Revista Brasileira de Física, Vol. 19, nº 3, 1989

Construction and study of exact ground states for a class of quantum antiferromagnets

M. **Fannes** [1]

Inst. Theor. Fysika, Universiteit Leuven, Leuven, Belgiurn

B. **Nachtergaele** [2]

Depto de Física, Universidad de Chile, Casilla 487-3, Santiago de Chile

R.F. Werner [3]

Dublin Inst. Adv. Studies, Dublin, Ireland

Received **29** August, 1989

Abstract We use techniques of Quaiitum Probability to construct the exact ground states for a class of Quantum Spin Systems in one dimension. This ciass in particular contains the antiferromagnetic models introduced by various authors under the name of VBS-models. The construction permits a detailed study of these ground states; as an example we compute expliciily the two-point correlation function for a family of one-dimensional models and we answer the question of Néel order for a set of models on a Cayley-tree.

1. Introduction

In this paper we study the ground states of a class of rotation-invariant nearest neighbour Hamiltonians for a quantum spin system on the one-dirnensional lattice or on a Cayley-tree. For this purpose we first explain a simplified version of a technique, borrowed from Quantum Probability[1,2], for constructing a class of 'trial'-states for Quantum Chains. We investigate in detail what kind of Hamiltonians have exact ground states in this ciass. These Hamiltonians are not

---

The present paper is an extended version of a communication by one of the authors (BN) at the Workshop on Phase Transitions, São Paulo, 25-28 July 1989.

[1] Bevoegdverklaard Navorser, N.F.W.O. Belgium

[2] Onderzoeker I.I.K.W. Belgium, on leave from Universiteit Leuven, Belgium

[3] On leave from Universität Osnabrück, F.R.G.

necessarily isotropic, nor nearest neighbour, but we will restrict our attention here to models which do have these properties.

Consider a one-dimensional spin system of spin s. This means that the algebra of observables consists of linear combinations of tensor products of $(2s+1)\times(2s+1)$ complex matrices; it is sufficient to consider simple tensors A of the type:

$$A = X_1 \otimes X_2 \otimes \cdots \otimes X_n \tag{1}$$

where $X_1, \ldots, X_n$ are observables at sites $1, \ldots, n$ of the lattice Z. The most important one-site observables certainly are the spin matrices S^x, S^y, S^z which are the generators of the $(2s+1)$–dimensional irreducible representation D_s of SU(2).

An isotropic and translation invariant Hamiltonian for such a spin chain is necessarily of the form

$$H = \sum_i \sum_{k=0}^{2s} J_k P^{(k)}_{i,i+1} \tag{2}$$

where $P^{(k)}_{i,i+1}$ denotes the orthogonal projection onto the subspace of $\mathbf{C}^{2s+1} \otimes \mathbf{C}^{2s+1}$ that carries the spin-k sector in the irreducible decomposition of the representation $D_s \otimes D_s$ of SU(2) on this space:

$$D_s \otimes D_s \cong D_0 \oplus D_1 \oplus \cdots \oplus D_{2s}$$

So, clearly

$$\sum_{k=0}^{2s} P^{(k)}_{i,i+1} = \mathbf{1}$$

The coupling constants J_k are real and without loss of generality they can be taken to be non-negative. The one-dimensional models considered in this paper are for integer values of the spin s, and the Hamiltonians are of the form (2) with $J_0 = J_1 = \cdots = J_{k_0} = 0$ and $J_{k_0+1} > 0, \ldots, J_{2s} > 0$. The ground state depends only on the value of k_0 (and of course on the value of the spin itself), and we require

$$k_0 > s \tag{3}$$

So for $s = 1$ there is essentially one model in our class of interest:

$$H = \sum_i P^{(2)}_{i,i+1} \tag{4}$$

which can be rewritten in terms of the spin-matrices as follows

$$H = \sum_i \left\{ \frac{1}{3} + \frac{1}{2}\vec{S}_i \cdot \vec{S}_{i+1} + \frac{1}{6}(\vec{S}_i \cdot \vec{S}_{i+1})^2 \right\}$$

In fact all interactions of the type (2) can be rewritten in terms of a polynomial in the Heisenberg-interaction $\vec{S}_i \cdot \vec{S}_{i+1}$.

The model (4) was first studied in[3] and more Hamiltonians of the type (2) are treated in[4], but with different and less general techniques than we will use here. Of course there exist generalizations of this type of Hamiltonians in higher dimensions (see e.g. refs. 3 and 5). The techniques we present below are in principle, also applicable to this higher dimensional models, though we will only discuss one dimension and Cayley-trees here.

2. Construction of the Ground States

A more general discussion of the mathematical techniques explained here can be found in refs.6 and,7 and there are also interesting applications to classical spin chains[8]. In this paper we restrict our attention to a particular case which will, however, be general enough to obtain the ground states for the model presented 'above.

Suppose we are considering a model of integer spin s. Then the Hilbert space for one site is $\mathbb{C}^{2s+1}$ on which the irreducible representation D_s of $SU(2)$ is acting. Furthermore we take an auxiliary representation D_j with j a half-integer ($1/2, 1, 3/2, 2, \ldots$) such that $2j \geq s > j$. The tensor product representation $D_s \otimes D_j$ can be decomposed in its Clebsch-Gordan series

$$D_s \otimes D_j \cong D_{s-j} \oplus D_{s-j+1} \oplus \cdots \oplus D_{s+j} \tag{5}$$

As we supposed that $0 < s - j \leq j$, the representation D_j appears exactly once in this direct sum. Define now a $(2s + 1)(2j + 1) \times (2j + 1)$-matrix V using the CG-coefficients of $SU(2)$ (for explicit expressions of -CG-coefficients see e.g ref.12):

$$V_{M,m;m'} = \langle s\, j, M\, m \mid j, m' \rangle \tag{6}$$

This matrix has the following properties:

$$V^* V = \mathbf{1}_{2j+1} \tag{7}$$

$$(D_s(g) \otimes D_j(g))V = V D_j(g) \quad \text{for all } g \in SU(2) \tag{8}$$

Because of (7) V is called an isometry and (8) is the intertwining property. In fact (7) and (8) define V uniquely up to a phase.

To define a state of our spin-s chain we essentially have two equivalent possibilities: either we define a density matrix for any finite number of sites, i.e. we give a positive matrix ρ_n for all $n = 1, 2, \ldots$ such that the expectation value of an observable of the form (1): $A = X_1 \otimes \cdots \otimes X_n$, is given by

$$\langle A \rangle = \mathrm{Tr}_{(\mathbb{C}^{2s+1})^n} \rho_n X_1 \otimes \cdots \otimes X_n \tag{9}$$

or we use the second possibility which is giving a formula for the computation of the expectation values (A) directly. Because we want to study the ground state in the thermodynamic limit (infinite volume) we cannot simply work with vectors in the Hilbert space. In our case the second possibility will turn out to be the most convenient way to present the construction, but we will also give a formula for the density matrix. In both cases the expectation values are defined for finite intervals only, so there will be a compatibility condition in order to assure that we are really defining expectation values for the infinite system. The observable

$X_a \otimes X_{a+1} \otimes \cdots \otimes X_{a+n}$ living on the volume $\{a, a+1, \ldots, a+n\}$ is identified with $X_a \otimes X_{a+1} \otimes \cdots \otimes X_{a+n} \otimes \mathbf{1}_{a+n+1}$ on thevolume $\{a, a+1, \ldots, a+n, a+n+1\}$. So the compatibility condition simply is that computing expectation values on $\{a, a+1, \ldots, a+n\}$ and on $\{a, a+1, \ldots, a+n, a+n+1\}$ should yield thesameresult for observables identified in this way. This requirement (and the analogous condition for adding sites on the left) will automatically be satisfied by our construction (e.g. see ref.6 for a proof of this).

So we have chosen an aiixiliary representation D_j and defined the matrix V as in (6). For any $X \in M_{2s+1}$ (the $(2s+1) \times (2s+1)$ complex matrices) we now define a linear transformation E_X of M_{2j+1} by putting for all $m \in M_{2j+1}$:

$$E_X(m) = V^*(X \otimes m)V \tag{10}$$

and then define the expectation values in our state $\langle \cdot \rangle^j$ by

$$\langle X_1 \otimes \cdots \otimes X_n \rangle^j = \frac{1}{2j+1} \mathrm{Tr}_{\mathbb{C}^{2j+1}} E_{X_1}(E_{X_2}(\cdots E_{X_n}(\mathbf{1}_{2j+1}))) \tag{11}$$

Of course, if necessary, we could consider the X_i as vectors (dimension $(2s+1)^2$) and the tranformations E_X as matrices. (11) then becomes a matrix element of a product of n matrices.

We now derive the density matrix ρ_n which corresponds to (11) such that (9) holds:

$$\begin{aligned}
&\mathrm{Tr}\, \rho_n X_1 \otimes \cdots \otimes X_n \\
&\quad = \langle X_1 \otimes \cdots \otimes X_n \rangle^j \\
&\quad = \frac{1}{2j+1} \mathrm{Tr}_{\mathbb{C}^{2j+1}} E_{X_1}(E_{X_2}(\cdots E_{X_n}(\mathbf{1}_{2j+1}) \cdots)) \\
&\quad = \frac{1}{2j+1} \mathrm{Tr}_{\mathbb{C}^{2j+1}} V^*(X_1 \otimes E_{X_2}(\cdots E_{X_n}(\mathbf{1}_{2j+1}) \cdots)V
\end{aligned}$$

$$= \frac{1}{2j+1} \mathrm{Tr}_{\mathbb{C}^{2j+1}} V^*(\mathbf{1} \otimes V^*) \cdots (\mathbf{1} \otimes \cdots \otimes \mathbf{1} \otimes V^*)$$

$$(X_1 \otimes X_2 \otimes \cdots \otimes X_n \otimes \mathbf{1})(\mathbf{1} \otimes \cdots \otimes \mathbf{1} \otimes V) \cdots (\mathbf{1} \otimes V)V$$

$$= \frac{1}{2j+1} \mathrm{Tr}_{\mathbb{C}^{2s+1}} \mathrm{Tr}_{\mathbb{C}^{2j+1}} VV^*(\mathbf{1} \otimes V^*) \cdots (\mathbf{1} \otimes \cdots \otimes \mathbf{1} \otimes V^*)$$

$$(X_1 \otimes X_2 \otimes \cdots \otimes X_n \otimes \mathbf{1})(\mathbf{1} \otimes \ldots \otimes \mathbf{1} \otimes V) \cdots (\mathbf{1} \otimes V)$$

$$= \frac{1}{2j+1} \mathrm{Tr}_{(\mathbb{C}^{2s+1})^n} \mathrm{Tr}_{\mathbb{C}^{2j+1}} (\mathbf{1} \otimes \cdots \otimes \mathbf{1} \otimes V) \cdots (\mathbf{1} \otimes V)$$

$$VV^*(\mathbf{1} \otimes V^*) \cdots (\mathbf{1} \otimes \cdots \otimes \mathbf{1} \otimes V(X_1 \otimes X_2 \otimes \cdots \otimes X_n \otimes \mathbf{1})$$

$$= \mathrm{Tr}_{(\mathbb{C}^{2s+1})^n} X_1 \otimes X_2 \otimes \cdots \otimes X_n \frac{1}{2j+1} \mathrm{Tr}_{\mathbb{C}^{2j+1}} (\mathbf{1} \otimes \cdots \otimes \mathbf{1} \otimes V) \cdots$$

$$(\mathbf{1} \otimes V)VV^*(\mathbf{1} \otimes V^*) \cdots (\mathbf{1} \otimes \cdots \otimes \mathbf{1} \otimes V^*)$$

Therefore

$$\rho_n = \frac{1}{2j+1} \mathrm{Tr}_{\mathbb{C}^{2j+1}} \{(\mathbf{1} \otimes \cdots \otimes \mathbf{1} \otimes V) \cdots (\mathbf{1} \otimes V)$$

$$VV^*(\mathbf{1} \otimes V^*) \cdots (\mathbf{1} \otimes \cdots \otimes \mathbf{1} \otimes V^*)\} \qquad (12)$$

From formula **(12)** it is immediately clear that ρ_n is a **positive** matrix and the property **(7)** of **V** implies that its trace is 1. So it is indeed a density matrix. It can **also** be **seen** that the states $\langle \cdot \rangle^j$ defined above are SU(2)-**rotation** invariant. **It** is **quite** evident that for doing explicit calculations formula **(11)** **is** going to be a lot more convenient than using **(12).**

Next we want to show that the states constructed above are orthogonal to the vectors belonging to high-spin representations. To be more precise, for all n, all vectors ψ, in the n-site Hilbert space and $\alpha = x, y, z$ one has

$$\|(S_1^\alpha + \cdots + S_n^\alpha)\rho_n\psi\| \leq 2j\|\rho_n\psi\| \tag{13}$$

where j is the half-integer parameter appearing in the construction of V in (6) above. We give the argument for $n = 2$; the general proof is a straightforward extension of this and can be found in ref.7. Denote by S^x, S^y, S^z the generators of D_s and by J^x, J^y, J^z the generators of D_j. They are normalized in such a way that

$$\|S^\alpha\| = s, \ \|J^\alpha\| = j \tag{14}$$

The intertwining property (8) in terms of the generators reads, for $\alpha = x, y, z$:

$$(S^\alpha \otimes \mathbf{1} + \mathbf{1} \otimes J^\alpha)V = VJ^\alpha \tag{15}$$

So, one also has

$$(S^\alpha \otimes \mathbf{1} \otimes \mathbf{1} + \mathbf{1} \otimes S^\alpha \otimes \mathbf{1} + \mathbf{1} \otimes \mathbf{1} \otimes J^\alpha)(\mathbf{1} \otimes V)V = (\mathbf{1} \otimes V)VJ^\alpha$$

Using (14) and $\|V\| = 1$ (which follows from (7)) one gets the estimate

$$\|(S^\alpha \otimes \mathbf{1} \otimes \mathbf{1} + \mathbf{1} \otimes S^\alpha \otimes \mathbf{1})(\mathbf{1} \otimes V)V\| \leq 2j \tag{16}$$

from which the desired result easily follows. Combining (16) with (12) (for $n = 2$) one concludes that $\rho_2 P^{(k)} = 0$ for all $k > 2j$. In particular

$$\langle H \rangle^j = 0$$

whenever H is of the form

$$H = \sum_i \sum_{k=2j+1}^{2s} J_k P_{i,i+1}^{(k)} \tag{17}$$

If all $J_k \geq 0$, then H $\geq$ 0 and so $\langle\cdot\rangle^j$ is a ground state for H. One can also prove that if $J_k > 0$ for $k = s+1,\ldots,2s$, the state $\langle\cdot\rangle^{s/2}$ is the unique ground state in the infinite volume limit which has the property that the energy is zero for every finite volume; in particular it is the unique ground state if one restricts oneself to translation invariant or periodic ground states. For a more complete discussion see ref. 8.

The states we constructed above have the property of being Finitely Correlated (they are FCS, Finitely Correlated States). For more details of this mathematical notion we refer the reader to a future publication[8]. The most important consequence of this property is the simple behaviour of the correlation functions. The two point correlation functions are given as a finite Iinear combination of pure exponentials. **A** particular example of this is given in the following theorem.

THEOREM 1.

Consider a spin s chain with Hamiltonian

$$H = \sum_i \sum_{k=s+1}^{2s} J_k P^{(k)}_{i,i+1}$$

where *all* coupling constants $J_k > 0$, $k = s+1, s+2,\ldots,2s$.

Then there *is* only one periodic ground state in the infinite volume limit, and it *is* given *by:*

$$\langle X_1 \otimes \cdots \otimes X_n\rangle^{s/2} = \frac{1}{s+1} Tr_{C^{s+1}} E_{X_1}(E_{X_2}(\cdots E_{X_n}(\mathbf{1}_{s+1})\cdots))$$

where the E_X are defined in (10).

If $J_k = 0$, for k $= 0,1,\ldots,2j_0$, with $s/2 \leq j_0 < s$, in the Hamiltonian (17) then also the states $\langle\cdot\rangle^j$, defined in *(11)*, with $j \leq j_0$ are ground states of **(17).**

The spin-spin correlation function of the states $\langle\cdot\rangle^j$ *is* given by the following formula:

$$\langle S_0^\alpha S_r^\beta\rangle^j = \frac{1}{6}\delta_{\alpha,\beta}\frac{s^2(s+1)^2}{s(s+1)-2j(j+1)}\left(\frac{2j(j+1)-s(s+1)}{2j(j+1)}\right)^{|r|} \qquad (18)$$

PROOF:

The fact that the states are ground states has been shown above. The uniqueness involves a quite elaborated argument that will be given in ref. 8. We are going to compute now the spin-spin correlation function of the states $\langle\cdot\rangle^j$.

Denote by S^z, S^y, S^z the generators of D_s and by J^z, J^Y, J^z the generators of D_j, normalized such that

$$\vec{S}\cdot\vec{S} = s(s+1) \tag{19}$$

$$\vec{J}\cdot\vec{J} = j(j+1) \tag{20}$$

One first checks that

$$E,(\vec{J}) = \lambda\vec{J}, \quad E_{\vec{S}}(\mathbf{1}) = (1-\lambda)\vec{J} \tag{21}$$

for some real λ. Using *(20)* and the intertwining property *(15)* it then follows that

$$V * \vec{S}\otimes\vec{J} + j(j+1)V = j(j+1)\lambda \tag{22}$$

(15) also implies

$$(\vec{S}\otimes\mathbf{1} + \mathbf{1}\otimes\vec{J})2V = V\vec{J}2$$

and so, using *(19-20)*

$$E_{\vec{S}}(\vec{J}) = -\frac{1}{2}s(s+1) \tag{23}$$

Combining *(22)* and *(23)* λ follows:

$$\lambda = \frac{2j(j+1) - s(s+1)}{2j(j+1)}$$

As $2j \geq s > j$ this is a real number of modulus strictly smaller than one, and λ completely determines the exponential decay of the correlation function:

$$\begin{aligned}
&\langle S_0^\alpha S_r^\beta\rangle^j \\
&= \frac{1}{2j+1}\operatorname{Tr} E_{S^\alpha}(E_1)^{r-1}E_{S^\beta}(\mathbf{1}) \\
&= \lambda^{r-1}\frac{1}{2j+1}\operatorname{Tr} E_{S^\alpha}(E_{S^\beta}(\mathbf{1}))
\end{aligned}$$

The factor $\delta_{\alpha,\beta}$ follows from rotation invariance and the same equations (19)-(23) can be used to derive that

$$\sum_\alpha \frac{1}{2j+1}\operatorname{Tr} E_{S^\alpha}(E_{S^\alpha}(\mathbf{1})) = -\frac{s^2(s+1)^2}{4j(j+1)}$$

This completes the verification of (18).

In the case where $j = s/2$ obviously $\lambda < 0$ and so the correlation function (18) exhibits antiferromagnetic behaviour.

3. Models on Cayley-trees

As we will only consider translation invariant or periodic states on Cayley-tree it is sufficient to consider one branch of such a tree; this means that one chooses an origin in the tree and then considers only these sites which are connected with this special site via $z-1$ of its z nearest neighbours, where z is the coordination number of the tree. We label the sites of such a branch in the following way: The origin has label (0), the $z-1$ nearest neighbours of the origin that we are including in our branch have labels $(1),(2),\ldots,(z-1)$, any of these sites (p) $(p=1,\ldots,z-1)$ has z nearest neighbours: of the origin (0) and $z-1$ new sites with labels $(p,1),(p,2),\ldots,(p,z-1)$ and so on. So we divide the branch into levels: level 0 contains one site: (0), level 1 contains $z-1$ sites, labeled $(1),\ldots,(z-1)$, level 2 contains the $(z-1)2$ next nearest neighbours of the origin, labeled by $(1,1),(1,2),\ldots,(z-1,1),\ldots,(z-1,z-1)$, the *nth* level contains the $(z-1)n$ n^{th}-neighbours of the origin, labeled by $(p,,p_2,\ldots,p_n)$, $1\le p_i\le z-1$.

The fundamental objects in the construction of Finitely Correlated States for chains (which is the case $z=2$) are the transformations E_X which in that case map an auxiliary algebra M_{2j+1} into itself. For $z > 2$ these will be replaced by linear transformations of a slightly more complicated type, one for any one-site observable X E M_{2s+1}:

$$E_X : M_{2j+1} \otimes \cdots \otimes M_{2j+1} \longrightarrow M_{2j+1} \tag{24}$$

This is the fundamental idea; the formula defining the local expectation values in the state has the same striicture as before (cf. (11)), although it might look slightly more complicated. We give the formula for observables liying on the first two levels:

$$\begin{aligned}&\langle X_0 \otimes (X_1 \otimes \cdots \otimes X_{z-1})\rangle^j \\ &= \frac{1}{2j+1} \mathrm{Tr}\, E_{X_0}(E_{X_1}(\mathbf{1} \otimes \cdots \otimes \mathbf{1}) \otimes \cdots \otimes E_{X_{z-1}}(\mathbf{1} \otimes \cdots \otimes \mathbf{1}))\end{aligned} \tag{25}$$

The general formula involving n levels is a straightforward generalization of (25): the level n, containing $(z-1)^n$ sites is mapped into the level $n-1$ (with $(z-1)^{n-1}$ sites) by application of a $(z-1)^{n-1}$-fold tensor product of mappings $E_{X_{p_1,\ldots,p_{z-1}}}$.

We now consider the following model on the Cayley-tree with coordination number z ($z \geq 2$). It is a spin $s = z/2$ model and the interaction is nearest neighbour:

$$H = \sum_{nn} P^{(z)}_{nn} \tag{26}$$

where as before $P^{(z)}_{nn}$ denotes the orthogonal projection onto the spin= z subspace of the tensor product of two copies of the irreducible representation $D_{z/2}$ of SU(2), located on two nearest neighbour sites.

A ground state for this class of models is now obtained by choosing the following üefinition for the E_X (depending on z of course):

$$\begin{aligned}&E_X : \underbrace{M_2 \otimes \cdots \otimes M_2}_{z-1} \rightarrow M_2 \\ &E_X(b_1 \otimes \cdots \otimes b_{z-1}) = V^* X \otimes b_1 \otimes \cdots \otimes b_{z-1} V\end{aligned} \tag{27}$$

where V is the unique intertwining isometry between

$$D_{z/2} \otimes D_{1/2} \otimes \cdots \otimes D_{1/2}$$

and $D_{1/2}$. This means that for all g $\in$ SU(2)

$$D_{z/2}(g) \otimes D_{1/2}(g) \otimes \cdots \otimes D_{1/2}(g)V = VD_{1/2}(g)$$

and

$$V^*V = \mathbf{1}$$

krguments the same kind of as we used to derive **(16)** for chains, permit one to derive a similar estimate in the case of trees, implying that the density matrix of this state, reduced to two nearest neighbour sites is formed by vectors in spin spaces of spin $s \leq z - 1$ only. So the energy of the state is again zero and as H $\geq$ 0, the state is a ground state.

The interesting question now is whether this is the unique ground state or not. More specifically whether'there is Néel order or not. Néel order means that this translation and rotation invariant ground state can be decomposed into states with periodicity 2 and with broken rotation symmetry such that one has the following situation: there is a non-zero Néel-order parameter n_0 such that for all $\alpha = x, y, z$ one has a decomposition

$$\langle\cdot\rangle = \frac{1}{2}(\langle\cdot\rangle^{\alpha,+} + \langle\cdot\rangle^{\alpha,-})$$

and

$$\langle S_r^\alpha\rangle^{\alpha,\pm} = (-1)^{r\pm 1} n_0$$

S_r^α denotes a spin operator on a (arbitrary) site of level r. The answer to the question of occurence of Néel order in ihe ground states constructed above, is stated in the following theorem.

THEOREM 2.

For the spin $s = z/2$ *model* on the Cayley-tree with coordination *number* z, $z \geq 2$, with the translation *invariant* nearest neighbour *interaction* given by *(26)*, the state constructed with the E_X defined in (27) and *(25) is* a ground state and

i) if $z = 2, 3, 4$ there is no Néel order.

ii) if $z \geq 5$ there is Néel order.

We will not give the full proof of this theorem here. Let us only mention that using general arguments together with some properties of SU(2), one can show that the question of Néel order is equivalent to the study of the invariant sets of a very simple one-dimensional 'dynamical system'.

Let us just give a sketch of this equivalent dynamical system. For all $z \geq 2$ we define a function $t^{(z)} : [0,2] \longrightarrow [0,2]$ by

$$\mathrm{t(z)}(x) = \frac{2}{z+2} \frac{\sum_{k=0}^{z-1} (z-k) x^k (2-x)^{z-1-k}}{\sum_{k=0}^{z-1} x^k (2-x)^{z-1-k}} \tag{28}$$

Néel order then corresponds to solutions $x_0 \neq 1$ of the equation

$$2 - x_0 = t^{(z)}(x_0) \tag{29}$$

If the only solution of (29) is $x_0 = 1$ then there is no Néel order. The asymptotic behaviour of the dynamics $(ot^{(z)})^n$ can be completely studied and the main implication of it is stated in Theorem 2. We still remark that except for the absence of Néel order in the case $z = 4$, this theorem was already derived with other techniques in ref. 3. The case $z = 4$ is marginal and the techniques used in ref. 3 could not determine whether there was Néel order or not, although absence of Néel order was conjectured on the basis of a numerical computation. Physically the question of Néel order is most relevant in models on the two-dimensiona! regular square lattice, where also $z = 4$ (related to a possible explanation of high T_c superconductivity in La_2CuO_4-type materials, see e.g. refs.10 and 13). One expects Néel order at least for some models on this lattice, but the problem is still open.

4. Conclusion and further prospects

We reported on a new technique to describe a class of exactly solvable finite range Hamiltonians (ground states) of Quantum Spin Systems. The technique could easily be extended to models on Cayley-trees and a generalization to real higher dimensional systems is in progress. It certainly applies to the study of strips and cylinders and can be used to extend the results in ref.11.

It is also clear that this techniques can be extended to other types of models, e.g. with other than SU(2)-symmetry (such as appear e.g. in ref. 14).

It is a pleasure to thank J. F. Perez, S. R. Salinas and W, F. Wreszinski for offering us such a nice and stimulating 'Workshop on Phase Transitions' in São Paulo, 25-28 july 1989. One of us (B.N.) also acknowledge valuable help by R. Benguria and C. Yarur.

References

1. L. Accardi, Phys. Rep. 77, 169 (1981).
2. L. Accardi and A. Frigerio, Proc. R. Ir. Acad. vol 831(2), 251 (1983).
3. I. Affleck, E.H. Lieb, T. Kennedy, H. Tasaki, Phys. Rev. Lett. 59, 799 (1987) and Comm. Math. Phys. 115, 477 (1988).
4. D.P. Arovas, A. Auerbach, F.D.M. Haldane, Phys. Rev. Lett. 60,531 (1988).
5. T. Kennedy, E.H. Lieb, H. Tasaki, J. Stat. Phys. 53, 383 (1988).
6. B. Nachtergaele, *Working with Quanturn Markov States and their Classical Analogues,* to appear in Springer Lecture Notes in Mathematics, Proceedings of Workshop on Quantum Probability and Applications, Heidelberg september 1988.
7. M. Fannes, B. Nachtergaele, R. Werner, *Exact Ground States of Quantum Spin Chains,* to appear in Europhys. Lett. 1989.
8. M. Fannes, B. Nachtergaele, R. Werner, in preparation.
9. M. Fannes, B. Nachtergaele, L. Slegers, *Functions of Markov Processes and Algebraic Measures,* preprint KUL-TF-89/10.
10. J. Chayes, L. Chayes, S. Kivelson, Comm. Math. Phys. **123,** 53 (1989).

11. P. L. Iske and W. J. Caspers, Mod. Phys. Lett. **B2**, 1223 (1988).

12. A.R. Edmonds, Angular Momentuin in Quantum Mechanics, Princeton University Press, New Jersey (1957).

13. P. Anderson, Science 235, 1196 (1987).

14. I. Affleck, Phys. Rev. Lett. 54,966 (1985).

Resumo

Utilizamos técnicas de probabilidade quântica para construir os estados fundamentais exatos para uma classe de sistemas de spin quâaticos em uma dimensão. Esta classe contém em particular os modelos antiferromagnéticos introduzidos por vários autores sob o nome de modelos VBS. A construção permite um estudo detalhado destes estados fundamentais; como exemplo, calculamos explicitamente a função de correlação de dois pontos para uma família de modelos uni-dimensionais, e respondemos a questão sobre a existência de ordem de Nkel para um conjunto de modelos na árvore de Cayley.