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A reduced covariant string model for the extrinsic string

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Abstract We study a reduced covariant string model for the extrinsic string by using Polyakov's path integral formalism. On the basis of this reduced model we suggest that the extrinsic string has its critical dimension given by 13. Additionally, we calculate in a simple way Polyakov's renormalization group law for the string rigidity coupling constant.

Some time ago it was proposed in literature a new kind of closed classical bosonic string depending in a explicitly way of the string's surface extrinsic curvature[1]. Such "elastic" string if quantized in a naive operator way has a non trivial perturbative renormalization effect: the inverse of the rigidity coupling constant obeys a renormalization asymptotic freedom law which signals this kind of string as a first effective string theory for the real $Q.C.D.(SU(\infty))$ string[2,3].

More recently it was showed by Kleinert[4] that a externa1 massive quantized scalar field interacting with the usual quantized Nambu-Polyakov string leads in the scheme of large mass to the above cited "elastic string" as a effective quantum theory.

Unfortunately a definite quantization scheme for the extrinsic string still be missing.

Our aim int this note is very modest: we write a covariant action for the elastic string and quantize in the Polyakov's path integral framework a truncated version of the covariant writen theory.

Let us start our study by considering the classical action for the elastic string in the conforma1 gauge.

$$S_0 = \frac{1}{2\pi\alpha}\int d^2 z\rho(2) + \gamma\int d^2 z\ \rho\Big[\Big(-\frac{1}{\rho}\partial^2 X\Big)^2 + i\frac{\lambda_{ab}}{\rho}\big(\partial_a X\partial_b X - \rho\delta_{ab}\big)\Big] \quad (1)$$

The string surface is described by $X = X(z)$, where X is the surface vector position in D Euclidean dimensions; z_a $(a=1,2)$ are the coordinates of the world sheet. The first term in eq. (1) is the Nambu term with the string tension equal to $1/2\pi\alpha$. The second term is the square of the extrinsic curvature with the rigidity coupling constant denoted by γ and $\lambda^{ab}(z)$ is a Lagrange multiplier which insures that the metric $(\rho\delta_{ab})$ coincides with the intrinsic metris $(\partial_a X\partial_b X)$.

Let us consider a covariant version of action eq. (1) by promoting $\rho(z) = g_{ab}(z)$ to be a dynamical field. This procedure yields the following action

$$S_1\big[X(z), g_{ab}(z), \lambda_{ab}(z)\big] = \frac{1}{2\pi\alpha}\int d^2 z\sqrt{g} + \int d^2 z\sqrt{g}\big[\gamma(-\Delta_g X)^2 + i\lambda_{ab}(g_{ab} - \partial_a X\partial_b X)\big] \quad (2)$$

Here $\sqrt{g(z)} = \mathrm{Det}(g_{ab}(z))$ and $\Delta_g = -\frac{1}{\sqrt{g}}\partial_a(g^{ab}\partial_b)$ is the Laplace-Beltrami operator associated to the intrinsic metric $g_{ab}(z)$.

In the Polyakov's path integral quantization framework the partition functional for the theory eq. (1) should be given by

$$Z = \int D^c[g_{ab}]D^c[X]D^c[\lambda_{ab}] \times \exp -S_1[X(z), g_{ab}(z), \lambda_{ab}(z)] \quad (3)$$

where the functional measures are the De-witt covariant functional measures[5].

Let us suppose that the constraint field is approximated by the intrinsic metric $\lambda_{ab}(z) = i < \lambda > g_{ab}(z)$ (The covariant version of the usual mean field approximation $\lambda_{ab}(z) = i < \lambda > \delta_{ab}$ with $< \lambda >$ a positive fixed value[6,7]. As a consequence of this hypothesis we get the truncated theory

$$Z_{(T)} = \int D^c[g_{ab}]D^c[X] \exp -S^{(T)}[g_{ab}(z), X(z)] \tag{4}$$

where the truncated action theory is written as

$$S^{(T)}[g_{ab}, X] = \frac{1}{2\pi\alpha}\int d^2 z\sqrt{g(z)} + \gamma\int d^2 z[(-\Delta_g X)^2] \\ + <\lambda> \int d^2 z\sqrt{g} g^{ab}\partial_a X \partial_b X + <\lambda> \int d^2 z\sqrt{g(z)} \tag{5}$$

For the evaluation of the X-functional integral in eq. (4) we consider the non-local variable change

$$X_\mu(z) = (-i(\Delta_g)^{1/2}\vartheta_\mu)(z) \quad \mu = 1, ..., D$$

Here $-i(\Delta_g)^{-1/2}$ is a well defined self-adjoint (pseudo-differential) operator. The truncated action takes the following form similar to a massive scalar field in the Z? domain:

$$S^{(T)}[g_{ab}, \vartheta] = \left(\frac{1}{2\pi\alpha} - <\lambda>\right)\int d^2 z\sqrt{g} + 2<\lambda>\int d^2 z\frac{1}{2}\vartheta^2 \\ + 2\gamma\int d^2 z\frac{1}{2}(\sqrt{g}\vartheta(-\Delta_g)\vartheta)(z) \tag{6}$$

The change in the (covariant) functional measure $D^c[x]$ is given by

$$D^c[x] = \left(\mathrm{Det}(-\Delta_g)^{-1}\right)^{D/2} \times D^c[v] \tag{7}$$

The main step in our calculation is to define the above written functional determinant as $\mathrm{Det}^{-D/2}(-\Delta_g)$. By chosing the conformal gauge $g_{ab} = e^\varphi\delta_{ab}$ and evaluating the covariant Gaussian ϑ-functional integral we obtain the partial result

$$Z_{(T)} = \int D[\vartheta]\exp\left(-\frac{26-D}{48\pi}\int\left[\frac{1}{2}(\partial_a\varphi)^2 + \mu_R^2 e*\right](z)d^2 z\right) \\ \mathrm{Det}^{-D/2}(-2\gamma\Delta_g + 2<l>) \tag{8}$$

where

$$\mu_R^2 = \lim_{\epsilon\to 0^+} \frac{2-D}{4\pi\epsilon} + \frac{1}{2\pi\alpha} - < l >$$

may be though as a renormalization of the bare string tension $1/2\pi\alpha$.

We analyze now the unrenormalized functional determinant

$$\exp - S_{EFF}[\varphi] = \mathrm{Det}^{-D/2}\left(- \Delta_g + \frac{< l >}{\gamma}\right) .$$

By defining it by a propertime prescription we obtain the counterterms of the above written action. Explicitly

$$S_{EFF}[\varphi] = \lim_{\epsilon\to 0} - \frac{D}{2}\int_\epsilon^\infty \frac{dT}{T}\mathrm{Tr}\left(\exp - T\left(- \Delta_g + \frac{< \lambda >}{\gamma}\right)\right) \tag{9}$$

Now it is well known that the counterterms of $S_{EFF}[\varphi]$ are determined by the asymptotic expansion of the diagonal part of massive Laplace-Beltrami operator which is tabulated[8]

$$\lim_{T\to 0}\mathrm{Tr}\left(\exp\left(- T\left(- \Delta_g + \frac{< \lambda >}{\gamma}\right)\right)\right)$$

$$= \int d^2 z\left\{\frac{e^\varphi}{2\pi}\lim_{T\to 0^+}\left(\frac{1}{T}\right) - \frac{1}{2\pi}\Delta\varphi + \frac{1}{2\pi}e^\varphi \cdot \frac{< \lambda >}{\gamma}\right\}(z) \tag{10}$$

By substituting eq. (10) in to eq. (9) we get straightforwardly the following counterterms associated to the two-dimensional intrinsince "mass" $< \lambda > /\gamma$

$$\frac{D}{2}\cdot\frac{\ell}{2\pi}\cdot\frac{< \lambda >}{\gamma}\lg\left(\frac{1}{\epsilon}\right)\int d^2 z e^{\varphi(z)} \tag{11}$$

So, on the basis of the counter term eq.(11) we have the following renormalization law for the inverse of the rigidity $\beta = 1/\gamma$ (by choosing $< \lambda >= 1$)

$$\frac{1}{\beta_R} = \frac{\ell}{\beta_0} - \frac{D}{2}\cdot\frac{1}{2\pi}\lg\left(\frac{1}{\epsilon}\right) \tag{12}$$

Eq.(12) yields the momentum dependence of the running coupling constant β [2,9])

$$\beta_R(p^2) = \beta_0(p^2 = 0) \Big/ 1 - \frac{D}{2}\frac{\beta_0}{2\pi} \cdot \lg\left(\frac{\epsilon}{p^2}\right) \tag{13}$$

It is instructive point out the $D/2$ factor in eq. (13) which appears in a natural way in our calculations (see the related comment in Polyakov's paper-ref.2).

Let us evaluate in a power series of $1/\gamma$ the Seff $[\varphi]$ in the $1/\gamma$ perturbative regime by considering the validity of the formal relationship

$$S_{\text{EFF}}[\varphi] = \text{Det}^{-D/2}(-\Delta_g) \cdot \text{Det}^{-D/2}\left(1 + \frac{<\lambda>}{\gamma}(-\Delta_g)^{-1}\right) \tag{14}$$

since γ is large we have the loop expansion

$$\text{Det}^{-D/2}\left(1 + \frac{1}{\gamma}(-\Delta_g)^{-1}\right) =$$
$$\exp\left(-\frac{D}{2}\sum_{N=3}^{\infty}\frac{N!}{\gamma^N}\int\prod_{i=1}^{N} d^2 z_i \left[\left(\prod_{i=1}^{N-1}(-\Delta_g)^{-1}(Z_i, Z_{i+1})\right)\right.\right.$$
$$\left.\left.(-\Delta_g^{-1})(Z_N, Z_1)\right]\right) \tag{15}$$

Since we have the relationship[10]

$$\left(-\Delta_{g=\rho\delta_{ab}}\right)^{-1} = \begin{cases} \dfrac{\lg\rho(z)}{2\pi} - \dfrac{1}{2\pi}\lg\epsilon & z_1 = z_2 \\ \\ -\dfrac{1}{4\pi}\lg|z_1 - z_2| & z_1 = z_2 \end{cases} \tag{16}$$

we can obtain all the (unrenormalized) coeficients of eq. (15). For instance in the case of one loop N = 2 we have exactly

$$\exp\left(-\frac{D}{2}\cdot\frac{2}{\gamma^2}\left[\int d^2 z\left(\frac{\lg\rho(z)}{2\pi} - \frac{1}{2\pi}\lg\epsilon\right)^2\right.\right.$$
$$\left.\left. + \int_{z_1 \neq z_2} d^2 z_1\, d^2 z_2\left(-\frac{1}{4\pi}\lg|z_1 - z_2|\right)\right]\right) \tag{17}$$

As an important remark let us point out that eq. (15) does not depends on the derivatives of the $\lg\rho(z)$ field as one can see from the form of eq. (15) and eq.

(16). So the whole dependence on the derivatives of the $\lg\rho(z)$ field in the Seff [(o] comes from the term $(\varphi(z) = \lg\rho(z))$.

$$\mathrm{Det}^{-D/2}(-\Delta_g) = \exp + \frac{D}{48\pi}\int \frac{1}{2}(\partial_a\varphi)^2(z)d^2z$$

$$\exp \lim_{\epsilon\to 0^+} \frac{2-D}{4\pi\epsilon}\int e^{\varphi(z)}d^2z \tag{18}$$

By combining eq.(8) with eq. (14) - eq. (18) we have that the conforma1 factor $\varphi(z)$ will not be dynamical at space-time dimension $D_c = 26/2 = 13$ which may be considered as the critical dimension for the truncated theory.

This result lead us to conjecture that the "elastic" string lives (in a quantum mechanical sence) in a space-time with dimensionality equals to 13.

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Resumo

Nesta nota estudamos um modelo covariante reduzido para a teoria das cordas com rigidez. Calculamos exatamente a dimensão crítica D_c para o modelo covariante reduzido, sendo $D_c = 13$. Este resultado nos leva a sugerir que a teoria das cordas com rigidez também possue a mesma dimensão critica do modelo proposto $D_c = 13$.