Revista Brasileira de Física, Vol. 19, nº 1, 1989

Ultimate temperature for laser cooling of two-level neutral atoms

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Received on September 15, 1988

Abstract We present a simple pedagogical method to evaluate the minimum attainable temperature for laser cooling of two-level neutral atoms. Results are given as a function of the laser detuning and intensity. We also discuss the use of this approach to predict the minimum temperature of neutral atoms confined in magnetic traps.

1. Introduction

There has been as increasing interest in laser cooling and trapping of neutral atoms during the past few years. This interest arises mainly from the need to reduce high order Doppler shifts in spectroscopic measurements and from the possibility of having a very hight density sample of extremally cold atoms, aiming the observation of collective quantum effects such as Bose-Einstein condensation and the study of collisions of cold atoms[1].

During the laser cooling process, the random nature of the spontaneous emission introduces a heating effect which limits the minimum attainable temperature. From the experimental point of view, the knowledge of this temperature and its relationship to the laser detuning and intensity is very important. Calculations based on the cross section of the anti-Stokes spontaneous Raman scattering describing the cooling process were already carried out by Wineland and Itano[2]. An alternative approach to this problem, based on the analysis of the atom diffusion in momentum space, was presented by C .Cohen-Tannoudji[3].

In this paper we describe a simple and pedagogical method to evaluate the ultimate attainable temperature as a iunction of laser detuning and intensity. Part of the method uses the diffusion of the atom in momentum space and is therefore similar to the one presented in ref. 3. However, the simplicity which characterizes this approach makes it suitable to analyse even trapped atoms subjected to magnetic fields.

2. Radiative forces

Two types of forces can be produced during the interaction of electromagnetic radiation with an atomic system. First, there is a spontaneous force which originates from the momentum transferred to the atom during the photon absorption followed by the spontaneous emission. Second, there is an induced force which comes from the interaction of the electric dipole induced during the electronic transition with gradients of the radiation intensity.

The usual approach to calculate the radiative force consists in treating the atoms as a two-level quantum system and the radiation as a classical electromagnetic field[4]. In this approach the radiation force is given by

$$\vec{F} = -\frac{\hbar\Gamma\Omega^2\nabla\Theta + \hbar(\Delta+\dot{\Theta})\nabla\Omega^2}{4(A+\dot{\Theta})^2+\Gamma^2+2\Omega^2} \tag{1}$$

where $\Omega = \mu E(\vec{r},t)/\hbar$ is the Rabi frequency, $\mathbf{A} = \omega - \omega_0$ is the detuning between the laser frequency and the atomic resonance frequency, Γ is the transition linewidth and $\boldsymbol{O}$ is the spatial phase of the electromagnetic field. In eq.(1) the term with the gradient of the phase represents the spontaneous force while the term with the gradient of the intensity for the induced force. When a plane wave is considered, we have just the spontaneous force given by

$$\vec{F} = \frac{\Gamma\Omega^2\hbar\vec{k}}{4(\Delta-\vec{k}\cdot\vec{v})^2+\Gamma^2+2\Omega^2} \tag{2}$$

where $\vec{k}$ is the wavevector and $\vec{v}$ is the atomic velocity.

The use of this force to cool neutra1 atoms was independently suggested by two groups[5,6]. Let us consider an atom with velocity $\vec{v}$ in the presence of two counter-propagating laser beams with the same frequency, as shown in fig. 1. For small velocities, the net force acting on the atom is

$$F_z = 2\Gamma\hbar k^2 \frac{s}{(1+s)^2} \frac{\Delta}{\Delta^2 + \frac{1}{4}\Gamma^2} v_z$$
$$F_x = F_y = 0 \tag{3}$$

where

$$s = \frac{\frac{1}{2}\Omega^2}{\Delta^2 + \frac{1}{4}\Gamma^2}$$

is the saturation parameter for a single beam. In the present treatment we will consider $s << 1$ (negligible saturation). Otherwise, effects such as stimulated emission have to be taken into account, which makes the situation very difficult to be analysed and more general treatment, such as the one presented in ref. 3, have to be used. For negative detunings eq. (3) represents a viscous force in the direction of the laser beams. Therefore, the configurations of fig. 1 enerates the so-called "one dimensional molasses" in analogy to the three dimensional molasses demonstrated by S. Chu and others[7]. This viscous force has ben used to cool atoms in atomic beams[S] and also trapped atoms .

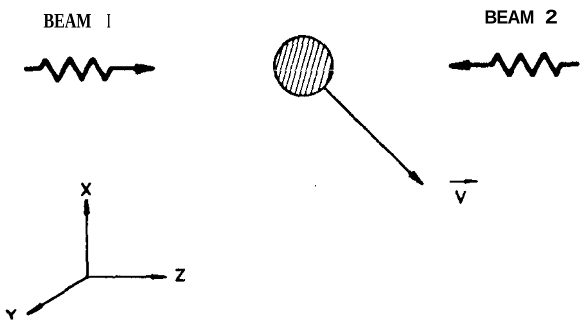

Fig.1 - Configuration for one-dimensional cooling with two counter-propagating plane waves.

3. Energy balance and ultimate temperature

Within the approach of considering the electromagnetic field as classical, one is left just with the viscous force given by eq. (3), which will bring the kinetic energy of the atom to zero. However we know that spontaneous emission, which appears naturally when the field is quantized, plays an important role in the cooling mechanism. Therefore, it is necessary to introduce an extra term into the rate equation for the kinetic energy. In the configuration of fig. 1, this rate equation is given by

$$\frac{dK_i}{dt} = v_i F_i + H_i \tag{4}$$

where $K_i = \frac{1}{2}mv_i^2$, F_i is the i^{th} component of the force given by eq.(3) and H_i represents a heating term associated to the spontaneous emission.

In order to find H_i one can look at the path followed by the atom in momentum space after one absorption-emission cycle. If the initial momentum is located at point 0 of fig.2, the absorption will take it either to point 1 or 2, which corresponds to a momentum transfer of $\hbar\vec{k}$. The spontaneous emission, assumed here to be isotropic, will take the atom to the ending point 0, due to the recoil of the emitted photon. We can see that this process can have an ending point anywhere on the spherical surface with radius $\hbar k$ shown in fig.2, which has rotational symmetry around the P_z axis. This leads to an average variation of the momentum components given by $<(\Delta P_x)^2>=<(\Delta P_y)^2>=(\hbar k)^2/3$ and $<\Delta(\Delta P_z)^2>=4(\hbar k)^2/3$. Therefore, the heating rate is given by

$$H_i = 2\frac{\Gamma s}{2}\frac{<(\Delta P_i)^2>}{2m} \tag{5}$$

where we have assumed $\Gamma s/2$ as the absorption and emission rate and the factor 2 arises due to the fact that we have two waves ($+z$ and $-z$ directions) and s is taken for a single wave. Eq. (4) can be solved for each K_i component and after adding them we get

$$K(t) = K_0 + \left(e^{-t/\tau} - 1\right)K_{0z} + \beta\left[t - \tau\left(e^{-t/\tau} - 1\right)\right] \tag{6}$$

where K_0 is the initial kinetic energy of the atom

$$K_{0z} = \frac{1}{2} m\, v_{0z}^2$$

$$\beta = \frac{2}{3} \Gamma s \frac{(\hbar k)^2}{m}$$

and

$$\tau = \frac{\left(\Delta^2 + \frac{1}{4}\Gamma^2\right) m}{4\Gamma \hbar k^2 s |\Delta|}$$

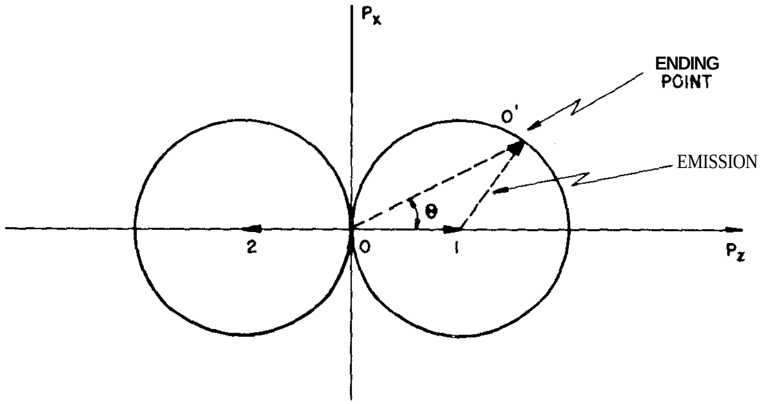

Fig.2 - Cross section of the surface in momentum space showing points where the atom may end after a cooling cycle.

A plot of eq.(6) is shown in fig. 3. As we can see, the best one can do with one-dimensional molasses is to cool the sample down to K_{min}. However, this has to be done in a time of the order of t_{min}, which usually is in the μs range.

The configuration discussed above is not unreal, since for many trao configurations the use of magnets to produce high fields may limit the optical access to the atoms. In this way, laser pulses may be a way to get cooler atoms using the Doppler cooling technique. If the time is much longer than t_{min}, the present configuration does not produce an effective cooling, since the energy is transferred from the laser beam direction to directions transverse to it. However, one can

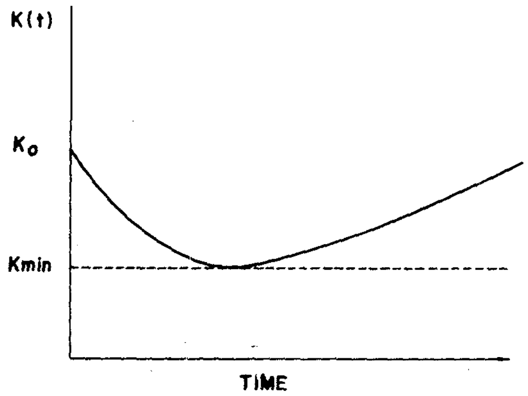

Fig.3 - Time evolution of the kinetic energy in the z-direction for the one-dimensional cooling.

think of alternating the laser among the three directions (x, y and z). We have performed calculations concerning this situation and the results show that for cooling times (time that the laser spends in a specific direction) shorter than t_{min}, we always get the minimum kinetic energy as in the case of six laser beams discussed below.

Another possible situation is the three dimensional configuration of laser beams shown in fig. 4, where six resonant laser beams illuminate the atom. In this case, the evaluation of the heating term is carried out by averaging the momentum in a surface made by six spheres with a common point. Fig. 5 shows a cross section of this surface. Now, the average momentum after one absorption-emission cycle is given by $<\Delta(\Delta P_i)^2>=2(\hbar k)^2/3$. Therefore, the rate equation for the kinetic energy is given by

$$\frac{dk}{dt}=2\Gamma\hbar k^2 s\frac{2\Delta}{\left(\Delta^2+\frac{1}{4}\Gamma^2\right)}\frac{k}{m}+3\Gamma s\frac{(\hbar k)^2}{m} \tag{7}$$

where the factor 3 present in the last term arises due to the fact that for six laser beams, one has three times more heating than in the two bearns (one-dimensional) configuration.

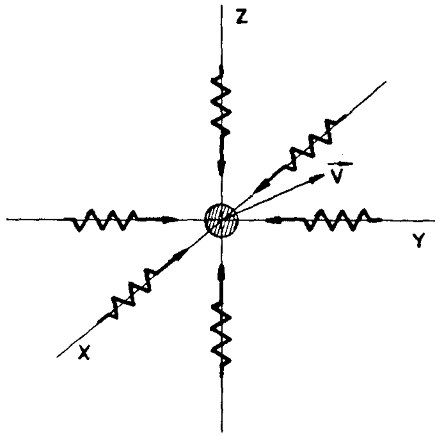

Fig.4 - Laser configuration for the three-dimensional cooling scheme.

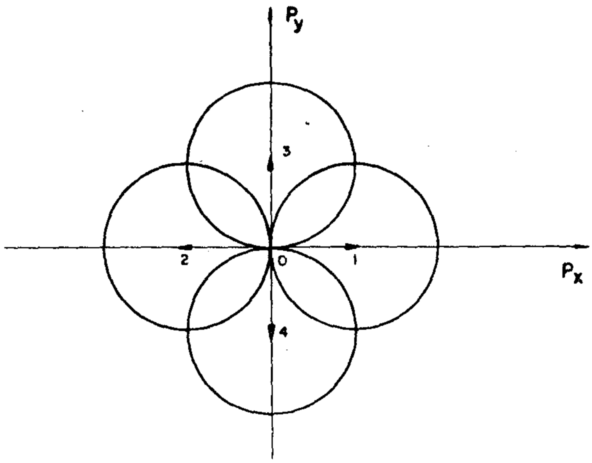

Fig.5 - Cross-section of the surface in momentum space showing possible positions for the atom after a cooling cycle.

In equilibrium we have $dK/dt = 0$ and this gives us the asymptotic kinetic energy for the process

$$K_{min} = \frac{3\hbar\left(\Delta^2 + \frac{1}{4}\Gamma^2\right)}{4|\Delta|} \tag{8}$$

The minimum kinetic energy has an optimum value for $\Delta = -\Gamma/2$, corresponding to $K_{min} = 3\hbar\Gamma/4$. This is in agreement with the previous result of Wineland and Itano[2]. Eq. (7) has as solution

$$K(t) = (K_0 - K_{min}e^{-t/\tau} + K_{min} \tag{9}$$

where K_0 is the initial kinetic energy and

$$\tau = \frac{\left(\Delta^2 + \frac{1}{4}\Gamma^2\right)}{4\Gamma\hbar k^2|\Delta|}\frac{m}{s}$$

is the cooling time.

The results presented above are valid for a free sample of two-level atoms submitted to a resonant laser light, in the form of a plane wave. However, it can also be used for several trap configurations including magnetic fields or light fields, because in these cases the atoms are so limited in space at the end of the cooling process that they do not suffer anly large variation of the resonance frequency due to these fields.

As a numerical example, let us consider sodium atoms, which have $\Gamma \cong 10MHz$. In this case, the minimum temperature $(3/2k\ T_{min} = K_{min})$ achievable under optimum conditions $(\Delta = -\Gamma/2)$ is 240μK. This has been observed experimentally . In the case of magnetically confined atoms[10] cooled by laser light, one has to pay attention to the fact that even at temperatures as low as 500 μK, the atom has enough energy to travel in a magnetic field gradient of the order to 10 G, which.gives an extra detuning of 15 MHz. In this case, since Δ may vary from $\Gamma/2$ to 2Γ, one does not operate with the optimum detuning and T_{min} is of the order of 2 mk, much higher than the previous result.

Finally, we may still find cases where trapped atoms are submitted to only two counter-propagating laser beams but the asymmetry of the trap field will produce a mixture of independent motions so that the effect is equivalent to the three dimensional cooling, but having a different cooling time.

The above calculations show that the limiting temperature for laser cooling is of order 240 μK for sodium atoms. This is only true in the special approximation of plane wave and a two-level atom. In fact, temperatures as lows as 40 μK have been observed[11] for optically trapped sodium atoms. Of course this shows that the model we have assumed above is too simple to describe the reality of the atom-field interaction. However it is useful due to its simplicity and applicability in some cases.

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Resumo

Apresentamos um método pedagbgico simples para calcular a temperatura mínima no resfriamento com laser de átomos neutros de dois níveis. Os resultados são apresentados como função da dessintonia e intensidade do feixe de laser. Discutimos o uso deste método para predizer a temperatura mínima de átomos neutros confinados em armadilhas magnéticas.