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Inclusion of the diffuseness in the schematic model of heavy ion collisions

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Abstract The schematic model of central heavy ion collisions developed by Swiatecki includes the Coulomb and surface contributions to the potential energy of the system and one-body dissipation. In this work the model is extended by considering the diffuseness of the nuclear surface; this has the implication that we must consider the proximity forces in the dynamics of the collisions. For the sake of simplicity we work with symmetrical systerns. The results of the model studied are compared with experimental data and with other theoretical calculations. We conclude that the detailed consideration of the diffuseness of the nuclear surfaces does not substantially change the results of the schematic model for sharp surfaces in which the diffuseness is considered only through the pararneters.

1. Introduction

Recent experimental data' confirm the results obtained by Swiatecki[2] with a schematic model, which considers one-body dissipation and the Coulomb and surface contributions to the potential energy of the system formed in a low energy nucleus-nucleus collision.

The calculation in ref. 2 do not include the diffuseness in a detailed form, but only in the chosen parameters. Here we consider that effect expecting to verify whether the agreement between theory and experiment is maintained. Only symmetrical systems are considered, in which projectile and target are identical nuclei with mass number A, and atomic number Z_1, and central collisions.

2. Parametrization of the system

When the nuclei come into contact they develop a neck between them. We parametrize that system schematically as in ref. 2, by two spheres each of radius R, whose centers are separated by a distance r, with a cylindrical neck between them of radius n, as shown in fig. 1. For a small neck whose volume can be neglected the radii of the spheres are approximately constant and can be calculated by the expression $R_1 = r_0 A_1^{1/3}$, if we admit that the nuclear fluid is incompressible.

We use Swiatecki's "elegant variables"[2] a and v defined by the equations

$$\begin{aligned} \rho^2 &= 1 + \sigma \\ \rho &= \frac{n}{2R_1} \\ \nu &= \frac{n}{R_1} \end{aligned} \tag{1}$$

the values of which are bounded by $-1 \leq \sigma < \infty$, $0 \leq \nu \leq 1$ and having the constraint imposed by the existence of a minimum value that the neck radius can have when the two spheres intersect. For a small neck this constraint can be written as $a \geq -\nu^2$. When we consider the diffuseness this inequality is substituted by eq. (17) of section 5.

The case in which $\sigma = 0$ and $\nu = 0$ corresponds to the configuration where the spheres are tangent (there is no neck), and when $a = -1$ and $\nu = 1$ it corresponds to the configuration where the system reduces to a single sphere (compound nucleus).

3. Potential energy of the Swiatecki model

In this model the nuclear system is considered as a macroscopic leptodermous (the surface thickness is small relative to the other relevant dimensions) system[3] in which the potential energy leading terms are the surface energy V, and the Coulomb electrostatic energy V_0.

The surface energy is proportional to the system's surface, the proportionality constant being the coefficient of surface tension v. The system's surface is equal

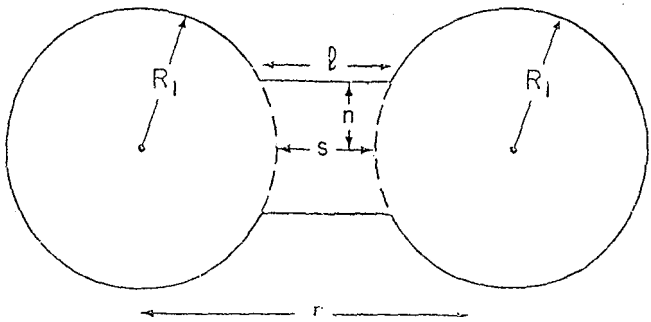

Fig.1 - Parametrization of the system.

to that of the two spheres excluding the surface of the caps delimited by the neck, plus the neck surface.

If we consider the case in which a and v are small of order ϵ^2 and ϵ respectively and we retain terms to the order ϵ^3 (see ref. 2), the radius of the spheres remains constant. The caps surface is approximately πn^2 for each of them and the neck surface is $2\pi n\ell$ where $\ell \cong s + \frac{n^2}{R_1}$ is the neck length and s is the separation between the tips of the spheres (see fig. *1)*. In this approximation

$$V_s = 2\pi R_1^2 \gamma(\nu\sigma - \nu^2 + n^3) \tag{2}$$

and the Coulomb potential energy is taken, along ref.3, as

$$2\frac{3}{5}\frac{(Z_1 e)^2}{R_1} + \frac{(Z_1 e)^2}{2R_1 + s}$$

and for small s, dropping constant terms, we obtain

$$V_c = -2\pi\gamma R_1^2 \frac{5}{6}\alpha x\sigma \tag{3}$$

where

$$x = \frac{3Z^2 e^2}{40\pi\gamma R^3}$$

is the fissility parameter, $Z = 2Z_1$ is the total electric charge of the system (in units of *e)* and $R = 2^{1/3} R_1$ is the compound nucleus radius. For convenience we add a factor a, $a = 1$ being deduced from the preceding equations.

A major assumption in the Swiatecki model is that it admits that equations 2 and 3 are valid even when a and v are not small quantities. That author takes $a = 6/5$, the result of this being that the saddle point of the potential energy $V = V_1 + V_2$ corresponds to the spherical configuration of compound nucleus $(a = -1, \nu = 1)$ when $x = 1$, in accord whith the classical theory of nuclear fission.

4. One-body dissipation

Another ingredient of the model is the dissipation produced by the fact that the particles experience collisions with the "walls" of the system giving energy to the individual particles' degrees of freedom. This process is know as one-body dissipation[3]. Simple formulae can be written in three particular cases as described below.

For a system without a neck we have the *wall dissipation* formula for the energy loss of the collective degrees of freedom[3]

$$\dot{Q} = \frac{dE}{dt} = \rho \bar{v} c \oint (v_n - D)^2 dS \tag{4}$$

where ρ is the nuclear mass density, $\bar{v}$ the mean nucleonic speed; v_n the normal velocity of the element dS of the system's surface and D is the normal component of the drift velocity for the particles in the vicinity of the system's surface, given by

$$D = (\mathbf{v} + \vec{\nabla} \times \mathbf{R}) \cdot \mathbf{N} \tag{5}$$

with v and $\vec{\nabla}$ being solutions of the equations

$$\oint \left[v_n - (\mathbf{v} + \vec{\nabla} \times \mathbf{R}) \cdot \mathbf{N} \right] dS = 0 \tag{6}$$

$$\oint \left[v_n - (\mathbf{v} + \vec{\nabla} \times \mathbf{R}) \cdot \mathbf{N} \right] (\mathbf{R} \times \mathbf{N}) dS = 0 \tag{7}$$

In these equations R is the vector from the system's center of mass to the surface element dS and N is a normal unit vector at that point of the surface.

The second situation is that in which we have a system composed by two rigid spheres in communication through a small window of area AS. In that case the ***window dissipation formula***[3] is applicable

$$\dot{Q} = \frac{1}{4}\rho\bar{v}\Delta S\left(u_t^2 + 2u_r^2\right) \tag{8}$$

where u_t and u_r are the tangential and radial components of the relative velocity of the two rigid portions of the system. In this work we consider only central collisions in which u_t vanishes.

The third situation is that in which the two parts described in the above paragraph are deformable. Here we apply the ***wall and window formula***[3]

$$\dot{Q} = \frac{1}{4}\rho\bar{v}\Delta S\left(u_t^2 + 2u_r^2\right) + \rho\bar{v}\int_1 \left(v_n - D_1\right)^2 dS + \rho\bar{v}\int_2 \left(v_n - D_2\right)^2 dS \tag{9}$$

where the subscripts *1* and 2 refer to the two parts of the system mentioned before.

We use eq. *(4)* in the case in which the neck is large and eq. (9) when it is small. As we do not have a formula applicable in the intermediate case we follow the prescription[2,4]

$$\dot{Q} = F(\nu)\dot{Q}\ (\text{eq.9} + (1 - F(\nu))\dot{Q}\ (\text{eq.4}) \tag{10}$$

in which $F(\nu)$ is a function subjected to the conditions $F(0) = 1$ and $F(1) = 0$. Swiatecki[2] uses a step function which goes from 1 to 0 when $\nu^2 = 1/2$, whereas the authors of ref. 4 take $F(\nu) = \cos^2 \frac{\pi\nu}{2}$. In this work we use the latter expression.

The procedure to follow is, as in the case case of the potential energy, to calculate $\dot{Q}$ from eqs. *(4)* and *(9)* for a small neck using the chosen parametrization, to express the result as a function of the ***elegant variables*** σ and ν and to use the result thus obtained for any neck[2].

In the case of the wall formula (eq.(4)) the indicated procedure gives

$$\dot{Q}_p = 2\pi\rho\bar{v}R_1^4\left(\frac{1}{3}\tilde{o}^2 + (u+\nu^2)\nu\dot{\nu}^2\right) \tag{11}$$

whereas the wall and window formula gives[4]

$$\dot{Q}_{p+v} = \frac{1}{2}\rho\bar{v}\Delta S u_r^2 + \sigma\bar{v}\int_1 (v_n - D_1)^2 dS + \rho\bar{v}\int_2 (v_n - D_2)^2 dS + \rho\bar{v}\int_3 (v_n - D_3)^2 dS$$

where the two spheres and the neck are denoted by the subscripts 1, 2, and 3 respectively. After performing the integration, we obtain

$$\dot{Q}_{p+v} = 2\pi\rho\bar{v}R_1^4\left(\frac{1}{4}\nu^2\dot{o}^2 + (o+\nu^2)\nu\dot{\nu}^2\right) \tag{12}$$

Following eq.(10) we take

$$\dot{Q} = 2\pi\rho\bar{v}R_1^4\left[(u+\nu^2)\nu\,\dot{\nu}^2 + \left(F(\nu)\frac{\nu^2}{4} + \frac{1}{3}(1-F(\nu))\right)\dot{o}^2\right] \tag{13}$$

with the Rayleigh dissipation function, $\Im$, given by

$$\Im = \frac{1}{2}\dot{Q} \tag{14}$$

in which we use eq. (13) for Q.

5. Kinetic energy

We approximate the kinetic energy by the expression[4]

$$T = \frac{1}{2}\mu\dot{r}^2 = \frac{1}{8}MR_1^2\dot{o}^2 \tag{15}$$

where $\mu = M_1/2$ is the reduced mass and M $= 2M_1$ is the system's total mass. (Swiatecki[7] takes this expression for $\nu^2 < 1/2$ and neglects the kinetic energy when $4 > \frac{1}{2}$).

6. Nuclear diffuseness

As we have said, in this work we include the effects produced by the diffuseness of the nuclear system in the model studied. There are two aspects of the problem to be considered; one is purely geometrical and consists in that if we follow Swiatecki's prescription[5] and parametrize the system along the contour with density one half the bulk density of the nucleus and maintain the notation of fig. 1 (now the lines are the half-density contours), a geometrical neck appears when $s = s_1 \simeq 1.73$ fm because that position is the beginning of the superposition of the $\rho/4$ density contours. (p is the bulk density of the nucleus). The radius of this geometrical neck is given by

$$n_g = \sqrt{R_1(s_1 - s)} \tag{16}$$

where we have neglected higher order terms in s/R_1. Because of this the configuration space is limited by the inequality

$$n \geq n_g \tag{17}$$

We remark that this neck does not correspond, generally, to the surface of a leptodermous system, because if the neck radius is small enough there is no region in the cylinder for which the density is equal to the bulk density of a nucleus. For that reason we define an effective neck length as

$$\ell_{eff} = \ell - s_1 \tag{18}$$

which we use to calculate the surface potential energy and the wall dissipation term of the neck. The eqs. (2) and (13) are replaced by

$$V_s = 2\pi\gamma R_1^2 \left[\nu\left(\sigma - \sigma_1 + \nu^2\right) - \nu^2\right] \tag{19}$$

$$= 2\pi\rho\bar{v}R_1^4 \left[(\sigma - \sigma_1 + \nu^2)\nu\,\dot{\nu}^2 + \left(F(\nu)\frac{\nu^2}{4} + \frac{1}{3}((1 - F(\nu))\right)\dot{\sigma}^2\right] \tag{20}$$

where

$$\sigma_1 = \frac{s_1}{R_1} \tag{21}$$

The choice of ℓ_{eff} as given by eq. (18) is similar, in some sense, to what Swiatecki calls a *proximity correction* in ref. 3 and it seems a reasonable guess, since in the initial instant of the collision where $s = s_1$ and n = 0 we have $\ell_{eff} = 0$, in agreement with the fact that in this instant the maximum density in the neck is $\rho/2$ (there is not a leptodermous neck). If additionally we suppose that the evolution of the system is such that $n = n_g$ the half-density contours touch when $s = 0$, in this instant begins the formation, in the neck, of a contour with density p which has a null length; this is consistent with the fact that in this configuration $\ell_n = 0$.

The second aspect related to the above considerations is that if we consider the nuclear diffuseness we must consider also the proximity potential energy, which can be written[5,6] .

$$V_p = \sum_{n=1}^{N} n c_n \xi_{n-1} \tag{22}$$

where

$$E_n(\zeta b) = 2\gamma b^{n+1} \Phi_n(\zeta) \tag{23}$$

the Φ_n functions being defined in ref. 5 where they are also tabulated for the first few values of n. The c_n coefficients are characteristic of the interacting surfaces, whereas b is the thickness of these surfaces.

In the parametrization in section 2 the contributions of the facing spherical parts are given by eqs. (22) and (23) with[6] N = 2, $c_1 = R_1 + \frac{1}{2}s'$, $c_2 = -1/4$ and $\zeta = \ell'/b$ where s' and ℓ' are defined by the relations

$$s' = s + 2\frac{b^2}{R_1} \tag{24}$$

$$\ell' = \ell + 2\frac{b^2}{R_1} \tag{25}$$

The second term on the right hand side in eqs. (24) and (25) appears because in the expressions for the proximity energy we must consider the central surfaces and not the equivalent sharp surfaces[6]. There is no contribution of the cylindrical neck to the proximity energy. All that gives us

$$V_p = 2\pi\gamma b\left[\left(R_1 + \frac{1}{2}s'\right)\Phi_0(\zeta) - \frac{b}{2}\Phi_1(\zeta)\right] \tag{26}$$

with

$$\zeta = \frac{R_1}{b}(\sigma + \nu^2) + 2\frac{b}{R_1} \tag{27}$$

and

$$s' = R_1\left(\sigma + 2\frac{b^2}{R_1^2}\right) \tag{28}$$

In eqs. (19), (20), (27) and (28) we are using the small neck approximation, but in the spirit of the Swiatecki model we use it for arbitrary neck sizes.

We will take Φ_0 from the approximate analytic expression indicated in ref. 6

$$\Phi_0(x) = \begin{cases} -1.7817 + 0.9270x + 0.01696x^2 - 0.05148x^3 & \text{if } 0 \le x \le 1.9475 \\ -4.41\exp(-x/0.7176) & \text{if } x \ge 1.9475 \end{cases} \tag{29}$$

but we only need the derivative of Phi_1 since it is that which appears in the equations of motion and it can be obtained from the relation

$$\frac{d\Phi_1(x)}{dx} = x\frac{d\Phi_0(x)}{dx} \tag{30}$$

which is easily deduced from the equations given in ref. 5.

For the Coulomb potential energy we use eq. (3) with $\alpha = 1$ since no attempt of a parameter adjust such as that indicated in section 3 has been made bacause the potential energy considered can be inappropriate for near spherical configurations.

7. Equations of motion and parameter values

The equations of motions are obtained from the Lagrange equations

$$\frac{d}{dt}\left(\frac{\partial \mathcal{L}}{\partial \dot{q}_i}\right) - \frac{\partial \mathcal{L}}{\partial q_i} + \frac{\partial \mathfrak{F}}{\partial \dot{q}_i} = 0; \quad i = 1,2 \tag{31}$$

where $q_1 = a$, $q_2 = \nu$, $\mathcal{L} = T - V$ is the Lagrangian of the system and $\mathfrak{F}$ is the Rayleigh dissipation function given by eq.(14).

The equations of motion obtained are

$$M\ddot{\sigma} + 8\pi\gamma\left(\nu - \frac{5}{6}x\right) + 8\pi\rho\bar{v}R_1^2\left[F(\nu)\frac{\nu^2}{4} + \frac{1}{3}(1 - F(\nu))\right]\dot{\sigma} + \left\{4\pi b\gamma\left[\frac{1}{R_1}\Phi_0(\zeta) + \frac{2-\nu^2}{b}\frac{d\Phi_0(\zeta)}{d\zeta}\right]\right\} = 0$$

$$\rho\bar{v}R_1^2(\sigma + \nu^2)\nu\,\dot{\nu} + \gamma(\sigma + 3\nu^2 - 2\nu) + \left\{\gamma\nu(2-\nu^2)\frac{d\Phi_0(\zeta)}{d\zeta} - \rho\bar{v}\sigma_1 R_1^2\nu\dot{\nu} - \gamma\sigma_1\right\} = 0$$

in which the terms in braces are those introduced to take into account the diffuseness. The initial conditions are now $\sigma(0) = a$, and $\nu(0) = 0$. In this work we use the following set of parameters[4,5,7]

$$r_0 = 1.16 \text{ fm} \tag{32}$$

$$\gamma = 0.951569(1 - 1.7826\, I^2); \quad I = 1 - \frac{2Z_1}{A_1} \tag{33}$$

$$b = 1 \text{ fm} \tag{34}$$

$$\rho\bar{v}c = 29.723 \quad (c \text{ is the speed of the light}) \tag{35}$$

Given the mass number A, of the nucleus in tbe collision we calculate their charge from the relation

$$Z_1 = \frac{1}{2}A_1\left(1 - \frac{0.4\,A_1}{A_1 + 200}\right) \tag{36}$$

so that the nuclei are in the beta stability valley.

8. Trajectories

We will show some trajectories in the configuration space. The possibility suggested in ref. 3 that initially the system goes into a *geometrical regime* in which $n = n_n$ does not happen in our model since the neck growth is very fast at the beggining of the reaction.

We take as an example the symmetric system in which $A_1 = A_2 = 104$. We show in fig. 2 a case in which the system is initially with zero radial kinetic energy. The neck is quickly developed but the distance between the centers of the spheres increases since the Coulomb repulsion predominates against the attractive nuclear forces. The neck obtains a maximum size, then decrease, and finally vanishes. This process lasts 2.0×10^{-21} sec.

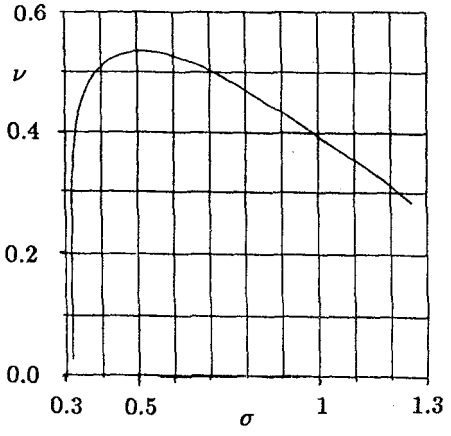

Fig.2 - Trajectory of the system in the a, v plane for the case A, = 104 with zero kinetic energy in the initial instant in which $\sigma \simeq 0.32$ and $\nu = 0$.

In fig. 3 the system evolution is shown when the radial kinetic energy is initially 2 MeV. The distance between the centers begins to decrease but the energy given to the system is not enough to pass the saddle point of the potential energy landscape and so arrive at the compound nucleus configuration. The duration of this process is longer than that in the preceding case: 5.2×10^{-21} sec.

In **fig. 4** we see a third case in which the initial kinetic energy (5 **MeV**) is enough to arrive at the compound nucleus configuration.

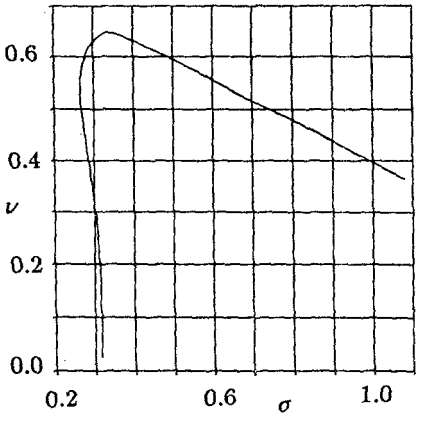

Fig.3 - Same as fig. 2 but for initial kinetic energy equal to 2 MeV.

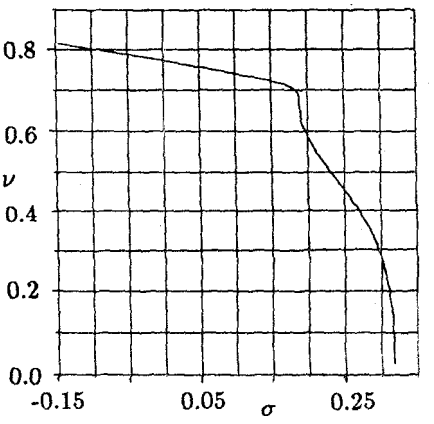

Fig. 4 - Same as fig. 2 but for initial kinetic energy equal to 5 MeV.

9. Results, discussion and conclusions

In this section we confront the model with experimental data and with other models. With this aim we define some quantities such as the threshold fissility parameter and the extra-push. energy.

The threshold fissility parameter, x_{th}, is the maximum value of the fissility parameter x defined in section 3 for which the nuclei in the collision arrive at the compound nucleus configuration when put in contact with zero relative velocity. When $x > x_{th}$ it is necessary to give energy to the system for the fusion to occur (with compound nucleus formation); the minimum value necessary for this to occur we call extra-push energy E_x. The function ϕ is defined[7] from the relation

$$E_x = E_{ch}\phi^2(x - x_{th}) \tag{37}$$

where E_{ch} is a characteristic energy of the system given by

$$E_{ch} = 0.0015202 A_1^{1/3} \left(\frac{Z^2}{A}\right)^2_{\text{crit}} \tag{38}$$

in the particular case of symmetric collisions, and

$$\left(\frac{Z^2}{A}\right)_{crit} = \frac{40\pi\gamma r_0^3}{3e^2} \tag{39}$$

Fig.5 is the plot of ϕ as a function of x obtained in this work; here we see that the slope of the curve increases with x.

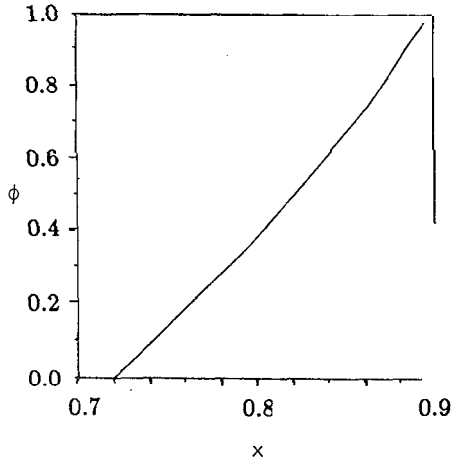

Fig.5 - Plot of the function $\phi(x - x_{th})$ as obtained in this work.

In table 1 we compare the experimental and calculated values of x_{th} from some models, including the one presently studied. The third column corresponds to the Swiatecki model described in sections 3,4 and 5 (sharp surfaces) whereas the fourth corresponds to a sharp surfaces model in which the system is represented by two spheres connected by a hyperbolic neck.

Table 1 – Comparison between experimental and calculated values of x_{th}

	exp (ref. 7)	exp (ref. 1)	theory (ref. 8)	theory (ref. 9)	theory (this work)
x_{th}	0.70 ± 0.02	0.61 - 0.64	0.584	0.723	0.720

The value of x_{th} of the model presented in this work agrees acceptably with the one calculated in ref. 9 and the experimental results of ref. 7 but not with the

more recent data in ref.1. The discrepancies between theory and experiment are not understood in a sufficiently clear way[1].

Another quantity of interest is the slope coefficient, a, defined by

$$a = \left.\frac{d\phi}{dx}\right|_{x=x_{th}} \tag{40}$$

but the comparison of this parameter for the different models is clouded by the fact that ϕ is not, in general, a linear function of x. Having this reservation in mind we indicate that the value obtained from our model, a = 4.6, is closer to the experimental value 7.31 ± 1.0 of ref. 1 than the one given in ref. 9 (a = 18) but it is farther than this from the experimental result in ref. 7 ($a = 12 \pm 2$).

To study the effects prodiiced by the inclusion of the nuclear diffuseness in the Swiatecki model we compare our results with that given in ref. 8. This is the sharp surfaces model described in sections 3, 4 and 5 which considers the diffuseness only through the chosen value of the parameter r_0. Then the equations of motion are those obtained in section 7 without the terms in braces and with the constants modified with respeçt to those used in this paper. We also recall that Çwiatecki uses a correction in the Coulomb potential by taking $\alpha = 6/5$, whereas no similar correction was made here. Then, in order to obtain his equations we must make also the substitution $x \to 6/5x$. We are interested in a comparison with the results of the Swiatecki model but without this last correction. As can be seen from the preceding considerations about the equations of motion these results can be obtained by multiplying the value of x_{th} in ref. 8 by 6/5, that is, $x_{th} = 6/5 \times 0.58 \cong 0.7$ whereas the slop parameter is not modified so that $a \cong 5$. If we compare these figures with ours, 0.72 and 4.6 respectively, we could conclude that it is sufficient to consider the diffuseness in an approximate form choosing appropriately the parameters in a model of sharp surfaces, at least in relation to the determination of x_{th} and a; the treatment we have given to it in this work does not substantially alter these paramters. But we must consider the possibility that this conclusion could be subjected to the validity of eq. (18) in defining the

effective neck length. To obtain a more definitive conclusion it will be necessary to consider a more detailed model in which the diffuseness is treated more carefully.

The author wishes to thank Dr. Raul Donangelo for suggesting this topic and for many useful discussions.

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Resumo

O modelo esquemático para colisões centrais de ions pesados desenvolvido por Swiatecki inclui contribuições coulombiana e de superfície à energia potencial do sistema e ao termo dissipativos de um corpo. Neste trabalho o modelo é estendido através da consideração da difusidade da superfície nuclear; a implicação disto é que devem ser levadas em conta as forças de proximidade na dinâmica das colisões. Por simplicidade, trabalhamos com sistemas simétricos. Os resultados do modelo estudado são comparados com dados experimentais e com outros cálculos teóricos. Concluimos que a consideração detalhada da difusidade das superfícies nucleares não muda substancialmente os resultados do

modelo esquemático para superfícies nitidamente definidas, no qual a difusidade é considerada apenas através dos parâmetros.