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Novel magnetotransport effects in a 1D periodically modulated two-dirnensional electron gas

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Abstraet The recently discovered magnetoresistance oscillations of a two-dimensional electron gas, modulated periodically and weakly in one direction, are fully accounted for with the help of a *quantum* Boltzmann equation. The bandwidth of the modulation-broadened Eandau levels, at the Fermy energy, oscillates with magnetic field giving rise to the observed oscillations. The magnetoresistance perpendicular to the modulation (ρ_{xx}) is dominated by *diffusive* current contributions, which increase with increasing bandwidth, while the magnetoresistance parallel to the modulation (ρ_{yy}) is dominated by *collisional* contributions which decrease with increasing bandwidth. The resistivity tensor is *asymmetric* and the components ρ_{xx} and ρ_{yy} are *out of phase as* observed. New oscillations are predicted for the Hall resistance, the cyclotron resonance position, and the linewidth.

Recently[1] a weak 1D modulation (taken along the x-direction) of a high mobility two-dimensional electron gas (2DEG) has been realized which leads to novel oscillations in the magnetoresistance. These oscillations are connected to the commensurability between the modulation period (a) and the diameter of the cyclotron orbit $2R_c = 2\sqrt{2\pi n_e l^2}$ at the Fermi energy with $l = \sqrt{\hbar/eB}$ the magnetic length and n_e the electron density. These oscillations[1-3]: 1) are periodic in $1/B$ like the Shulnikov-de Haas (SdH) oscillations; 2) the periodicity depends on the electron density as $\sqrt{n_e}$ while the SdH have a n_e-dependence; 3) the amplitude of these oscillations has almost no temperature dependence in contrast with that of the SdH oscillations; 4) they show up most clearly at small magnetic fields, because at higher fields they are obscured by the SdH oscillations; 5) Weiss et al.[1-4] also found oscillations in ρ_{yy} which are much weaker in amplitude and are out of phase with the oscillations in p,, .

Different theoretical models have been given which are able to explain the oscillations in ρ_{xx}. Gerhardts et al.[2] presented a quantum mechanical calculation based on a Kubo-type formula. Theoretically no noticeable oscillations in p,, and R_H were obtained. Winkler et al.[3] calculated the diffusive contribution to p,, in the high temperature and classical (large Landau level index) limit. This approach led to a simple expression for the oscillations which agreed very well with the experimental results in the very small magnetic field limit but for higher magnetic fields the theoretical result did not recover the SdH oscillations. Beenakker[5] presented an alternative explanation for the oscillations in p,, on the basis of a classical picture, in which a resonance between the periodic cyclotron orbit motion and the induced (by the periodic potential) oscillatory motion of the center of the orbit leads to oscillations in p,,. Because the theory is classical the transition to SdH oscillations in ρ_{xx} for larger magnetic field is not obtained. No oscillations are found in the other components of the resistivity tensor indicating that the weak oscillations in p,, have a pure quantum mechanical origin.

At present no explanation is available for the anti-phase oscillations in p,, . This has motivated us to investigate this problem in more detail and to calculate the complete resistivity tensor. The anti-phase oscillations in ρ_{yy} are explained and new oscillations in the Hall resistance and the cyclotron resonance position and linewidth are predicted. In this paper we demonstrate that a quantum Boltzmann equation[6] derived in the framework of Kubo's linear response formalism, accounts well for all the observations mentioned above.

Consider a two-dimensional electron gas, in the (x,y) plane, in the presente of a magnetic field B along the z-axis, and periodically modulated in the x-direction by the potential $U(x) = V_0 \cos(Kx)$ with $K = 2\pi/a$, a being the modulation period. To evaluate the resistivity tensor $\rho_{\mu\nu}$ $(\mu, \nu = x, y)$ we will use the components $\sigma_{\mu\nu}$ of the conductivity tensor in the standard expression:

$\rho_{xx} = \sigma_{yy}/S$, $\rho_{xy} = \sigma_{xx}/S$, and $\rho_{yx} = -\sigma_{yx}/S$ where $S = \sigma_{xx}\sigma_{yy} - \sigma_{xy}\sigma_{yx}$ with $S \approx \sigma_{xy}^2 = n_e^2 e^2/B^2$ in the experiments under consideration.

From the outset we remark that there are two different contributions to the current. First there is the usual *diffusive* current which leads to the conductivity[6a]

$$\sigma_{\mu\nu}^{d} = \beta\frac{e^2}{\Omega}\sum_{\varsigma} f_\varsigma(1-f_\varsigma)\tau(E_\varsigma)v_\mu^\varsigma v_\nu^\varsigma \ , \tag{1}$$

and which is valid for elastic or quasi-elastic scattering. Here, f_ς is the Fermi-Dirac function, $\beta = 1/k_b T$ and $\tau(E_\varsigma)$ is the relaxation time for an electron in state $|\varsigma>$ which has an energy E_ς. Second, there is the collisional current which leads to the dc-conductivity (we consider only $\mu = \nu$)[6a]

$$\sigma_{\mu\nu} = \frac{\beta e^2}{2\Omega}\sum_{\varsigma\varsigma'} f_\varsigma(1-f_{\varsigma'})W_{\varsigma\varsigma'}(\alpha_\mu^\varsigma - \alpha_\mu^{\varsigma'})^2 \ , \tag{2}$$

where $W_{\varsigma\varsigma'}$ is the transition rate between the states $|\varsigma>$ and $|\varsigma'>$. This is the well-known hopping-type formula for transport in the presence of a magnetic field[8,9]. Conduction occurs by transitions through spatially separated states from $\alpha_\mu^\varsigma =<\varsigma|r_\mu|\varsigma>$ to $\alpha_\mu^{\varsigma'} =<\varsigma'|r_\mu|\varsigma'>$.

For the Hall resistance the nondiagonal part of the current density is the relevant quantity and leads[6] to the dc-conductivity

$$\sigma_{\mu\nu}^{nd} = 2\frac{i\hbar}{\Omega}\sum_{\varsigma\neq\varsigma'} f_\varsigma(1-f_{\varsigma'}) <\varsigma|v_\mu|\varsigma'><\varsigma'|v_\nu|\varsigma> \frac{1-e^{\beta(E_{\varsigma'}-E_\varsigma)}}{(E_\varsigma - E_{\varsigma'})^2} \ . \tag{3}$$

The total conductivity is then given by the sum $\sigma_{\mu\nu} = \sigma_{\mu\nu}^{d} + \sigma_{\mu\nu}^{nd}$. The above formulas have been successfully applied to many situations[6,7]: hopping conduction and magnetophonon resonances, quantum Hall effect, Aharonov-Bohm effect etc.

To apply eqs.(1-3) to the present problem we need the eigenfunctions and eigenvalues of the one-electron Hamiltonian $H_0 = (\mathbf{p} - e\mathbf{A})^2/2m^* + U(x)$ where p is the momentum operator, $\mathbf{A} = (0, Bx, 0)$ the vector potential and $U(x)$ the spatial 1D modulation. The exact eigenstates of this Hamiltonian with the modulation are difficult to obtain. In the experimental systems under study the amplitude of the modulation is small and we may evaluate the correction to the energy levels by first-order perturbation theory[2] using the unperturbed wave functions of the $U(x) = 0$ system. This gives

$$E_{n,k_y} = \left(n + \frac{1}{2}\right)\hbar\omega_c + V_0 \cos(Kx_0)e^{-u/2}L_n(u) \ , \tag{4}$$

where $u = K^2 l^2/2$ and L, (u) is a Laguerre polynomial[10]. We see that the modulation lifts the k, degeneracy of the unperturbed Landau levels which are broadened into **bands** with a bandwidth that oscillates with band index n and magnetic field. What will be important is the bandwidth at the Fermi energy which is illustrated in **fig.1.** The calculated results are not a continuous function of the magnetic field because we have to assume that the electron is in a definite Landau level $n = E_F/\hbar\omega_c - 1/2$.

When the modulation is absent the diffusive contribution to the current vanishes identically because v_x^ς and v_y^ς are zero. The only current contribution left for transport along the electric field ($\mu =$ v) is the "collisional" one, as given by eq. (2). However, in the presence of the modulation the carriers acquire a mean velocity in the y-direction $v_y = -\partial E_{n k_y}/\hbar\partial k_y = -(2V_0/\hbar K)\sin(2k_y u/K)ue^{-u/2}L_n(u)$, whereas v_x^ς is again zero. Thus a,, has only a collisional contribution while **a,,** will have two contributions: one collisional and the other one diffusive. This already implies that the resistivity tensor is asymmetric. In the following we will concentrate on the dominant correction to the conductivity due to the spatial modulation of the 2D electron gas.

For the evaluation of formulas (1)-(3) we assume that the electrons are scattered elastically by randomly distributed impurities. This is a very good approximation for the experimental temperatures $T < 10K$. To evaluate Ap,, we have to calculate the correction to the conductivity $\Delta\sigma_{yy}$ due to the modulation which is dominated by a diffusive component. To leading order in V_0 we find

$$\Delta\sigma_{yy} = \frac{e^2}{h}\frac{2\pi^2}{\hbar}\tau V_0^2\frac{l^2}{a^2}e^{-u}\sum_{n=0}^{\infty}[L_n(u)]^2\left(-\frac{\partial f(e)}{\partial E}\right)_{E=E_n} . \tag{5}$$

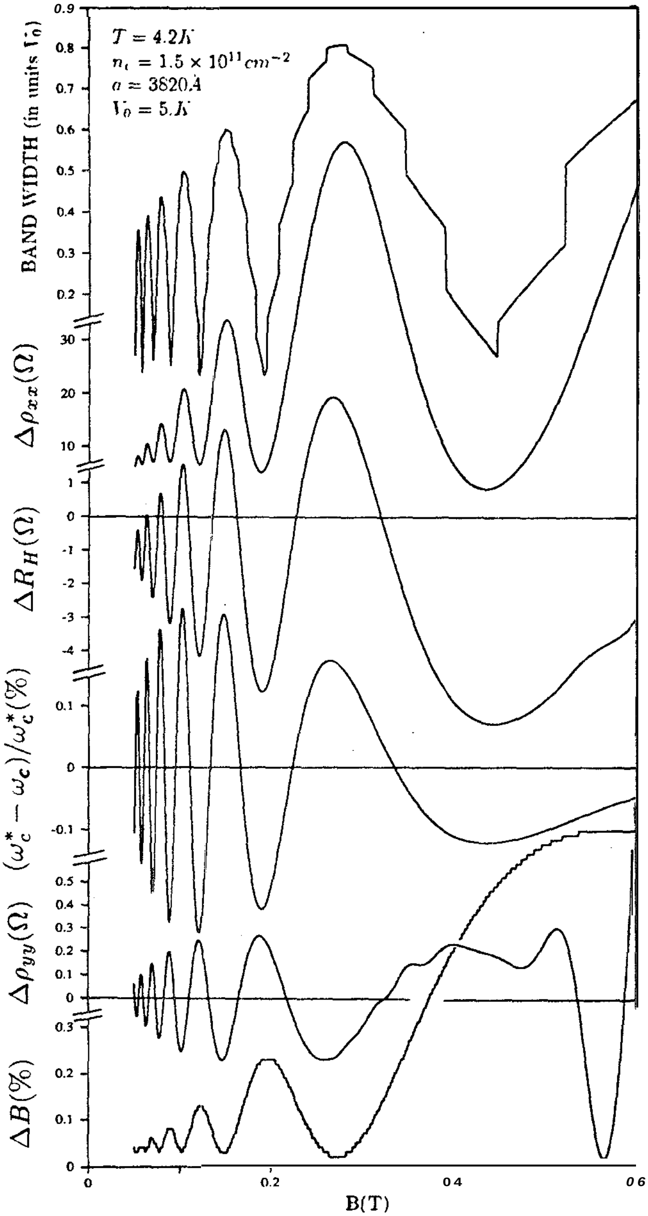

Fig.1 - The bandwidth at the Fermi energy, the correction to the magneto-resistances ρ_{xx} and ρ_{yy} and to the Hall resistance, the shift in the cyclotron resonance frequency and the oscillations in the linewidth are shown as function of the magnetic field. The 1D modulation of the 2DEG is along the x-direction.

with τ an energy-independent relaxation time which we have approximated by $r = \mu m^*/e$ where μ is the mobility of the 2DEG. It is evident from eq.(5) that $\Delta\sigma_{yy}$ and thus ***Ap,,*** is proportional to the square of the bandwidth at the Fermi energy. This is a generalization of the result of ref. **3** to arbitrary temperature which is the reason why **eq.** (5) also contains the SdH oscillations''. The result of eq. (5) leads to ***Ap,,*** which is shown in fig. 1; we took $\mu = 1.3 \times 10^6 cm^2/Vs$, $n_e = 1.5 \times 10^{11} cm^{-2}$ and a 1D modulation with period $a = 3820$Åand amplitude $V_0 =$ ***5.K.*** For $T = 4.2K$ and $B < .6T$ the SdH oscillations are not yet visible in ***Ap,,***. For $T = 4.2K$ the SdH oscillations appear for $B > .6T$. When $T < 4.2K$ they are also present for $B < .6T$ as seen clearly in fig. *2*. With increasing temperature the SdH oscillations weaken and disappear whereas the novel ones remain almost unaffected. The position of the extrema of the novel oscillations are accurately described by the condition $2R_c/a = n + 1/4$ for the maxima and $2R_c/a = n - 1/4$ for the minima as derived[3] from the asymptotic expression of L, (u) for large n.

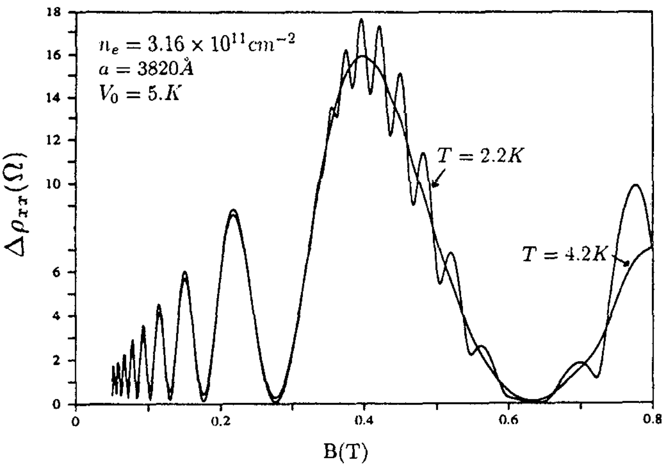

Fig.2 - The correction $\Delta\rho_{xx}$, due to the modulation V_0, to the magnetoresistance ρ_{xx} as function of the magnetic field B. Two kinds of oscillations are clearly seen for $T = 2.2K$. Those with the short period are the SdH oscillations which weaken with increasing temperature. Those with the long period are the novel oscillations.

The magneto-resistance alang the modulation $\rho_{yy} \approx$ **a,,** $/\sigma_{xy}^2$ is proportional to the conductivity σ_{xx} which has only collisional contributions and can be evaluated from eq. *(2)*

$$\sigma_{xx} = \frac{e^2}{h}\frac{N_I U_0^2}{\pi\Gamma}\frac{1}{a}\sum_n (2n+1)\int_0^{a/l^2} dk_y \beta f_{nk_y}(1-f_{nk_y}) \ , \tag{6}$$

where N_I is the impurity density with $U_0 = 2\pi e^2/\epsilon k_s$ the impurity potential in Fourier space in the limit $k_s >>$ q; k_s is the screening wavevector and ϵ the dielectric constant. In the absence of modulation eq. (6) gives the standard two-dimensional result. In the following we will calculate the correction due to the modulation: **Ap,,** $= \rho_{yy}(V_0) - \rho_{yy}(V_0 = 0)$ which is shown as the second curve from the bottom in fig. 1; the SdH oscillations are the short-period oscillations evident for B $\gtrsim .27T$. The conduction along the modulation occurs through hopping between the Landau states. This type of conduction is smallest (and thus also *p,,)* when the bandwidth at the Fermi level is largest, because then the electron is able to scatter into a broad band of states. This explains why the modulations in ρ_{yy} are out of phase with those of ρ_{xx}. The fact that the oscillations in ρ_{yy} are much weaker is also evident because they are only a consequence of small perturbations on the collisional current which is also present without the modulation. This is different from ρ_{xx} where the modulation opens up an *extra* conduction mechanism. The collisional contribution to the resistance *p,,* should also exhibit these out-of-phase oscillations but because they are at least an order of magnitude smaller than the diffusive channel, they will not be observable.

The Hall conductivity is evaluated from eq. **(3)** along the lines of ref. 6b. The results is

$$\sigma_{yx} = \frac{e^2}{m^*\omega_c \pi a}\sum_{n=0}^{\infty}(n+1)\int^{a/l^2} dk_y \frac{f_{n,k_y} - f_{n+1,k_y}}{\left[1+\lambda_n \cos\left(2u\frac{k_y}{K}\right)\right]^2} \ , \tag{7}$$

where

$$\lambda_n = \frac{V_0}{\hbar\omega_c} e^{-u/2} L_{n+1}^{-1}(u).$$

In the absence of the modulation and for strong magnetic fields eq. (7) leads to the integral quantum Hall effect[Gb]. $\Delta R_H = R_H(V_0) - R_H(V_0 = 0)$ versus the magnetic field is plotted in fig. 1. The oscillations are in phase with those of $\Delta\rho_{xx}$. The oscillations in R_H can be understood as follows. The quantization of the Hall resistance is a consequence of the fact that there is only one extended state in the center of the Landau level and that the Fermi level is pinned in the region of localised states between successive Landau levels. On the other hand the modulation leads to a band of extended states whose width oscillates with magnetic field. This leads two oscillations in σ_{yx}, σ_{xx}, and a,, . However, since σ_{xx} and a,, oscillate ***out of phase*** we have $\rho_{xy} \approx -1/\sigma_{yx}$, i.e. the (small) oscillations of a,, ***alone*** give rise to those of the Hall resistance. Consequently the latter are seen not to be connected with the Fermi level being no longer in a gap of extended states, as indicated by the finite σ_{xx} and σ_{yy}.

Previously Chaplik[12] has predicted that the cyclotron resonance of electrons in a lateral superlattice in a strong field perpendicular to the growth axis exhibits a two-peak structure due to the singular nature of the density of states (DOS) at the band edges. Up to now the experiments[13] have shown only a broadening of the linewidth. The present system under study is the weak modulation limit of the system studied in ref. 12. We found the following expression for the cyclotron resonance power spectrum

$$P(\omega) = \frac{e^2}{h}\frac{E^2}{2}(\hbar\omega_c)^2\frac{l^2}{a}\sum_{n=0}^{\infty}(n+1)$$
$$\int_0^{a/l^2} dk_y \frac{f_{n,k_y} - f_{n+1,k_y}}{\Delta_{n,k_y}} \frac{\Gamma}{(\Delta_{n,k_y} - \hbar\omega)^2 + \Gamma^2} . \qquad (8)$$

with $\Delta_{n,k_y} = \hbar\omega_c[1 + \lambda_n \cos(2\pi k_y l^2/a)]$, and E the strength of the oscillating electric field with frequency w. For the broadening a typical value of $\Gamma = 2$K was used, but we have checked that the numerical conclusions do not depend on the value of Γ. The numerical results for the percentage change in the position of the cyclotron resonance frequency and the linewidth are shown in fig. 1. The position of the cyclotron resonance frequency oscillates around the unperturbed value, it reaches a maximum at maximum bandwidth and is minimum at zero bandwidth correction. The width of the cyclotron resonance peak oscillates in phase with those of ρ_{yy}.

In conclusion we have presented a full quantum mechanical calculation of the resistivity tensor for a 2DEG in a weak 1D periodic potential. All available experimental data can be explained by our model. An interpretation of the antiphase oscillations in p,, has been given. New oscillations in the Hall resistance, the cyclotron resonance frequency and the cyclotron resonance linewidth are predicted. We found numerically that the amplitude of the oscillations increases quadratically with the amplitude of the modulation potential V,. Furthermore, lowering the electron density also increases the amplitude of the oscillations.

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10. The exact spectrum obtained numerically in ref. **3** is very close to that given by eq. **(6)** for n $\gtrsim$ **3.** In the experiments n $\gtrsim$ **10.**
11. **Eq. (5)** is valid only for $V_0 \neq 0$. If $V_0 = 0$ the diffusive contribution to the current vanishes and the SdH oscillations show up only in the collisional contribution given by eq. **(6)** for $V_0 = 0$, by symmetry then $\sigma_{xx} = \sigma_{yy}$.
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Resumo

O fenômeno recentemente descoberto, de oscilações da magnetoresistência de um gás de elétrons bidimensional, com modulação fraca e periódica em uma dimensão, é completamente explicado com a ajuda de uma equação de Boltzmann *quântica.* A largura de banda dos níveis de Landau alargados pela modulação, na energia de Fermi, oscila com o campo magnético dando origem às oscilações observadas. A magnetoresistência perpendicular à modulação (ρ_{xx}) é dominada por contribuições *difusivas* à corrente, as quais crescem com a largura de banda, enquanto a magnetoresistência paralela à modulação (ρ_{yy}) é dominada por contribuições *colisionais*, as quais decrescem quando a largura de banda aumenta. O tensor de resistividade é *assimétrico* e as componentes ρ_{xx} e ρ_{yy} estão fora *de fase*, tal como observado. Novas oscilações são previstas para a resistência de Hall, a posição da ressonância de cíclotron, e a largura da linha.