D_{fi}(\vec{k}-\vec{k}')$ and the operator $a''(\vec{k})$ as the Fourier transforms of $D_{fi}(\vec{r})$ and $a^{\alpha}(\vec{r})$, respectively. With these definitions, the distorted momentum amplitude (eq.(3)) may be expressed in the form

$$g'^{\alpha}_{m_f m_i}(\vec{k}) = \int D_{fi}(\vec{k} - \vec{k}') < A-1; j_f m_f |a^{\alpha}(\vec{k}')| j_i m_i; A > d^3k' \quad . \tag{4}$$

In this note we shall consider $(\vec{p}, 2p)$ quasi-free processes in a coplanar geometry. In this case the axial vector effective polarization, which is orthogonal to the scattering plane, is given by[10,13]

$$P_{eff}(\vec{k}) = \frac{\sum\limits_{m_f m_i} (|g'^{(+)}_{m_f m_i}(\vec{k})|^2 - |g'^{(-)}_{m_f m_i}(\vec{k})|^2)}{\sum\limits_{m_f m_i} \sum\limits_{\mu} |g'^{\mu}_{m_f m_i}(\vec{k})|^2} \tag{5}$$

As was pointed out in ref. 1, the effective polarization is caused by the fact that the kinematical conditions of the experiment, through the distortion, may destroy the isotropy of the momentum overlap function $g'^{\mu}_{m_f m_i}(\vec{k})$ with respect to μ. (See also the discussion following expression (9) below.) The quasi-free scattering appears to be performed on a proton with momentum $\vec{k}_3$ and spin wave function $\sum g'^{\mu}_{m_f m_i}(\vec{k}) S^{\mu}$. The expectation value of the polarization of such an ensemble of protons, obtained by summing over the orientations of the initial and final nuclei, is given by eq. (5). The factorization assumption implies, in particular, that the spin-orbit part of the optical potential is neglected; explicit calculations[17] have shown that this is a good approximation in many case, in particular, at the maxima of the distorted-momentum distributions. In this case the effective polarization of the knocked-out nucleon can be determined by measuring the asymmetry

$$A = \frac{d\sigma^{(+)}/d\Omega - d\sigma^{(-)}/d\Omega}{\sigma^{(+)}/d\Omega + d\sigma^{(-)}/d\Omega} \ , \tag{6}$$

where the + and - signs indicate the direction of polarization of the incoming beam and $d\sigma^{(\pm)}/d\Omega$ is a short notation for $d^6\sigma^{(\pm)}/dE_1 d\Omega_1 dE_2 d\Omega_2$.

The free nucleon-nucleon cross section may be written as[18]

$$d\sigma^{fr}/d\bar{\Omega}(T, \bar{\Theta}, P_0, P_3) = I_0(T, \bar{\Theta})(1 + (P_0 + P_3)P(T, \bar{\Theta}) + P_0 P_3 C_{NN}(T, \bar{\Theta})), \tag{7}$$

where P_0 and P_3 are the initial polarizations of the collising protons and $I_0(\mathrm{T},\tilde{\mathrm{O}})$, and $C_{NN}(\mathrm{T},\bar{\Theta})$ are the usual functions from phase shift analysis[18]. Substituting eq.(7) into eq.(1) and the result on eq.(6), the distorted-momentum distribution and the kinematic factors drop out and we find

$$A = \frac{P(T,\bar{\Theta}) + P_3 C_{NN}(T,\bar{\Theta})}{1 + P_3 P(T,\bar{\Theta})}, \tag{8}$$

$$P_{eff}(\vec{k})(=P_3) = \frac{\mathrm{A} - P(T,\bar{\Theta})}{C_{NN}(\mathrm{T},\bar{\Theta}) - AP(T,\bar{\Theta})} \,. \tag{9}$$

The effective polarization can be in general quite large. This can be seen easily in a simple geometrical picture[2] which is similar to one applied earlier to deuteron stripping[19] (see fig. 2). Consider, as an example, the quasi-free knocking-out of a pshell proton of ^{18}O in a nonsymmetrical geometry. The total path in the nucleus of the nucleon from the lower event in fig. 2 is shorter than that of the upper one and, therefore, because of absorption, the former nucleon is more probable to be knocked-out than the latter. With the choice of $\vec{k}_3$ in fig. 2 the protons with clockwise angular momenta will contribute more to the process than the counterclockwise ones. If the protons is in a $p_{3/2}$ state its spin will be predominantly down; for a $p_{1/2}$ state proton, its spin will be mostly up. As a result, the knocked-out proton will have an effective polarization that is opposite for the $j = 1/2$ and the $j = 3/2$ states. This is a very simple sheli model prediction; from expression (3) it is clear that for a given geometry, the effective polarization (see expressions (4) and (5)) depends on subtle details of the structure of the nucleus.

In general, we may test any model for the initial and final nuclear states by calculating the expected quasi-free cross sections for polarized protons and by comparing them with experiment. Though the normalizations of these cross sections are rather uncertain, their shapes and, in particular, the measured asymmetries can be quite reliable and characteristic.

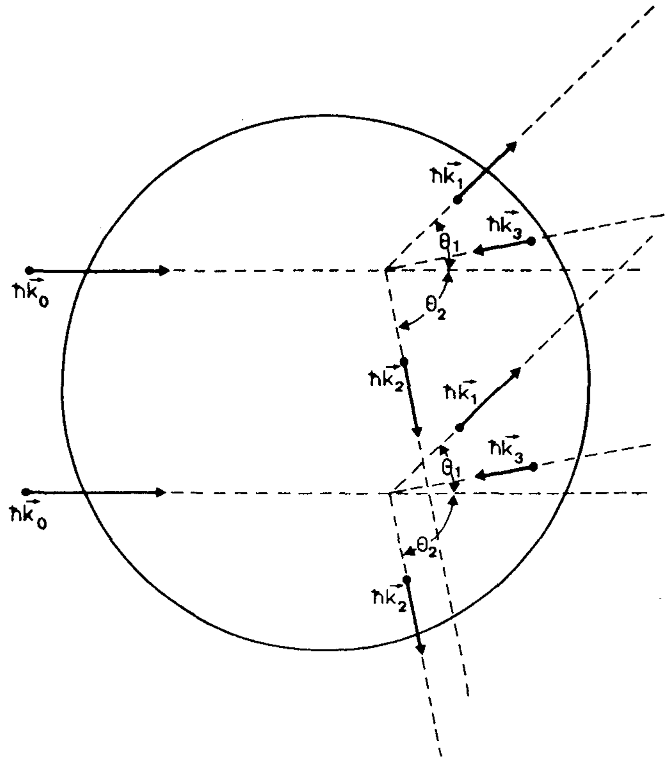

Fig.2 - **The qualitative explanation of the effective polarization.**

3. Quasi-free reactions and the single-particle shell model

In this section we basically follow ref. 21. We shall first consider the simplest case of a knock-out from a closed shell, such as in the reaction $^{16}O(\vec{p},2p)\ ^{15}N$. In this case the extreme single particle shell model should be a good starting point and we choose a suitable shell model potential to expand a" $(\vec{k})$ in terms of its eigenfunctions in the following form

$$a^{\alpha}(\vec{k}) = \sum_{n\ell jm} g_{n\ell jm}(\vec{k}) a^{\mu}_{n\ell jm}\,,$$
$$j = \ell \pm 1/2, \quad \mu = + \text{ or } - \,. \tag{10}$$

Each operator $a_{n\ell jm}$ now annihilates a proton in the shell model state characterized by the quantum numbers, n, ℓ, j and m.

In the following we assume that for an optimal choice of the shell model potential and for a certain j_f-value of the final nucleus, only one value n in expression (10) contributes to the'overlap integral

$$< A-1; j_f m_f | a^{\pm}(\vec{k}) | j_i = m_i = 0; A > , \tag{11}$$

in expression (4). We then can show that expression (2) reduces to

$$\sum_{m_f} |g'^{\mu}_{m_f m_i=0}(\vec{p})|^2 = \frac{|\gamma_j^0|^2}{2j+1} \sum_m \left| \int D_{f_i}(\vec{p}-\vec{k}) g'^{\mu}_{jm}(\vec{k}) d^3k \right|^2 . \tag{12}$$

Of course, nuclear correlation will lead to multiple particle excitations and depth the pure one-hole states. We represent this efect by the real factor $|\gamma_j^0|/(2j+1)^{1/2} \leq 1$, which we suppose to be state independent for each shell. In case the initial nucleus is not closed but has a vanishing spin the reduction factor γ_j^0 is to be replaced by γ_j , which in addition contains the fractional parentage coefficient for the probability of finding the residual nucleus in the ground state of the initial one.

For an initial nucleus with a non-vanishing spin we obtain for the effective polarization

$$P_{eff}(\vec{k}) = \frac{\sum_j |\gamma_j|^2 |g'_j(\vec{k})|^2 P_j(\vec{k})}{\sum_j |\gamma_j|^2 |g'_j(\vec{k})|^2} , \tag{13}$$

with

$$\begin{aligned} < j_f m_f | a_{jm} | j_i m_i > &= < j_f \parallel a_j \parallel j_i > C(j_f j; m_f m | j_i m_i), \\ &= \bar{\gamma}_j C(j_f j; m_f m; | j_i m_i), \end{aligned} \tag{14}$$

and $|\gamma_j| = |\bar{\gamma}_j|/(2j+1)^{1/2}$. In expression (13), $P_j(\vec{k})$, which can be calculated from eqs. (5) and (6), is the effective polarization caused by a,,; furthermore, $|g'_j(\vec{k})|^2$ is the distorted momentum distribution of the j-subshell normalized to the occupation number $2(2j+1)$. From this expression it follows that measurements of P_{eff} (and of $P_j(\vec{k})$ and $|g'_j(\vec{k})|^2$) in nearby zero-spin nuclei will give information on the $|\gamma_j|^2$.

From the geometrical picture of fig. (2), it is expected that nucleons ejected from two one-particle states differing only by their spin-orbit couplings (for example, the $p_{3/2}$ and $p_{1/2}$ states in ^{16}O) should have opposite effective polarizations. We may derive a quantitative relationship hetween these effetive polarizations by the following simple argument. We assume, as in the preceding, that the distortion has no spin-orbit components and, furthermore, that the effects of the nuclear binding energy on the effective polarization may be disregarded in a first approximation. In this case, for a spherical initial nucleus with vanishing spin we have, using a single particle basis with $j = \ell + 1/2$ and $j = \ell - 1/2$ and taking into account the occupation number of the (completely filled) j-subshells, N_j, with ℓ fixed,

$$\begin{aligned} P(\vec{k}) &= \sum_j N_j P_j(\vec{k}) \\ &= 2(2(\ell+1/2)+1)P_{\ell+1/2}(\vec{k}) + 2(2(\ell-1/2)+1)P_{\ell-1/2}(\vec{k}) \end{aligned} \tag{15}$$

and, since in this case $P(\vec{k}) = 0$, we obtain

$$P_{\ell+1/2}(\vec{k}) = -\frac{\ell}{\ell+1} P_{\ell-1/2}(\vec{k}) \ , \tag{16}$$

a result which is independent of any kinematics and distortion. If we now turn to the more realistic case of a non-spherical nucleus with a small spin-orbit splitting, the effective polarizations change only by modifications in the wave functions due to the difference in binding energies of the two subshells. Calculations have shown* that the effective polarizations are quite insensitive to such changes, and, therefore eq. (16) is still expected to be valid to a good approximation.

In fig. (3) we show some typical measured asymmetries for the knock-out from the two p-states of ^{16}O (the curves represent factored DWIA calculations'). Fig. 4 shows effective polarizations calculated with eq. (9)'. In the figure are plotted the values $P_{eff}(j = 1/2)$ and $-2P_{eff}(j = 3/2)$ which, according to eq.(16), should be equal. The agreement is excellent for the symmetrical angles, but only qualitative

* P.G.Roos, private communication, cited in ref.1.

for the asymmetrical ones. The deviation of the calculations from the experimental results may have a very simple and interesting interpretation: it seems that the nucleon-nucleon cross section in the nuclear medium is modified in such a way that $P(T,\bar{\Theta})$ (see expression *(7)*) is reduced to approximately zero. (See also ref. 1 for a more complete discussion of these points.)

4. Angular momentum quasiparticle approach to cluster states in nuclei

A very important question now arises: how should expression (13) change in a cluster model for the nucleus? To answer this question we proceed as follows.

The annihilation operator for an A nucleons state, defined in terms of a single-particle $[jm]$ shell-model basis, is given by

$$a_{jm} = \sum_{1\ldots A-1} |1,...,A-1><jm;1,...,A-1|, \tag{17}$$

with $\eta = [j_\eta m_\eta]$ for $\eta = 1...A-1$.

We assume for simplicity that the motion of the A nucleons is dinamically correlated in such a way that their behaviour is described by a two-cluster model (the extension of the following discussion to more than two clusters can be done in a very easy and direct way). Furthermore, we require that this model be independent of the motion of the total centre-of-mass (this avoids non-physical (spurious) collective excitations of our system). To garantee translational invariance, we introduce a more consistent representation, namely a cluster model basis, defined in terms of internal and relative motion coordinates (see, for example, ref. 22 for the usual definitions of these new coordinates). In this representation, the internal motions of the clusters 1 and 2 are characterized by the quantum numbers j_{ρ_k} and m_{ρ_k} (with the k's ranging from 1 to ξ (=number of internal coordinates) for the cluster 1 and from $\xi+1$ to $A-3$ for the cluster 2 (recall that the new coordinates

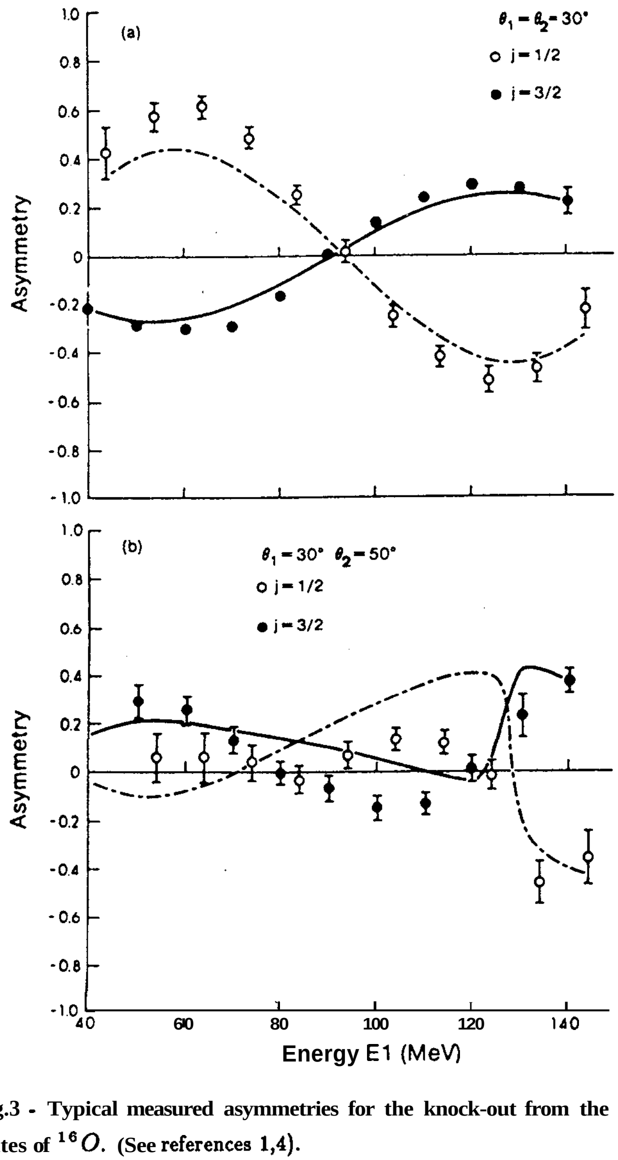

Fig.3 - Typical measured asymmetries for the knock-out from the two p states of ^{16}O. (See references 1,4).

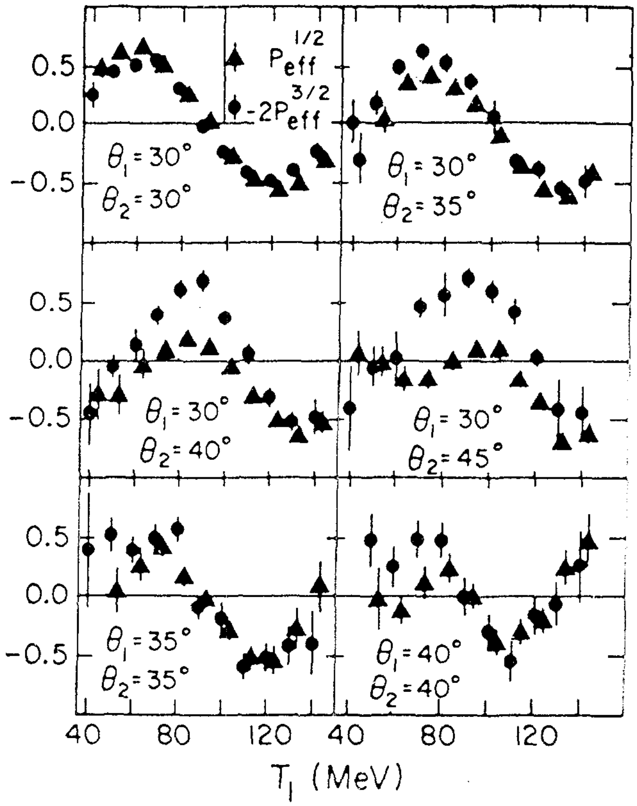

Fig.4 - Effective polarirations for the reaction $^{16}O(\vec{p},2p)^{15}N$ at 200 MeV calculated with eq.(16)[1]. (Error bars arise from errors in experimental measurements of asymmetry.)

are not linearly independent)) and the relative motion by the quantum numbers j_R and m_R.*In this model the wave function of the initial state is then given by

$$|\Psi_i> = \sum_{j_{\rho_i} m_{\rho_i}} \sum_{j_{R_i} m_{R_i}} C(j_{\rho_i} m_{\rho_i}; j_{R_i} m_{R_i} | j_i m_i)$$
$$\cdot |\psi_{j_{\rho_i} m_{\rho_i}} [\varphi(1)\varphi(2)] \chi_{j_{R_i} m_{R_i}}(R) >, \tag{18}$$

* As was pointed out in ref. 23 the new quantum numbers are not really good ones if the wave functions are properly antisymmetrized. We require, however, for the sake of simplicity, that the clusters be so well separated that these quantum numbers maintain a literal real meaning in the limits of the assumed separation. In this sence, they are asymptotically good quantum numbers.

where $\chi_{j_{R_i} m_{R_i}}(R)$ describes the relative motion of the two clusters, R denoting the relative coordinate between their centres-of-mass. The internal motion of the clusters is described by

$$|\psi_{j_{\rho_i} m_{\rho_i}}[\varphi(1)\varphi(2)]> = \sum_{j_{\rho_a} m_{\rho_a}} \sum_{j_{\rho_b} m_{\rho_b}} C(j_{\rho_a} m_{\rho_a}; j_{\rho_b} m_{\rho_b} | j_{\rho_i} m_{\rho_i})$$
$$\cdot |\varphi_{j_{\rho_a} m_{\rho_a}}(\rho, \rho_1 ... \rho_\epsilon)\varphi_{j_{\rho_b} m_{\rho_b}}(\rho_{\epsilon+1} ... \rho_{A-3}) >; \quad (19)$$

in this expression the symbols $j_{\rho_a} m_{\rho_a}$ and $j_{\rho_b} m_{\rho_b}$ characterize, respeciively, the total internal angular momenta and their projections for the clusters 1 and 2. The final state wave function reads

$$|\Psi_f> = \sum_{j_{\rho_f} m_{\rho_f}} \sum_{j_{R_f} m_{R_f}} C(j_\rho m_\rho; j_R m_R | j_f m_f)$$
$$\cdot |\psi_{j_{\rho_f} m_{\rho_f}}[\varphi(1')\varphi(2')]\chi_{j_{R_f} m_{R_f}}(R') >, \quad (20)$$

in which $|\chi_{j_{\rho_f} m_{\rho_f}}[\varphi(1')\varphi(2')]>$ can be expanded similarly as in eq. **(19).** In these equations the C's denote Clebsch-Gordan coefficients, coupling in a first step the internal angular momentum and projection quantum numbers of each cluster to give $j_{\rho_i} m_{\rho_i}$ and $j_{\rho_f} m_{\rho_f}$, respectively; in a second step coupling with the corresponding relative quantities $j_{R_i} m_{R_i}$ or $j_{R_f} m_{R_f}$, yielding $j_i m_i$ and $j_f m_f$, respectively.

Introducting this change of basis the a_{jm} operator (eq.(10)) may be written accordingly as

$$a_{jm} =$$
$$\sum[u_{jm} M^{jm}(\rho ... R')|\rho'_1 ... \rho'_\epsilon; \rho'_{\epsilon+1} ... \rho'_{A-3}; R' >< \rho, \rho_1 ... \rho_\epsilon; \rho_{\epsilon+1} ... \rho_{A-3}; R|$$
$$+ v_{jm} M_{jm}(\rho ... R')|\rho, \rho'_1 ... \rho'_\epsilon; \rho'_{\epsilon+1} ... \rho'_{A-3}; R' >< \rho_1 ... \rho_\epsilon; \rho_{\epsilon+1} ... \rho_{A-3}; R|] \, . (21)$$

In this expression, the sum extends from p, $\rho_1 ... \rho_{A-3}, R$ to $\rho'_1 ... \rho'_{A-3}$, R' and the ρ's and R's represent short notations for the quantum numbers $j_{\rho_k} m_{\rho_k}, j_R m_R$ and so on; the M^{im} are the transformation coefficients from the old basis to the new one; finally, the primes are remainders of the fact that the internal coordinates are not linearly independent whereas the $\sim$ indicate the usual time-reversed operators defined as $a_{J\tilde{M}} = (-1)^{J-M} a_{J,-M}$, with $J = j_{\rho_k}, j_R$ and $M = m_{\rho_k}, m_R$.

Inserting in this expression the unit operator defined in terms of the new coordinates and assuming that the A nucleon Hamiltonian is separable in these coordinates, we have, in a short notation,

$$a_{jm} = u_{jm} \mathrm{L}_{jm} + v_{jm} \mathrm{L}^{\dagger}_{jm}, \tag{22}$$

where

$$\mathrm{L}_{jm} = \sum_{j_\rho m_\rho} \sum_{j_R m_R} C(j_\rho m_\rho, j_R m_R | jm) \alpha_{j_\rho m_\rho} \otimes W_{j_R m_R} \tag{23}$$

In this latter expression, $\alpha_{j_\rho m_\rho}$ represents an effective annihilation operator annihilating a single p-quasiparticle with quantum numbers $j_\rho m_\rho$. This operator is defined in terms of an expansion of a single pquasiparticle annihilation operator coupled with a pairwise combination of the annihilation and creation operators for the $2(A-3)$ remaining p-quasiparticles of the initial and final states. The operator $W_{j_R m_R}$ on the other hand is defined by the angular momentum expansion of the annihilation and creation operators $W_{j_{R_i} m_{R_i}}$ and $W^{!}_{I_{R_f} m_{R_f}}$. Frorn these equations we see that we have expanded the one-particle annihilation operator a_{jm} as a set of new operaton, i.e., $\alpha_{j_\rho m_\rho}, a^{\dagger}_{j_\rho m_\rho}, W_{j_{R_i} m_{R_i}}$, and $W^{\dagger}_{j_{R_f} m_{R_f}}$ which in this model act on the internal and relative motion vectorial states, respectively, annihilating or creating states with definite angular momenta and projections. The new creation and annihilation operators are defined as linear combinations of the old creation and annihilation operators and the number of particles is not conserved. However, due to the structure of the initial and final states in our model (see expressions (18) and (20))) the resulting second quantization Hamiltonian, expressed in terms of this set,

$$\begin{aligned}
\mathcal{H} = &\sum_{\alpha\beta} (u_\alpha \mathrm{L}^{\dagger}_\alpha + v_\alpha \mathrm{L}_\alpha) < \alpha|\mathcal{T}|\beta > (u_\beta \mathrm{L}_\beta + v_\beta \mathrm{L}^{\dagger}_\beta) \\
&\frac{1}{2} \sum_{\alpha\beta\gamma\delta} (u_\alpha \mathrm{L}^{\dagger}_\alpha + v_\alpha \mathrm{L}_\alpha)(u_\beta \mathrm{L}^{\dagger}_\beta + v_\beta \mathrm{L}_\beta) < \alpha\beta|\mathcal{V}|\gamma\delta > \\
&(u_\delta \mathrm{L}_\delta + v_\delta \mathrm{L}^{\dagger}_\delta)(u_\gamma \mathrm{L}_\gamma + v_\gamma \mathrm{L}^{\dagger}_\gamma),
\end{aligned} \tag{24}$$

describes in the initial and final states the dynamical behaviour of one R– and, respectively, $A-2$ and $A-3$ p-quasiparticles. We could associate the so called R-quasiparticles with collective excitations in the nucleus. In this picture, the destruction of a nucleon in the conventional single- particle description could be interpreted, when translated to our cluster model, as the destruction of an interna1 pquasiparticle acompanied by a change in the collective excitation of the system. Since, in this picture the spin character is cai-ried over only by the pquasiparticles, the poperators are fernion-like and the corresponding R-quasiparticles are boson-like objects. In eq. (22), the coefficients u_{jm} and v_{jm} are assumed to be real and spherically symmetric. The condition $u_{jm}^2 + v_{jm}^2 = 1$ ensures, in particular, that the fermion-like quasiparticle annihilation and creation operators, L_{jm} and $L_{jm}^\dagger$, respectively, satisfy the anticommutation relation $[L_\alpha, L_\beta^\dagger]_+ = \delta_{\alpha\beta}$ as do the original single-particle operators a_{jm} and $a_{jm}^\dagger$.

Combining these definitions with expressions (5) and using the orthogonality properties for the Clebsch-Gordan coefficients and the Wigner-Eckart theorem we obtain an expression of the effective polarization similar to eq.(13) in the form

$$P_{eff}(\vec{k}) = \frac{\sum_{j_\rho} |\gamma_{j_\rho}|^2 [\sum_{j_R} |G'_{j_\rho j_R}(\vec{k})|^2] P_{j_\rho}(\vec{k})}{\sum_{j_\rho} |\gamma_{j_\rho}|^2 [\sum_{j_R} |G'_{j_\rho j_R}(\vec{k})|^2]}, \tag{25}$$

or

$$P_{eff}(\vec{k}) = \frac{\sum_{j_\rho} |\gamma_{j_\rho}|^2 |G'_{j_\rho}(\vec{k})|^2 P_{j_\rho}(\vec{k})}{\sum_{j_\rho} |\gamma_{j_\rho}|^2 |G'_{j_\rho}(\vec{k})|^2}, \tag{26}$$

In this expression the γ_{j_ρ} denote reduction parameters (reduced matrix elements) obtained from direct application of the Wigner-Eckart theorem to the matrix elements for the knocking-out of a ρ-quasiparticle (see eq.(14)); $G'_{j_\rho}(\vec{k})$ is the distorted momentum amplitude defined in the context of the cluster model. As the angular momenta and projections are coupled in the form:

$$\begin{aligned} \vec{j}_i &= \vec{j}_{\rho_i} + \vec{j}_{R_i}, \quad m_i = m_{\rho_i} + m_{R_i}, \\ \vec{j}_f &= \vec{j}_{\rho_f} + \vec{j}_{R_f}, \quad m_f = m_{\rho_f} + m_{R_f}, \end{aligned} \tag{27}$$

and since $\vec{j} = \vec{j}_i - \vec{j}_f$ together with $m = m_i - m_f$, we have

$$\vec{j} = \vec{j}_\rho + \vec{j}_R, \quad m = m_\rho + m_R, \tag{28}$$

with $\vec{j}_\rho = \vec{j}_{\rho_i} - \vec{j}_{\rho_f}, m_\rho = m_{\rho_i} - m_{\rho_f}, \vec{j}_R = \vec{j}_{R_i} - \vec{j}_{R_f}$ and $m_R = m_{R_i} - m_{R_f}$.

The comparison of P_{eff} calculations with expcrimrntal results could give information on the γ_{j_ρ} parameters. From the Wignor-Eckart thcorem (see expression (14)) we see that these coefficients are sensitive to the coupling of the wave functions and could give when compared to experiment, rather direct informations on cluster correlations in nuclei.

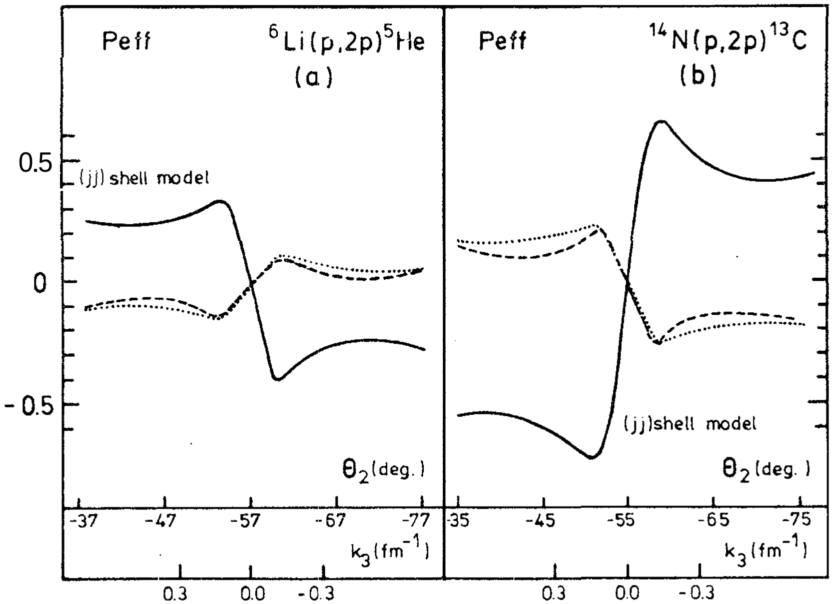

Fig.5 - Effective polarization calculated for the nuclear reaction $^6Li(\vec{p},2p)^5$He *(a)* and $^{14}C(\vec{p},2p)^{13}C$ *(b)* at a lab energy of 320 MeV. In the figure, the full lines correspond to the shell model, the dotted lines to the two-body cluster model; $\vec{k}$ is the linear momentum of the knocked-out proton and Θ is the scattering angle.

At this point it would be interesting to compare the results of calculations of the effective polarizations for the reactions $^{14}N(\vec{p},2p)^{13}C$ and $^{6}Li(\vec{p},2p)^{5}He$ (see fig. 5). Recent predictions from a two-body cluster model are compared with previous findings obtained for the single-particle shell model with **jj** coupling. In the shell model calculations we have used harmonic oscillator wave functions with a width parameter fitted to reproduce the rms radius of the initial nucleus measured by electron scattering[24]. The ground states of the initial nuclei, ^{14}N and ^{6}Li, are considered to have (both) one valence neutron and one valence proton in the $j = 1/2$ and $j = 3/2$ states, respectively. The protons couple their angular momenta with their cores, $^{13}C(j_f = 1/2)$ and $^{5}He(j_f = 3/2)$, respectively, to give $j_i = 1$. The final states are ^{13}C and ^{5}He, each one with one neutron outside a closed shell or subshell. In the cluster model calculations, we have assumed that the target nuclei are constituted by the inert spectator cores ^{12}C and ^{4}He and two correlated valence particles which form a deuteron cluster. In this model, the wave functions for the initial states may be written as

$$\Psi_i = A[\psi(core)\phi(d)\chi(R)], \tag{29}$$

where A is the antisymrnetization operator. The function $\phi(d)$, which describes the internal behaviour of the deuteron cluster, is chosen to be of the Gaussian type,

$$\phi(d) = \exp\Big(-\frac{1}{2}\alpha^2\sum_{i=1}^{2}(\vec{r}_i - \vec{R})^2\Big)\varsigma(\sigma,\tau), \tag{30}$$

R denoting the coordinate of the deuteron centre-of-mass. The spherically symmetric function $\chi(R)$ which describes the dynamical behaviour of the relative motion we have taken proportional to $R^2\exp(-\beta^2R^2)$, with the factor R^2 of the cluster model prescription for valence particles in the *p*-shell[22]. $\varsigma(\sigma,\tau)$ is the spin-one-isospin-zero wave function. The core wave functions of the initial states as well as of the corresponding final ^{13}C and ^{5}He ground states are assumed to be identical to the ones of our shell model calculation. The size parameter α of the deuteron was varied between 1.9 fm (the free value) and 1.4 fm, as from fits to cluster calculations[25] there seens to be some evidence for a contraction of the deuteron in the nucleus. For each value of a, the parameter β of the wave function of the centre-of-mass of the deuteron was computed to reproduce the same density as in the shell model. Because of the values of the parameters and the shape of the wave function $\chi(R)$, there is on the average only a small core-deuteron overlapping, so that exchange effects between these can be neglected. In the computations we have assumed that the total centre-of-mass of the system coincides with the centre-of-mass of the core and is not affected by the knock-out process (inert-core approximation). This assumption results in a strong simplification in the calculations but causes an error of order A^{-1} in the squared overlap integrals; in particular, in the case of the ^{6}Li this may not seem to be a good approximation. However, for calculations of effective polarization this error is considerably reduced as only ratios between momentum distributions enter. For our present purposes, this approximation is sufficient because the qualitative behaviour of the effective polarization is certainly not changed. The distorted wave calculations were performed in the WKB approximation with a square-well spin-independent nuclear optical potential with real and imaginary parts calculated as in ref. 20; experiment has shown that for the present energies and geometries (taken also as in ref. 20), the effective polarizations are only slightly affected by such approximations. (See references 1, 21, 27 and also 2,8.)

Typical results of our calculations are shown in fig. 5. Evidently, the effective polarization for the two models show striking differences: they differ both in sign and size*; both effects are fairly stable against reasonable parameter variations.

In the jj coupled shell model the effective polarization is caused by the correlations, in the initial nucleus, of the spin and angular momentum of the proton which is knocked-out from it. This is a result[2] of the nuclear spin-orbit coupling combined with distortion effects in the selected asymmetric geometry of the experiment. For the cluster model, one could expect, "a priori*, that the effective polarization should be equal to zero since it seems, at a first glance, that there are no correlations of this type in. the initial nuclei because the internal and relative motion wave functions of the deuteron surrounding the core both represent, in this very simple example, L = 0 states. In fact this is not so, as shown in fig. 5. The effective polarizations in the ^{6}Li case are about - 1/2 times the corresponding ones for the ^{14}N. This result is in agreement, in the case of the single-particle shell model with jj coupling, with the anticipated trend (see eq.(16)) for the effective polarization of the $1p_{3/2}$ and $1p_{1/2}$ single-particle states.

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* In the cluster model calculations, the numerator of expression (5), for the particular case of a proton knock-out from ^{14}N, is given by

$$\Delta_{cm} = |f'_{\ell,\ell}(\vec{k})|^2 - |f'_{\ell,-\ell}(\vec{k})|^2$$

where

$$f'_{\ell,\pm\ell}(\vec{k}) = (factors)\int d^3r \exp(-i\vec{k}\cdot\vec{r} - \Lambda^2 r^2)D(\vec{r})(a - br^2)rY_{\ell\pm\ell}(\hat{r}).$$

In this expression Λ denotes the width parameter for the cluster model potential and a and b are two constants defined to fit the rms radius for the initial nucleus. Assuming for simplicity

$$\int d^3r \exp(-i\vec{k}\cdot\vec{r}-\Lambda^2 r^2)D(\vec{r})r^3 Y_{\ell,\pm\ell}(\hat{r}) \simeq R^2\int d^3r \exp(-i\vec{k}\cdot\vec{r}-\beta^2 r^2)D(\vec{r})rY_{\ell,\pm\ell}(\hat{r})$$

in which β is the shell model width parameter and R is a cut-off radius, Δ_{cm} can be expressed in the approximate form $\Delta_{cm} \simeq (a - bR^2)^2\Delta^{fes}_{sm}$ where Δ^{fes}_{sm} represents the corresponding numerator of expression (5) in the shell model calculations for the knock-out of a proton from the first excitated state of ^{14}N [26]. Δ^{fes}_{sm} has, according to expression (16), the opposite sign of the corresponding expression for the ground state of the ^{14}N nucleus. A similar relations is obtained for the knock-out of a proton from the ^6Li nucleus.

The differences between the results of the shell and cluster models are not difficult to understand (see references 20 and 27). Because of the spin wave function of the deuteron with $S = 1$, the spin of the knocked-out proton is paralell to the spin of the remaining valence neutron. The momentum of this neutron has, however, a tendency to be opposite to that of the knocked-out proton, because both momenta are anticorrelated in the deuteron. Using the distortion arguments of section 1 but now applied to the final nucleus, one finds again that the remaining neutron is effectively polarized but oppositely to the knocked-out proton in the shell model, because of its opposite internal momentum*. This polarization carriers over to the knocked-out proton throughout the mentioned $S = 1$ correlation, which explains the difference in sign of the effective polarization in the two models. The quantitative difference of the polarization is caused by the fact that in the cluster model the momentum of the remaining valence neutron is not exactly opposite to the one of the knocked-out proton but is smeared out by the centre of mass motion of the deuteron. We show in the following that these results could be obtained, in the case of the cluster model, directly from eq.(26).

We construct an "effective internal spin-orbit coupling" of the type $a\vec{\ell}_\rho \cdot \vec{s}_\rho$ for the p-quasiparticles with a=constant and where $\vec{\ell}_\rho$ and $\vec{s}_\rho$ characterize, respectively, their (conserved, in our model) internal angular momentum and spin and define the quantum numbers $\vec{j}_\rho = \vec{\ell}_\rho + \vec{s}_\rho$ and $m_\rho = m_{\ell_\rho} + m_{s_\rho}$. In the case in which we could associate to a given nucleus only one value of $\vec{j}_\rho$ this would correspond, in our model, to a "pure configuration" with $j_\rho = \ell_\rho + 1/2$ or with $j_\rho = \ell_\rho - 1/2$ for that nucleus. For a nucleus characterized by configuration mixing with a fixed value of ℓ_ρ, from eq. (26) we have

$$P(\vec{k}) = \frac{P_{\ell_\rho+1/2}(\vec{k}) + \eta_\rho P_{\ell_\rho-1/2}(\vec{k})}{\eta_\rho + 1}, \tag{31}$$

* **Crudely speaking, due to the peculiar structure of the wave functions of the initial state and assuming sheil model wave functions for the final state, we obtain overlaps approximately of the form $\simeq \chi^*_{\uparrow\downarrow}(2)\chi_{\uparrow\downarrow}(2)\chi_{\uparrow\uparrow}(1) = \chi_{\uparrow\uparrow}(1)$ for ^{14}N or of the form $\simeq \chi^*_{\uparrow\uparrow}(2)\chi_{\uparrow\uparrow}(2)\chi_{\uparrow\downarrow}(1) = \chi_{\uparrow\downarrow}(1)$ for ^6Li in the momentum distributions; the arrows denote the relative directions of angular momenta and spins which are different from the corresponding ones in the single-particle shell model with jj coupling.**

where

$$\eta_\rho = \frac{|\gamma_{\ell_\rho-1/2}|^2}{|\gamma_{\ell_\rho-1/2}|}. \tag{32}$$

In the simplest case of spherical initial nucleus with vanishing spin ($j_i = 0$) from the definitions of the p-quasiparticle operators, of the p-vacuum, and of the γ_{j_ρ} coefficients,

$$\begin{aligned}\gamma_{j_\rho} &= (2j_\rho+1)^{-1/2}C^{-1}(j_{\rho_f}m_{\rho_f};j_\rho m_\rho|j_{\rho_i}m_{\rho_i}) < j_{\rho_f}m_{\rho_f}|\alpha_{j_\rho m_\rho}|j_{\rho_i}m_{\rho_i}>, \\ &= (2j_\rho+1)^{-1/2} < j_{\rho_f} \parallel \alpha_{j_\rho} \parallel j_{\rho_i} >,\end{aligned} \tag{33}$$

we have, taking fixed values of $j_{\rho_f} = j_\rho = \ell_\rho \pm 1/2$, and $m_{\rho_f} = -m_\rho$, and from eq.(32), with $j_\pm = \ell_\rho \pm 1/2$ and $m_\pm = m_{\ell_\rho} \pm 1/2$,

$$\eta_\rho = \frac{C^2(j_+,j_+;m_+,-m_+|00)}{C^2(j_-,j_-;m_-,-m_-|00)}, \tag{34}$$

or

$$\eta_\rho = \frac{\ell_\rho}{\ell_\rho+1}. \tag{35}$$

In this case $P(\vec{k}) = 0$, and from eq.(31) and eq.(35) we obtain a relation between the effective polarizations $P_{\ell_\rho+1/2}(\vec{k})$ and $P_{\ell_\rho+1/2}(\vec{k})$ associated to the "internal configurations" with $j_\rho = \ell_\rho + 1/2$ and $j_\rho = \ell_\rho - 1/2$ similar to expression (16)

$$P_{\ell_\rho+1/2}(\vec{k}) = -\frac{\ell_\rho}{\ell_\rho+1}P_{\ell_\rho+1/2}(\vec{k}). \tag{36}$$

We consider now the most general case of non-spherical initial nucleus with a non-vanishing spin. For simplicity, in order to compare our results with the corresponding ones for the single particle shell model with jj coupling, we normalize the cluster model momentum distributions with respect to the "occupation number" $2(2j_\rho+1)$ so we have $2(2\ell_\rho)|G'_{\ell_\rho+1/2}(\vec{k})|^2 = 2(2\ell_\rho+2)|G'_{\ell_\rho-1/2}(\vec{k})|^2$. In this case, eq. (31) should be replaced by

$$P(\vec{k}) = \frac{(\ell_\rho+1)P_{\ell_\rho+1/2}(\vec{k}) + \eta_\rho\ell_\rho P_{\ell_\rho+1/2}(\vec{k})}{(\ell_\rho+1)+\eta_\rho\ell_\rho} \tag{37}$$

Combining these definitions with expression *(36)* we obtain for the effective polarization, in the general case of a state with "configuration mixing",

$$P(\vec{k}) = \frac{(\eta_\rho - 1)\ell_\rho}{(\ell_\rho + 1) + \eta_\rho \ell_\rho} \cdot P_{\ell_\rho + 1/2}(\vec{k}). \tag{38}$$

A change of basis from the $j_\rho m_\rho$ basis to the $L_\rho S_\rho$ one with $\vec{L}_\rho = \sum \vec{\ell}_\rho$ and $\vec{S}_\rho = \sum \vec{s}_\rho$ gives for the knock-out of a ρ-quasiparticle from the deuteron cluster in the model [*core* + *deuteron]* for a fixed value of L, and S_ρ

$$|\gamma_{j_\rho}|^2 = [(2j+1)(2S_\rho+1)(2L_\rho+1)] \begin{Bmatrix} 1/2 & \ell_\rho & j \\ 1/2 & \ell_\rho & j_\rho \\ S_\rho & L_\rho & j_i \end{Bmatrix}^2 , \tag{39}$$

in which the $\{\}$ represent Wigner $9j$-coefficients and j is the total angular momentum of the remaining nucleon. For the state with L, $= 0$ and $S_\rho = 1$ and from the definition of the η_ρ parameters, expression (39) gives $\eta_\rho > 1$ for $j = 3/2$ and $\eta_\rho < 1$ for $j = 1/2$. When combined with eq. (38) these results show that the sign of the effective polarizations in the *[core +* $deuteron]_{L_\rho=0,S_\rho=1}$ cluster model for the reactions $^6Li(\vec{p},2p)^5He$ and $^{14}N(\vec{p},2p)^{13}$C should indeed be different from the corresponding ones in the single-particle [jj]shell model.

In fact the initial state is not a pure L, $= 0$, $S_\rho = 1$ one because, as was pointed out earlier due to the smearing out of the centre of mass motion, the momentum of the remaining valence neutron is not exactly opposite to the momentum of the knocked-out proton. Taking into account the contribution of the L, $= 2$, $S_\rho = 1$ components to the effective polarization gives, for $|\gamma_{j_\rho}|^2$

$$|\gamma_{j_\rho}|^2 = \left(\sum_{L_\rho S_\rho} a_{L_\rho S_\rho} \sqrt{(2j+1)(2S_\rho+1)(2L_\rho+1)} \begin{Bmatrix} 1/2 & \ell_\rho & j \\ 1/2 & \ell_\rho & j_\rho \\ S_\rho & L_\rho & j_i \end{Bmatrix} \right)^2 , \tag{40}$$

where $a_{L_\rho S_\rho}$ denotes the admixture coefficient for the L, $= 0,2$; $S_\rho = 1$ components. Estimates range from 4% to 8% probability that L, $= 2$ [28]. This gives, respectively, $a_{2,1} = \sqrt{4/100}$ (or 20%) and a,,, $= \sqrt{8/100}$ (or 28%). With these values we have obtained, as in the preceding case, $\eta_j > 1$ for $j = 3/2$ and $\eta_j < 1$ for $j = 1/2$. When combined with expression *(38)* these results show that, even taking the $L_p = 2$ contribution into account, the sign of the effective polarization in the $[core + deuteron]_{L_p S_p}$ cluster model for the reactions $^{6}Li(\vec{p}, 2p)^{5}He$ and $^{14}N(\vec{p}, 2p)^{13}C$ should be different from the corresponding one in the single-particle $[jj]$ shell model.

Concluding remarks

We have shown that the effective polarization can give information on cluster correlations in nuclei. In the particular case of an initial nuclei modeled as consisting of an inert core surrounded by a deuteron cluster, we have shown that the triplet spin correlation in the deuteron results in an effective polarization that has the opposite sign from the one expected from the single-particle $[jj]$ shell model for the same nucleus. In this case, the observation of the asymmetry might shed light on the two models. In general, we may test any model for the initial and final nuclear states by calculating the expected quasi-free cross sections for polarized protons and by comparing them with experiment. As we have remarked earlier (see also ref. 1), the normalization of these cross sections can be rather uncertain, but their shapes and, in particular, their measured asymmetries, can be quite reliable and characteristic. Finally, we would like to point out that, for nucleon-nucleus quasi-free reactions, the impulse approximation is on a much stronger footing than for cluster knock-out reactions. Cluster deformations inside the nucleus, the possibilities of pick-up and stripping processes before, during, or after the knock-out process, as well as the large size of the clusters, complicate the reaction mechanism and cast doubt on the use of the DWIA and on the interpretation of the results.

The author is grateful to Prof. Dr. M. Dillig (Erlangen) and to Prof. Dr. F. Fernandez (Salamanca) for comments, sugestions and also for their kind hospitality.

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Resumo

Um método simples e direto para a obtenção de informações sobre agregados nucleares, através de reações quase-livres à energias médias, é apresentado. No caso em que o núcleo-alvo tem spin diferente de zero, a polarização efetiva quase-livre pode ser expressa em termos de uma combinação linear de polarizações efetivas definidas no espaço de momentum angular em um modelo de quase-partículas.