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Theory of the quantized Hall effect

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Abstract This paper presents a theory of the quantized Hall effect based on flux quantization. The strong magnetic field divides the device into a large numbers of square orbitals. The orbital is the elementary dynamical system, which is a correlation domain, where a few correlated electrons interact with the same quantum of magnetic flux. Acting upon each one of those correlation regions, there is an effective gate voltage, which is proportional to the device's gate voltage, and whose variation will make the orbital electron number change. But since this number is quantized, it must change by steps, and this yroduces the normal effect sequence of plateaus. The coupling of the orbital's electric flux to an external source is what makes a current pass through. The electric flux also couples to the operator that describes electron number fluctuations. The finite temperature behavior of a standard device is studied. The Hall voltage and the longitudinal voltage are obtained as functions of gate voltage, magnetic field, temperature and device's current. The specific heat is computed as a function of magnetic field. The results compare well with what is observed in quantized Hall effect experiments. A preliminary analysis about the anomalous effect is also presented.

1. Introduction

This paper proposes a theory of the quantized Hall effect, which uses the quantum flux method introduced by one of the authors[1].

The quantized Hall effect was discovered in 1980 by von Klitizing, Dorda and Pepper[2], during measurements of the Hall voltage and longitudinal voltage

in a MOSFET (**metal-oxide-semiconductor-field**effect-transistor) operated at liquid Helium temperature, and under a high magnetic field. They observed the formation of Hall resistance plateaus, with variation of the gate voltage.

An important step for development and comprehension of quasi two-dimensional electronic systems, had been taken in the work of Fowler, Fang, Howard and Stiles[3], who, in 1966, observed the Shubnikov-de Haas effect during measurements of conductance in a field effect device, with an electron gas formed on its Si/SiO_2 interface.

In 1982, Tsui, Stormer and Gossard[4] discovered the so called anomalous or fractional Hall effect, in a device of the heterojunction type: GaAs(AlGa)As.

In the development of the present work, in order to interpret our numerical results, we made frequent comparisons with experimental results shown in the reports by Paalanen, Tsui and Gossard[5], and von Klitzing, Tausendfreund, Obloh and Herzog[6] on the normal Hall effect; with those by Tsui, Stormer and Hwang[7] on the anomalous effect, and also with the analysis made by Gornik, Lassing, Strasser, Stormer, Gossard and Wiegman[8] about the system's specific heat. We have also used informations contained in the review papers by Ando, Fowler and Stern ; Souillard, Toulousse and Voss[10] and Stormer[11].

Now we pass to some theoretical references about the quantized Hall effect:

As early as 1967, Stern and Howard[12] had already computed the electron gas sub-bands, taking into account the average potential of neighbouring centers of charge, and also the screening effect.

The quantization of electronic motion along the direction normal to the inversion layer had been predicted by Schrieffer[13] in 1957.

In 1971, Ohta[14] noticed that electron scattering by impurities should lead to an broadening of Landau levels, and observed that this is an essentialy two-dimensional phenomenon. He also observed that the longitudinal resistance peaks at low temperature, should be related to that broadening of levels.

His theory was then improved and extended by Ando and Uemura[15], and Ando[16], who assumed that the electron gas should be viewed as a disordered system. Ando developed many-body techniques more complex than those used by

Ohta, and obtained results which, in the single-site approximation were consistent with Ohta's ones.

The theory motivated a lot of theoretical studies, and has been serving as a guide for the interpretation of some experimenta! aspects of the phenomenon. However, the theory is complex, and frequently presents methodological difficulties typical of many-body problems.

On the other hand it is difficult to conciliate the high numerical precision of Hall resistance plateaus, with the idea of a system that should be essentially disordered.

In 1981, concerned with this problem, and under the light of certain results by Ando, Laughlin[17] concluded that the high precision of the Hall effect plateaus, should be due to some fundamental principle. He then proposed that gauge invariance should play a fundamental role in the description of the phenomenon. Laughlin took the vector potential as a classical field, but used the Dirac-London flux quantization rule $\left(\oint \vec{A} d\vec{\ell} = \frac{2\pi}{e} p\right)$ when making gauge transformations in extended states wave functions.

In order to present his ideas in a clear manner, Laughlin conceived an idealized Hall effect device: a ring with many Landau orbitals circulating in parallel. His ideas soon found a large number of followers[1,18-21].

Imry[20] and Lindelof and Hansen[al] proposed schemes to explain the quantized Hall effect, supposing that flux quantization should play a central role, and stablishing a connection with the Josephson effect.

In 1984, Zawadzki and Lassing[22] obtained the electron gas specific heat, assuming a phenomenological density of states with the form of a sum of gaussians.

Also in 1984, Ventura' introduced a canonical method of flux quantization that extends the traditional method of Dirac-London to the description of mixed states of flux. Ventura proposed that every Landau orbital should have independent dynamics. The action of global external sources, like the gate voltage and current source, on a given orbital, should be introduced by means of external source terms in the orbital Hamiltonian. He also noticed that, from the viewpoint

of an orbital, the two main observables, the Hall voltage and the longitudinal voltage, are respectively proportional to the elements of a canonical pair of fluxes: the magnetic flux ϕ_B and the electric flux ϕ_E.

Reference (1) also shows a calculation of an orbital with well defined electron number, which provides an explanation for the existence of fractional plateaus, and describes how the transition from a plateau to the next one occurs, with variation of the magnetic external source. The peak in longitudinal voltage that appears in transition regions, is explained as a direct consequence of the non commutativity of electric and magnetic fluxes.

Ventura argues that there should be an effective gate voltage at every orbital, which, by forcing the variation of electron number, should lead to the sequence of plateaus of the normal effect. He also proposes that the orbital Hamiltonian must contain operators to describe the effect of electron number fluctuation, although it was still unclear that, as we will show here, electron number fluctuations and momentum fluctuations should be coupled. Polarization flux is introduced in the final version of Reference (1).

However, that theory is a good explanation for the existence and behavior of plateaus, but only for the idealized system, that is, a Landau orbital in Laughlin device. It was still necessary to apply those theoretical developments to the direct study of the real transistor, at finite temperature, and also to explain in detail the important role played by the device's electrostatic energy. And this is just the purpose of this paper.

In reference (23) we analyze the statistical mechanics of the canonical quantum flux theory.

The plan of the paper is as follows:

In section 2 we review Landau theory.

In section 3 we study the device's eleçtrostatics. We elected the MOSFET as our theoretical framework, so that, discussions of section 3 more directly refer to such a kind of devices,where the number of carriers is fixed by a gate voltage; although, with minor modifications, they can be extended and applied to the

description of other devices. A brief discussion about devices without gate is presented in section 10.

The quantized effective ***electrostatics*** of individual orbitals is treated in sections 4 and 5. It is determined by the requirements of global electrostatics. At this stage one is already able to understand the existence of normal Hall effect plateaus.

In section 6 we deal with flux quantization, accordingly with the proposal of Reference (1).

The kinetic Hamiltonian is considered in section 7, where we also construct a perturbation Hamiltonian (proportional to the electric flux operator) which makes current pass through the orbital.

In section 8 we discuss the effect of fluctuations of the number of electrons of an orbital.

The solution of the Harniltonian is carried out in section 9, together with the construction of the corresponding statistical mechanics.

Section 10 is the longest one. There we describe the normal Hall effect, and discuss in detail the following topics: orbitals' geometry and dimension; behavior of the Hall resistance and longitudinal voltage, with the variation of the gate voltage, at constant magnetic field; estimate of the device's gate voltage variation needed for the system to cross a plateau; estimate of the temperature region where the effect becomes clearly seen; a standard device; top electron dominance; constant total current regime; results on the Hall resistance and longitudinal voltage, under magnetic field variation; Ohmic behavior; dissipation mechanism; and specific heat.

Section 11 is dedicated to a preliminary analysis of the anomalous effect.

2. The electron in a magnetic field

The motion of ***one*** electron in a magnetic field $\vec{B}$ is described by the Schrödinger equation[24]

$$i\,\partial_t \psi = \frac{1}{2M}\left[\hat{\vec{p}} - e\,\vec{A}^{ext}\right]^2 \psi \qquad (1)$$

where $\hat{\vec{p}}$ is the generalized momentum, e is the electron charge, and $\vec{A}^{ext}$ a classical vector potential, whose rotational is the $\vec{B}$ field.

Let $\vec{B}$ be a uniform field parallel to the z-axis. Thanks to gauge invariance there is a large continuous set of potential vectors which reproduce the same $\vec{B}$.

In the sequence of the paper, we will introduce an externa1 electric source along the z-direction, which will break the system's symmetry, leading to the formation of orbitals along the x-axis. Thus it is convenient to choose from now on a special classical vector potential, so as to form Landau orbitals parallel to the x-axis:

$$\vec{A}^{ext} = B\, y\, \vec{i}\,. \tag{2}$$

With this vector potential choice, the electron's equation of motion turns out to be:

$$i\,\partial_t \psi = -\frac{1}{2M}\,\frac{1}{\partial y^2}\,\psi + \frac{1}{2M}\left[-i\frac{\partial}{\partial x} - e\,By\right]^2 \psi \tag{3}$$

In devices where the quantized Hall effect is observed, the electron is bound to an attractive well $V(z)$ varying in the z-direction. We will not treat this z-direction motion in detail. The effect shall be seen when only the first few levels of the potential $V(z)$ are populated[12]. Here we suppose that its first excited state energy is so high, that we can ignore the possibility of occupation of this level[12].

The electronic wave function is thus $f_0(z)\,\psi(x,y)$, where $\psi(x,y)$ is an eigen-solution of Eq. (3), and $f_0(z)$ is the ground state wave function of $V(z)$.

Let us take the system to be a box of length L, in the z-direction, and width H, in they y-direction. The electron's eigenstates are functions $\psi_{m\,n}(x,y)$ that solve Eq.(3) in this box:

$$\psi_{m\,n}(x,y) = e^{ik_m x}\,\phi(y - y_m) \tag{4}$$

where the wave number k, is

$$k_m = \frac{2\pi}{L}\, m \qquad (m = 0, \pm 1, \pm 2, \text{etc}) \tag{5}$$

and $\phi_n(y - y,)$ is the $\mathbf{n}^{\text{th}}$ eigensolution of the harmonic oscilator:

$$\varepsilon_n \phi_n = -\frac{1}{2M}\,\frac{d^2}{dy^2}\,\phi_n + \frac{e^2 B^2}{2M}\,(y - y_m)^2\,\phi_n \tag{6}$$

centred around y_m, with

$$y_m = \frac{\text{k}}{eB} = \frac{2\pi m}{LeB} \tag{7}$$

The energy of state $\psi_{m\,n}(x, y)$ does not depend on m and is given by:

$$\varepsilon_n = \left[n + \frac{1}{2}\right]\,\frac{eB}{M} \tag{8}$$

m is the index denoting a particular orbital.

So, we have a set of identical orbitals. One electron of energy E in a magnetic field B, has frequency $w = eB/M$, and its motion is over a circle of radius $R = 2E/|eB|$, centred at an origin orgin 0. In quantum theory, in turn, the wave function factor $e^{ik_m x}$ is interpreted as reflecting an uncertainty in the position of the origin, although such an origin must be at rest.

In this regard, the expectation value of the vector potential in the electron state $\psi_{m\,n}(z, y)$ is

$$< A^{ext} >_{mn} = By_m = \frac{1}{\text{L}}\,\frac{2\pi}{\text{e}}\, m\,, \tag{9}$$

and, using Eq.(7), we deduce that, if $\hat{p}_x$ is the generalized momentum along the z-direction then

$$< \hat{p}_x - \text{eA}^{\text{ezt}} >_{mn} = \text{k}\quad - eBy_m = 0\,, \tag{10}$$

and we see that the expectation values of z-direction momentum and current vanish.

The coupling of a z-direction externa1 electric source to the system, makes a current to flow in this direction, as it will be discussed in sections 7 and 10..

3. Many-electron systems

In a MOSFET, the region filled with the electron gas is a plane charged because of a gate voltage V_g. After turning on the magnetic field, the system's ground state is such that electrons are distributed among a large number of orbitals, like those discussed in the former section.

Classical estimate of the orbital number of electrons, at given values of magnetic field and gate voltage:

(i) From the analysis of the previous section, one concludes that the number of orbitals is the same as the total number K of magnetic quanta of flux flowing across the system area $S = LH$ (this is true as far as the gate voltage is high enough to keep at least one electron per orbital).

Since BLH is the total magnetic flux, and $2\pi/e$ the magnitude of a quantum of magnetic flux, we get

$$K = \frac{e}{2\pi} BLH = \frac{e}{2\pi} BS \tag{11}$$

as the relation between the number of orbitals and the magnetic field.

(ii) $E(B, V_g, n)$ is the system's total energy as a function of magnetic field, gate voltage and orbital number of electrons, n. We assume that the system is in "electrons bath", i.e., it may have as many electrons as it needs to minimize the energy; so that by minimizing $E(B, V_g, n)$ one can obtain n.

That energy has three parts that we pass to analyze:

(iii) One of them refers to the coupling to the magnetic field.

For our purpose here, the complete treatment of the spin variable is not essential, and we will assume that every electron has spin up, for instance. Hence, there is only one electron per orbital level. The first electron enters in the orbital with energy $\frac{1}{2}\frac{eB}{M}$, the second one with $\frac{3}{2}\frac{eB}{M}$, and so on.

Every n-electron orbital will then have an energy

$$\frac{1}{K} E^1(n) = \sum_{n'=1}^{n} \left[n' - \frac{1}{2}\right] \frac{eB}{M} = \frac{1}{2} \frac{eB}{M} n^2 \tag{12}$$

due to the direct coupling to the field B.

(iv) Another term in total energy corresponds to the electrostatics repulsion

$$E^2(K,n) = \frac{e^2}{2}\,\frac{(Kn)^2}{C} \tag{13}$$

where C is the device capacity, and K n the *total* number of electrons in the electron gas.

(v) Finally, we must consider the electrostatic coupling to the externa1 gate voltage:

$$\mathrm{E}^3(\mathrm{K},\mathrm{n}) = -eV_g Kn\,, \tag{14}$$

(vi) So that the total energy ($\mathrm{E}^1 + \mathrm{E}^2 + \mathrm{E}^3$) will be

$$E(B,V_g,n) = K\left[\frac{1}{2}\,e^2\,\frac{K}{C}\,n^2 + \frac{eB}{2M}\,n^2 - eV_g n\right]. \tag{15}$$

(vii) And, supposing for a while that n is a continuous classical variable, then by minimizing Eq. (15) (use Eq.(11) also) we can get the value of n corresponding to the lowest energy state

$$n = \frac{M^* V,}{B}\,. \tag{16}$$

The constant M^* is such that

$$\frac{1}{M^*} = \frac{1}{M} + \frac{e^2 S}{2\pi C} \sim \frac{e^2 S}{2\pi C}\,, \tag{17}$$

and the approximation made in the last step holds in general.

4. Quantized orbital 'electrostatics'

Now we suppose that an orbital is a correlation region: electrons belonging to a given orbital are all interacting with the same quantum of magnetic flux, and we expect that electronic motion is correlated together with quantum electromagnetic field fluctuations.

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Based on the material of section 2, we also suppose that orbitals are very similar,so that, by treating one of them theoretically, one is already able to describe the behavior of the ensemble of orbitals.

The electrostatic action of the device's gate voltage and of other orbitals upon an individual orbital is described by means of an effective gate voltage $\tilde{V}_g$. $\tilde{V}_g$ must be such that, at the classical level, every orbital in equilibrium should have the same number of electrons as given by Eq.(16), since this number is determined by the global electrostatics of system.

The effective *electrostatics* of a correlation region with n electrons will then be given by the static energy:

$$\tilde{E}(n) = \frac{1}{2}\,\frac{eB}{M}\,n^2 - e\tilde{V}_g n \tag{18}$$

where the first term is the interaction with the magnetic field (see Eq.(12)); and $\tilde{V}_g$ is such that, if we minimize the above expression relative to n, then, at the point of minimum, the number of electrons of Eq.(16) should result. Therefore, the orbital effective gate voltage must be

$$\tilde{V}_g = \frac{M^*}{M}\,V_g \cong \frac{2\pi}{e^2}\,\frac{C}{SM}\,V_g\ . \tag{19}$$

It is remarkable that this relation depends only on geometric constants, S and C, having no dependence on the magnetic field B.

Eq.(18) has no term of electrostatic repulsion among electrons belonging to the same orbital, since electrostatic interaction, which was already considered in the study of the global device, is a long range effect. On the other hand the coupling to the external magnetic field must take part in orbital dynamics, because this is a local effect.

Plateaus of the normal Hall Effect: up to now we have considered the number of electrons n as a continuous classical variable, in order to construct the effective orbital "electrostatics". The next step is to quantize it.

We must then start from a Hamiltonian operator, which has a classical limit compatible with the energy formula $\tilde{E}(n)$, that is

$$H_{\text{charge}}(\hat{n}) = \frac{1}{2}\,\frac{eB}{M}\,\hat{n}^2 - e\tilde{V}_g\hat{n}\,. \tag{20}$$

Now $\hat{n}$ is the electron number operator. Its eigenvalues are the non-negative integers ($n = 0, 1, 2, \ldots$, etc).

At this point one can already explain the existence of plateaus in the so-called normal Hall effect.

The energy of state $|n>$ is

$$E_n = \frac{1}{2}\,\frac{eB}{M}\,n^2 - e\tilde{V}_g n\,, \tag{21}$$

and the ground state $|n_0(\mathrm{B}, \tilde{V}_g)>$ corresponds to the integer $n_0(\mathrm{B}, \tilde{V}_g)$ that minimizes Eq. (21). n_0 is the integer in the interval:

$$\frac{M\tilde{V}_g}{B} - \frac{1}{2} < n_0 < \frac{M\tilde{V}_g}{B} + \frac{1}{2}\,. \tag{22}$$

One then sees that, if B increases at constant V_g (or vice versa) the number of electrons varies in steps.

On the other hand, if S is the system's surface and N the number of carriers, then the classical description of the Hall effect (that must hold when the ensemble of orbitals is taken together) means that the Hall resistance is given by the relation between the magnetic flux across surface S and the total carriers charge

$$R_{\text{Hall}} = \frac{V_{\text{Hall}}}{I} = \frac{BS}{eN}\,, \tag{23}$$

where V_{Hall} is the transverse Hall voltage and I the longitudinal current.

Then, since $\mathrm{B\,S} = 2\pi K/e$ and $N = n_0 K$, we get

$$R_{\text{Hall}} = \frac{2\pi}{e^2 n_0} \tag{24}$$

so that, in the normal Hall effect case, the Hall resistance depends only on the number of electrons per orbital.

From inequality (22), one concludes that when the ratio $M\tilde{V}_g/B$ is in the interval

$$n_0 - \frac{1}{2} < \frac{M\tilde{V}_g}{B} < n_0 + \frac{1}{2} , \tag{25}$$

then the Hall resistance is given by Eq. (24). Therefore, it is clear that, under continuous variation of magnetic field or gate voltage, the Hall resistance varies in steps.

5. Correlation among the orbital electrons

Owing to the external induction field electrons move in tubes along the x-direction. The boundaries of a certain tube are approximately defined by the transverse wave function factors, $\phi_n(y - y_m)$ and $f_0(z)$.

After the orbitals are filled with their n_0 K electrons, Pauli's principle forbids an electron to leave its orbital spontaneously. A given electron could not for instance, move away from a state in the m^{th} orbital to occupy the same level in the k^{th} orbital, since this state is already filled. Therefore, that electron can only leave its orbital by receiving a certain amount of energy, so as to make its energy larger than $\varepsilon(n_0)$ (see Eq. (8)).

Any orbital has then a particle cluster with n electrons. The collective motion of this cluster is ruled by the Hamiltonian

$$\bar{H}_2(\hat{p}) = \frac{1}{2M_\theta}\,\hat{p}^2 \tag{26}$$

where $\hat{p}$ is the collective s-momentum and M_θ the cluster total mass:

$$M_\theta = M\,n \simeq \frac{M^* M}{B}\,V_g , \tag{27}$$

so that, Hamiltonian (26) describes the center of mass motion of the electron cluster.

In orbital dynamics, there is a situation where the motion of an internal excitation can be more important than the center of mass mode: it occurs in the transition region between plateaus, when the binding energy of the (electron-type) quasi-particle at the top of the orbital becomes much smaller than the binding

energies of other quasi-particles. In this case, both the orbital polarization and its current shall be determined mostly by the motion of the quasi-particle at the top. This effect is considered in section 10.

Polarization Operator

Let Vol= $L\tilde{S}$ be the fraction of the device's volume ascribed to an orbital. L is its length along the x-direction, and $\tilde{S}$ its transverse section. (This volume is a rectangular box larger than the region where the electrons belonging to that orbitai move).

For every correlation region, we then define the polarization operator:

$$\hat{p} = \frac{enx}{\mathrm{Vol}} \,. \tag{28}$$

In this very section and in the next one, we treat the variable n, as if it were a c-number.

Variable P is a measure of how much the electronic cluster center of charge is displaced from its original position at the center of the orbital.

6. The quantum flux, the polarization, and the momsntum

Let $A_1(\vec{x})$ be the x-cornponent of the vector potential, defined within the volume ascribed to an orbital. Assuming periodic boundary conditions, one can decompose $A_1(\vec{x})$ as follows

$$A_1(\vec{x}) = A_0 + \sum_{n \neq 0} a_n \, \psi_n(\vec{x}) \,, \tag{29}$$

where the first mode operator A_0 is $a_0/\sqrt{\mathrm{Vol}}$.

The set $\{\psi_i\}$ is orthonormal and complete; whereas the field variables a_i are canonically conjugated to another set of variables $\{b_j\}$, so that $[a_i, b_j] = i\,\delta_{ij}$.

By using the operators b_j and the orthonormai set {ri,), we then construct another local field

$$-\tilde{E}_1(\vec{x}) = -\tilde{E}_0 + \sum_{n\neq 0} b_n\ \psi_n(\vec{x}) \tag{30}$$

with the first mode $\tilde{E}_0$ given by $-b_0/\sqrt{\text{Vol.}}$

We call this field the *x*-component of the pure electromagnetic electric field. It is the A, $(\vec{x})$ field canonical momentum: [A, (Z), $-E_1(\vec{y})] = \mathrm{i}\ \delta(\vec{x}-\vec{y})$. In classical theory, and in the gauge ϕ =constant, we have $\vec{E} = -\vec{A}$, so that the two fields indeed form a canonical pair.

In order to simplify the presentation, we shall take only the first mode A_0 and $\tilde{E}_0$ to represent respectively the *x*-direction vector potential and pure electromagnetic electric field, since, in the retangular geometry, A_0 and $\tilde{E}_0$ are the operators more closely related to fluxes. The complete treatment of fluxes has been presented in Reference (1).

A_0 and $\tilde{E}_0$ are proportional to the elements of the canonical pair (a_0, b_0), so that the commutator is $[A_0, \tilde{E}_0] = -i/L\tilde{S}$, or, in the form of a commutator of fluxes:

$$[LA_0, \tilde{S}\tilde{E}_0] = -i\,. \tag{31}$$

We define the true electric field as the difference between the pure electromagnetic electric field and the polarization

$$\hat{E} = \tilde{E}_0 - P\ . \tag{32}$$

And the corresponding electric flux is given by

$$\hat{\phi}_E = \tilde{S}\hat{E} = \tilde{S}\tilde{E}_0 - en\frac{x}{L} \tag{33}$$

The canonic conjugate to the electric flux is another hybrid operator[1]

$$\hat{\phi}_B = \left[LA_0 + \frac{i}{en}\frac{\partial}{\partial x}\right], \tag{34}$$

which results proportional to the physical momentum of the electron cluster

$$\hat{p}_f = -i\partial_x - enA_0 = -\frac{en}{L}\,\hat{\phi}_{at\,B} \ . \tag{35}$$

In order to keep our treatment as close as possible to tradtional methods, we choose to work with the cluster momentum $\hat{p}_f$ and its canonic conjugate X:

$$X = -\frac{\mathrm{Vol}\,\hat{E}}{2en} = \frac{1}{2}\left[x - \frac{L\,\tilde{\phi}_E}{en}\right] \tag{36}$$

instead of the pair of fluxes $(\hat{\phi}_E, \hat{\phi}_B)$. One must however have in mind that X is proportional to the orbital electric field.

Given the commutator $[X, \hat{p}_f] = i$, and the domain of the variable X : $-\frac{L}{2} < X < \frac{L}{2}$ (notice that $-\frac{L}{2} < x$; $[\frac{L\tilde{\phi}_E}{en}] < \frac{L}{2}$), we can obtain the physical momentum eigenfunctions

$$\psi_p(X) = \frac{1}{\sqrt{L}}\,\exp i\left[\frac{2\pi}{L}\,pX\right] \tag{37}$$

and eigenvalues $p_{f,p} = \frac{2\pi}{L}\,p$ (with $p = 0, \pm 1, \pm 2, \ldots$, etc.).

7. Motion of the center of mass

After introducing electromagnetic variables, one modifies Hamiltonian (26), according to the rule of minimal coupling,

$$H_2(\hat{p}_f) = \frac{1}{2M_\theta}\,\hat{p}_f^2 \ . \tag{38}$$

This Hamiltonian has eigenvalues $2\pi^2 \mathbf{p}^2 / M_\theta L^2$.

The devices where the quantized Hall effect is observed are coupled to current sources. We will describe this effect by means of a self-consistent electric source.

Every orbital is an *open* system of finite volume. We suppose that the device's current source plus neighboring orbitals act together on a given orbital, tending to create there an electric field.

In classical physics, we can describe the effect of externa1 sources producing a uniform electric field in a open volume Vol (which has no charge), by adding a term $-E_{ext}\,\mathrm{Vol}$ to the system's Hamiltonian. This term combined with the

quadratic electric field energy $\frac{1}{2}$ E^2Vol, leads to the expected value $E = E_{min}$, after minimization in E.

The, we suppose that also in our quantum system the action of a weak electric source is given by perturbation Hamiltonian – Vol $E_{ext}\hat{E}$; although we discard any Hamiltonian dependence on quadratic terms such as $\hat{E}^2$ or X^2, etc. (We recall that the orbital electric field is not confined into the tube where electrons move, but extends along the device in the z-direction).

Writing that perturbation Hamiltonian in terms of the variable X, we get

$$H_I^1(X) = -2|e|nE_{ext}X\,. \tag{39}$$

The value of E_{min} can be determined self-consistently in terms of the current.

At this point we would like to add a note about the passage from classical electrodynamics to the construction of a quantum electrodynamics of fluxes. Consider the interaction term in the classical action of an electron interacting with an externa1 field $S_A = e\int dt\, A_i^{ext} v_i$.

Formally, it can be integrated by parts leading to an action term of the form $S_B = -e\int dt\, \dot{A}_i^{ext} x_i = e\int dt\, E_i^{ext} x_i$, which is analogous to the corresponding classical action associated with Hamiltonian (39).

However, in the construction of a quantum electrodynamics suitable for the description of nonperturbative/low energy/large distance phenomena (like many effects belonging to quantum electronics, for instance), where fluxes play an important role, the type of interaction to choose, S_A or S_B is still somewhat ambiguous. This certainly depends on the nature of the system we want to describe.

In our case here, of open finite volume orbitals, formed because of the existence of a magnetic field, we made the (justified) choice S_B; although the choice S_A would lead to a qualitatively similar picture,and to results of nearly the same order of magnitude.

Notice also that the classical $\vec{A}$ field has already been used to introduce the effect of the classical magnetic field; and this is one more indication that choice S_B should be the right one in the present case.

A study about the construction of a quantum electrodynamics of fluxes is being carried out[25], and shall be published elsewhere.

8. Electron number fluctuations

One more effect must be considered, in order to complete our descritpion of the quantized Hall effect: the effect of electron number fluctuation in an orbital.

Starting with a classical argument, we observe that sudden alterations of externa1 conditions may lead to the appearancç of perturbative forces able to change the number of electrons in an orbital. Besides, an oscillatory longitudinal electric field might also make the electron number change. For example, variations of electric field can make an electron move from an orbital to the other.

Another possible cause of electron number fluctuations could be the presence of impurities in the material[1]. An electron weakly bound to an impurity, may in its ground state have a "small fraction" of its wave function in the orbital region. In an excited state, in turn, the electron will be with higher probability in the orbital volume. Variations of electric field might then produce oscillations between these two states, so leading also to fluctuations in the orbital electron number.

On the other hand, when an orbital gains an additional electron, even if transitorily, the electron cluster momentum will change, reciprocally.

Then, we are led to conclude that there must exist a coupling between the electric flux operator (which brings magnetic flux/momentum fluctuations) and the operator related to electron number fluctuations.

If the operators a+ and a are such that $[a^+, a] = -1$ and $a^+ a = \hat{n}$, fluctuations of electron number can be introduced by means of the hermitian operator $a^+ + a$, since this operator does úot commute witli $\hat{n}$.

Then, in order to describe the mutual effect of fluctuations of particle number and momentum, we construct a perturbation Hamiltonian of the type:

$$H_I^2(X, a^+ +, a) = \lambda\, X(a^+ + a)\ . \tag{40}$$

In this work we treat low energy phenomena at low temperature, so that only a few lowest energy levels contribute to the motion. Therefore, in this case, the quantum analysis, made in terms of Hamiltonians (20), (38), (39) and (40) is already complete, in the sense that it is impossible to have a more local description for what happens inside an orbital. This limitation comes from the uncertainty principle. Since the modes are extended within the orbital, the description given by Hamiltonians H_I' and H_I^2 for their respective effects is the most detailed.

9. Solving the Hamiltonian

In order to solve the system's Hamiltonian, we first choose its main part

$$H_0 = H_{\text{charge}}(\hat{n}) + H_2(\hat{p}_f) \,, \tag{41}$$

which is already diagonalized; and let the other two terms

$$H_I = H_I^1(\mathrm{X}) + H_I^2(\mathrm{X}, a^+, a) \tag{42}$$

to be treated as perturbations.

At given values of magnetic field and gate voltage the ground state of H_0 is a state $|p = 0, n>$, with n given by inequality (22). The perturbation Hamiltonian then changes this pure H_0 ground state into a mixed state, that nevertheless is still dominated by the state $|0, n>$.

The perturbation effect is introduced by coupling that state to its nearest neighbors, as we explain below:

(i) Besides the state $|0, \mathrm{n}>$, with n given by inequality (22), we also include the state $|-1, \mathrm{n}>$ to form the mixed ground state.

It would be more appropriate to take in account the state $|1, n>$ also. However, in order to simplify the calculations of results presented in this paper, we did not consider this latter state.

Taking $E_{ext} > 0$, one observes that perturbation H_I' produces a ground state with $< \mathrm{X} >> 0$, which means a positive electric field. On the other hand, it also

leads to a mixture of p = 0 and p = –1 states, that corresponds to a negative average momentum, and, in consequence, to a positive current.

(ii) Every plateau defined by inequality (25) is separated into two parts: the upper region, in the interval $a < \frac{M\tilde{V}_g}{B} < n + \frac{1}{2}$; and the lower region in the interval $n - \frac{1}{2} < \frac{M\tilde{V}_g}{B} < n$.

(iii) When the system is in the upper region of the n^{th} plateau, then only the states $|-1, n>$ and $|0, n>$, and their nearest neighbors above, $|-1, n+1>$ and $|0, n+1>$, will give relevant contribution to the system's dynamics. We have therefore, a four-state dynamics, with a mixed ground state, where the state $|0, n>$ prevails.

However, in a transition region between plateaus, when $M\tilde{V}_g/B$ is in a small neighborhood of the value $n + \frac{1}{2}$, and the energy $E_0(0, n)$ is close to $E_0(0, n+1)$, (see the energy formula below) then the ground state is nearly a fifty-fifty mixture of states $|0, n>$ and $|0, n+1>$.

(iv) In the lower region, states $|-1, n>$ and $|0, n>$ are in turn mixed with nearest neighbors below, $|-1, n-1>$ and $|0, n-1>$; and we proceed as we do in the other case.

(v) The next goal is to obtain the matrix elements of the Hamiltonian that rules the four states' approximated dynarnics. A generic state shall be represented by (with index i running from 1 to 4)

$$|\psi_i>^n = \alpha_1^i |0, n> + \alpha_2^i |0, n+1> + \alpha_3^i |-1, n> + \alpha_4^i |-1, n+1> \quad . \qquad (43)$$

(vi) By numbering the H_0 eigenstates from 1 to 4, in the order in which they appear in formula (43), we can write the Kamiltonian matrix that gives the four states' dynamics. It is the hermitian matrix:

0	0	-ign	-if $\sqrt{n+1}$
0	Δ_1	-if $\sqrt{n+1}$	ign
ign	if $\sqrt{n+1}$	Δ_2	0
if $\sqrt{n+1}$	-ign	0	$\Delta_1 + \Delta_2$

where Δ_1, Δ_2, g and f are given by:

$$\Delta_1 = \frac{eB}{M}\left[n + \frac{1}{2}\right] - e\tilde{V}_g \ , \tag{45}$$

$$\Delta_2 = \frac{2\pi^2}{M_\theta L^2} = \frac{2\pi^2}{ML^2 n} \ , \tag{46}$$

$$g = \frac{e}{\pi} L \cdot E_{ext} \ , \tag{47}$$

and

$$f = \frac{\lambda L}{\pi} \ . \tag{48}$$

(vi) After diagonalizing matrix (44), we determine the energy eigenvalue ε_i, and the average electron number n_i for every state $|\psi_i>$; we determine also the average electric field $-enX_i/\mathrm{Vol}$; the average orbital current $-e(p_f)i/ML$; etc.

(vii) Then we develop the statistical mechanics of the ensemble of orbitals, similarly to what we did in Reference (23). For example, the average electron number at temperature T is

$$< n >= \frac{1}{Z}\sum_{i=1}^{4} n_i e^{-\varepsilon_i/T} \tag{49}$$

where the partition function Z is

$$Z = \sum_{i=1}^{4} e^{-\varepsilon_i/T} \tag{50}$$

The electric field average value at finite temperature will in turn be given by

$$< E >= \frac{en}{\mathrm{Vol}} \frac{1}{Z}\sum_{i=1}^{4} X_i \, e^{-\varepsilon_i/T} \tag{51}$$

and so on.

10. The normal Hall effect

The quantized Hall effect of the normal type corresponds to the formation of electron number plateau states. Under variation of external conditions, transitions

between these states occur, without pronounced alteration in the number of quanta of flux ascribed to each orbital.

In this case the Hall resistance can be obtained by means of formula (24), but now replacing n_0 by the finite temperature average electron number

$$R_{\text{Hall}} = \frac{2\pi}{e^2 < n >} . \tag{52}$$

The other important observable is the longitudinal voltage V, (the product of the electric field by the system length), with the related longitudinal resistance $R_\ell = V_x / I\text{total}$, i.e., the ratio between the longitudinal voltage and the total current in the device.

(a) On the geometry and length of orbitals

In an orbital, electrons are strongly correlated, and are interacting with the same quantum of magnetic flux. On the other hand, there is the semiclassical interpretation, accordingly to which, those electrons should be rotating, due to the magnetic force, around a common orign of uncertain position[24].

Both facts indicate that electrons belonging to the same orbital must be close, and that the orbital form must therefore be squared.

Taking then a squared orbital, and considering that there is a quantum of flux per orbital, one can have an estímate of the orbital length L, by using the relation

$$BL^2 \simeq \frac{2\pi}{e} . \tag{53}$$

For example, if the magnetic field strength is near 100 kG, then the orbital length L_0 shall be around $\sim$ 200 A.

Equation (53) leads also to an interesting relation between the energy coefficients that appear in the gap equations (45) and (46). Multiplying Eq.(53) by $\pi e/M$, we get:

$$\frac{2\pi^2}{ML^2} = \pi \left[\frac{eB}{M}\right] \tag{54}$$

This equation has been used in every calculation on the normal effect, whose results we now pass to discuss.

(b) Varying the gate voltage at constant magnetic field

The behavior of the Hall resistance and longitudinal voltage with the variation of the gate voltage, at finite temperature and constant magnetic field, is illustrated in Figure 1. This first example is from a calculation made with electric source E_{yy} kept constant, and with the effective electron mass in the material taken equal to the very electron mass.

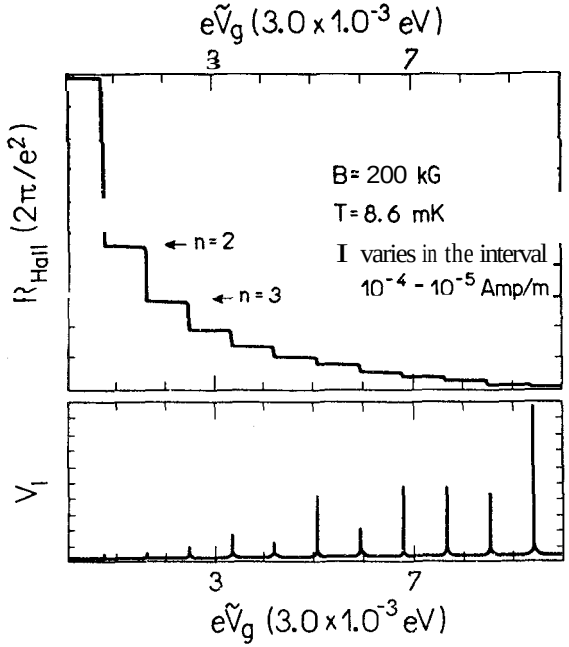

Fig.1 – Hall resistance and longitudinal voltage as functions of the orbital effective gate voltage, at $B = 200$ kG, $T = 8.6$ mK, and with $M = M_s$ and the electric source E_{yy} kept constant.

The result is very similar to the ones from quantized Hall effect experiments. One sees the Hall resistance plateaus, and, in the transition regions between plateaus, the longitudinal voltage peaks. Since the temperature is low, and the electric source weak, the longitudinal voltage becomes very small at the central

region of the plateau, which is just the region where the Hall resistance is quite well defined near the ideal values of $2\pi/en$.

However, in the experimental situation, it is the device's total current that is in general kept constant. In the sequence of the paper we will show reaults of calculations where the electric source strength has been varied concomitantly with the magnetic field, so as to keep constant the device's current (see Figures (3) and (4)).

In this paper the constant λ of Hamiltonian H_I^2 enters as a phenomenological number. In principle it would be possible to estimate this number; but it is nevertheless a complicate estimate.

In view of this difficulty, we decided to take a phenomenological constant A that grows with $\sqrt{B}$ ($A = \lambda_0\sqrt{B/B_{max}}$, where B_{max} is the highest B value, in each exhibited graph). We will not justify our particular choice for the form of A, but we notice that it corresponds to writing the perturbation H_I^2 in the form $\rho\,\hat{\phi}_E(a^+ + a)$, with a constant p. In the calculation that resulted in Figures 3 and 4, λ_0 is such that $\lambda_0 = \xi eB_{max}/ML(B_{max})$; $L(B_{max})$ being the orbital length when $B = B_{max}$; and ξ is a small number fixed in a value in the interval $0.0075 < \xi < 0.075$.

(c) On the temperature region where the effect starts being observed

The temperature value around which the quantized Hall effect becomes visible in experiment, can be estimated in terms of the typical energy of Hamiltonians H_{charge} and $H_{\ldots}$, that is about eB/M.

If we take the magnetic field to be near 100 kG, then that characteristic gap shall be $eB/M \sim 10^{-3}$eV (or $\sim 12^0$K).

On the other hand, if the effective electron mass assumes the value that supposedly it must have[11] in the device's material ($M \sim 0.10M$), then the above estimate will result larger by an order of magnitude.

Therefore, Hall effect plateaus will get a sharp definition, only after the device is cooled down to $\sim 0.1^0$K or 1.0^0K. And this is in fact the temperature region

where the effect starts being clearly seen, in most experiments[2,4,6], when $B \sim$ 100 kG.

(d) The length of a plateau under variation of the gate voltage

Let $\Delta\tilde{V}_g$ be the effective gate voltage variation needed to put one more carrier in every orbital. And let $\Delta\bar{V}_g = (e^2SM/2\pi C)\Delta\tilde{V}_g$ be the corresponding variation of the devices gate voltage (see Eq.(19)). If one knows the device geometry, and its dielectric characteristics, then one can estimate $\Delta\bar{V}_g$, for every value of B.

In a MOSFET, the capacity is nearly given by $C \in KS/d$, where S is the devices surface, and d is the distance between the electron gas plane and the gate. d is the distance along which the global gate voltage is applied ($d \sim 10^{-6}$m, typically). K is in turn the relative dielectric constant of the material that fills that space. From the relations above, and by inspecting Eq.(21), one deduces that $\Delta\bar{V}_g \in Be^2d/2nK$.

So, at an induction field of $\sim$ 100 kG, the global gate voltage variation needed to make the system run through a plateau, or to vary by one the orbital electron number, is about 1.0 Volt to 10.0 Volt, depending upon the insulator's dielectric properties and width. This is also the approximate plateau length, in quantized Hall effect experiments done under those conditons[2,6].

(e) A standard device

Figure 2 shows a standard MOSFET, suitable for the observation of the quantized Hall effect. It is made up of a 1.0 μm thick insulator, grown on a body of semiconductor material with thickness of about 30 μm. On top of the insulator material there is a thin metallic gate.

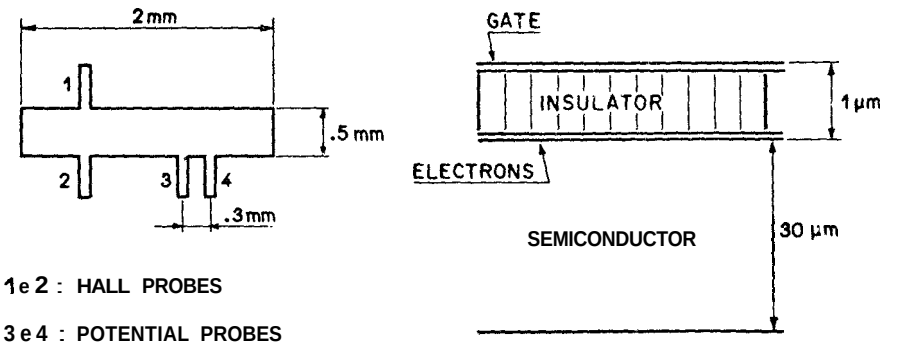

Fig.2 – Standard device

Electrons move in a thin layer ($\sim$ 50 A) around the insulator/semiconductor interface.

The device is 2.0 mm long and has 0.50 mm in width. The distance between potential probes is 0.30 mm.

All numerical results shown in Figures 3 anà 4 refer to this standard device.

After solving the Hamiltonian, and constructing the statistical mechanics of the ensemble of orbitals, we compute the average value of the variable X. In order to get the longitudinal voltage from that, we then take $V_\ell = \ell < \mathrm{E} >$, where $\ell = 0.30$ mm is the distance between the potential probes, and $< \mathrm{E} >$, the electric field average value, can be obtained from $< \mathrm{X} >$ by means of Eq.(36).

In this regard, we recall that the volume Vol appearing in that equation is the orbital volume (the fraction of the device's volume ascribed to each orbital): Vol= L $\times$ L $\times$ D, where D $\simeq 31\mu m$ is the device thickness.

Even though we have chosen the MOSFET as our theoretical framework, the theory presented here is general. It can be easily modified for application to any kind of device where the quantized Hall effect has been observed. All one needs to know is what effect, in the device, plays a role analogous to that of a gate voltage, fixing the system's number of carriers.

Let us then briefly discuss a device without gate. Consider, for instance, a device similar in form to the standard device of Figure 2, but with the insulator

replaced by a thin layer of dopped semiconductor; and suppose that the impurity concentration is small, and also the temperature is very low. Let L be the device length, I its total current, N the number of carriers (electrons), v the average electron speed, and M their effective mass.

For a given value of the total current, which we suppose to be small, $I \simeq eNv/L$, the electronic kinetic energy whill be $\frac{1}{2} N\bar{M}v^2 \simeq \bar{M}(LI/e)^2/2N$.

Now, if ε_0 is the energy needed to ionize an impurity and produce a carrier, then the electron gas has also an extra ionization energy $\varepsilon_0 N$; so that the total energy shall be:

$$E = \frac{1}{2}\bar{M}\left[\frac{LI}{2}\right]^2 \frac{1}{N} + \varepsilon_0 N \ . \tag{55}$$

This is a nonconservative system. When the current is constant the current source continuously supplies the necessary energy, to compensate dissipation, and keep mechanical energy constant. In spite of that, and considering that the device is at very low temperature, we will use the universal criteria of minirnizing its mechanical energy, in order to determine its preferred configuration.

Minimizing Eq.(69), we then conclude that the system's number of carriers is current dependent: $LI\sqrt{\bar{M}}/e\sqrt{2\varepsilon_0}$. Taking for example: $M \simeq M/10$, $\varepsilon_0 \simeq 10^{-5}$eV and $I \sim 100\ \mu A$, one gets a value for the number of carriers, which is close in magnitude to the typical number of carriers in quantized Hall effect devices.

Another very important conclusion one gets in the realm of this model, is that the electron's velocity results independent of the current value but depends only on the ionization energy, and on the electron effective mass: $v \simeq \sqrt{2\varepsilon_0/\bar{M}}$.

Notice that, in cases where ε_0 is very small, the electron speed will also be very small, so that the passage of low currents through the device will cause little dissipation.

Consider the limiting case when the impurity concentration is very high, and the binding energy ε_0 very small. In this case, thanks to the tunnel effect and(or) to temperature effect, electrons may originally form a sort of gas, even before the onset of current.

It seems to us that this device without gate we briefly discussed, might be of some use, at least as a preliminary and rough model for the heterojunction-like devices.

Anyway, the number of electrons in the system is defined somehow. The effect that, in devices other than MOSFET, fixes the total number of carriers, is called here the *effective gate voltage* of the device.

(f) Dominance of the electron on top of the orbital

Now we pass to examined the case in which orbital polarization is mostly due to the dislocation of the (electron-type) quasi-particle on top of the orbital. Such an effect is expected to occur in the transition region between plateaus.

For a first qualitative discmsion about the effect it is convenient to return to Landau theory, because of its simplicity.

Before the introduction of perturbations, orbitals are all filled with the same number of electrons, and, thanks to Pauli's principle, a particle cannot move to another orbital, unless it gains a certain amount of energy, that will be bigger the deeper is the level occupied by that particles in its original orbital.

A small perturbation of the type 7, (analogous to Hamiltonian (44)) will mix the wave function of an electron of a given orbital with nonpopulated high energy levels of neighboring orbitals. But it is the quasi-particle on top of the orbital that will be mostly affected by the effect, specially in the transition region between plateaus, where the gap that inhibits that electron on top from moving away becomes much smaller than the binding energy of other quasi-particles.

So, in the transition region, the orbital polarization is mainly due to the dynamics of the quasi-particle on top, and not to the center of mass dynamics. Under these circumstances, and only for computational purposes, one can suppose that the orbital has a center of charge, corresponding to the $(n-1)$ charges belonging to the $(n-1)$ deeply bound quasi-particles; plus one more quasi-particle of very small binding energy, which is just the one that couples to fluxes, and whose motion shall be the main responsible for the polarization effect.

Now, in order to introduce this effect into tbe model, we construct the dynamical variables by combining pure electromagnetic variables, with only the position x and generalized momentum $-i\partial_x$ of the quasi-particle on top. The orbital electric flvv and momentum shall then be given by (see Eqs. (33), (34) and (35) for comparison): $\widetilde{S}\widetilde{E}_0 - e(x/L)$ and $-i\partial_x - eA_0$.

One must also replace the mass M_θ in Hamiltonian (38) by the electron mas M.

Because of electron indistinguishability it is impossible to distinguish which one gained the average momentum ascribed to the top quasi-particle. Therefore all orbital electrons must be counted as carriers; or alternatively, one should count the orbital as a whole as a single carrier of charge ne.

(g) The system at constant current

At a given magnetic field strength, the choice of the total current fixes the orbital momentum, in a self-consistent way. Neighborhoods create a self-consistent external source E_{ext} at any individual orbital, so as to induce there the same average momentum required by the global current source.

When computing the results shown below, we adopted the top quasi-particle approximation, and, in order to have a theoretical picture closer to the experimental situation, we then modified the electric source value E_{ext}, concomitantly with the magnetic field variation, so as to produce the same total current in the device, for every value of B.

If I^{total} is the device current, the average orbital current shall be $i = LI^{\text{total}}/H$. On the other hand this same average current depends on the orbital average momentum in the following way: $i = e < p_f > /ML$; where the average momentum can be roughly estimated to be $(gM)^2 L^3/2\pi^3$ (in this estimate we took into account only Hamiltonians $H_2(\hat{p}_f)$ and H_f^1). Combining all this with formulae (47) and (53), we obtain the relation between electric source E_{ext} and device current:

$$E_{\text{ext}} \approx \frac{\pi^2}{e}\left[\frac{e}{2}\right]^{1/4} B^{3/4}\sqrt{\frac{I^{\text{total}}}{HM}}\,. \tag{56}$$

This approximated formula is reasonable only at low temperature. When treating the system at high temperatura, one can only obtain the function $E_{ext}(\mathbf{I}^{total})$, by means of a complex self-consistent numerical calculation, which we did not do.

Figures (3) and (4) show the Hall resistance and longitudinal resistance (or conductance) which resulted from calculations of the standard device behavior. In these calculations we took the effective electron mass in the device material to be one tenth of the electron mass ($M = M/10$). In order to obtain the Hall resistance in ohms, one must multiply R_{Hall} by 25812.8Ω.[2]

Figure 3a shows what happens at the relatively high temperature of 13^0K, when the effective gate voltage is varied, but the magnetic field is kept fixed at 200 kG, and the current at $0.4\mu A$. In this case, temperature is so high that there is no Hall resistance plateau yet; although one can observe a trend towards formation of plateaus, at the indicated regions. One notices also, the oscillations in longitudinal resistance, which resembles those of the Shubnikov-de Haas effect.

The next figure, Figure 3b, shows the system's behavior at a temperature one order of magnitude lower, and at a current of 4.5 μA. Now, one clearly sees the Hall resistance plateaus, as well as the longitudinal resistance characteristic peaks.

In Figure 3c, which corresponds to a still lower temperature ($.15^0$K), and current of 12 μA, plateaus become somewhat sharper. There one notices also the longitudinal conductance, with its typical behavior.

Figure 4a and 4b show the device behavior when we vary B, at constant temperature (2.0^0K and 0.60^0K), constant gate voltage ($V_g = 30$ mV), and constant current (0.95 μA and 0.82 μA, respectively). The picture is similar to the other case, in the sense that plateaus gain definition, and the longitudinai resistance peaks become sharper, as the temperature goes down.

Figures 4c, which corresponds to conditions similiar to those of Figure 4b, shows the Hall resistance and longitudinal conductance, at the relatively low current of 0.064 μA. And finally Figure 4d shows the system's behavior at very low temperature and current.

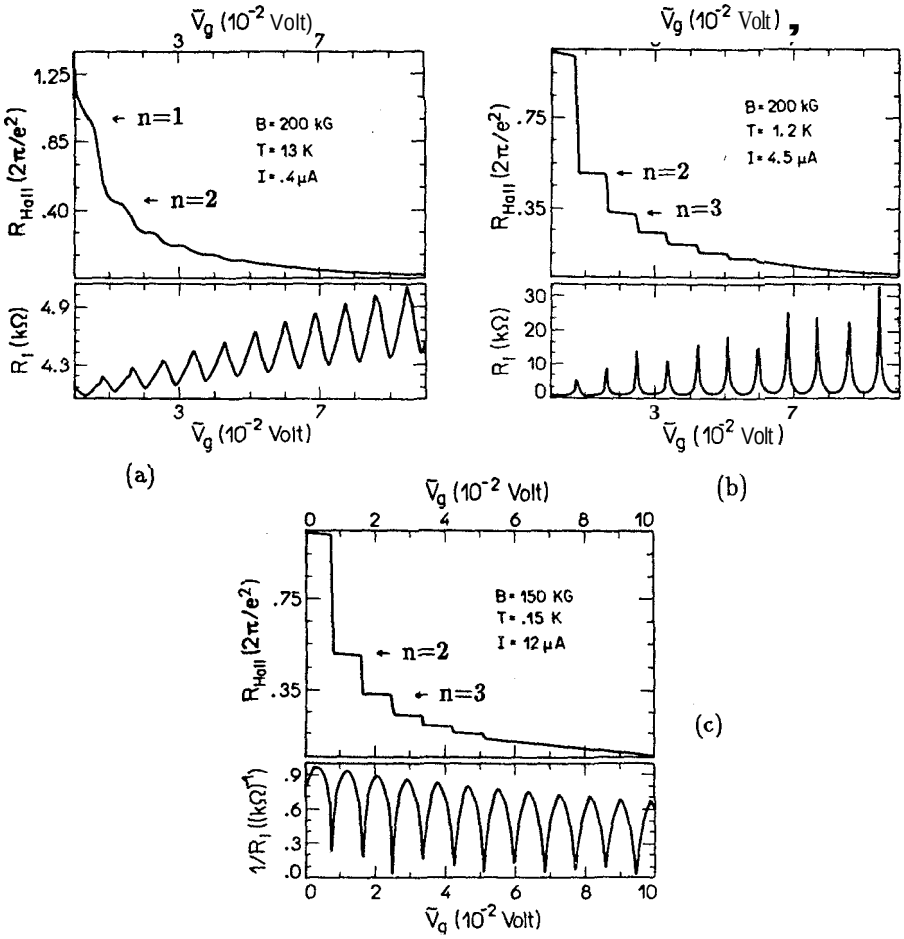

Fig.3 – Hall resistance and longitudinal resistance of the standard device as functions of the orbitals effective gate voltage, for $M \simeq 0.10M$, and (a) $B = 200$ kG, $T = 10^0$K and $I = 4.5\mu A$; (b) $B = 200$ kG, $T = 1.2^0$K and $I = 4.5\mu A$; and (c) Hall resistance and longitudinal conductance, for $B = 150$ kG, $T = 0.15^0$K and $I = 12\mu A$.

Both for the order of magnitude of physical variables (magnetic field, gate voltage, Hall resistance, longitudinal resistance, temperature and current), as well as for the general behavior of the results, the example's shown here, compare well with experimental results[2,3,5,7].

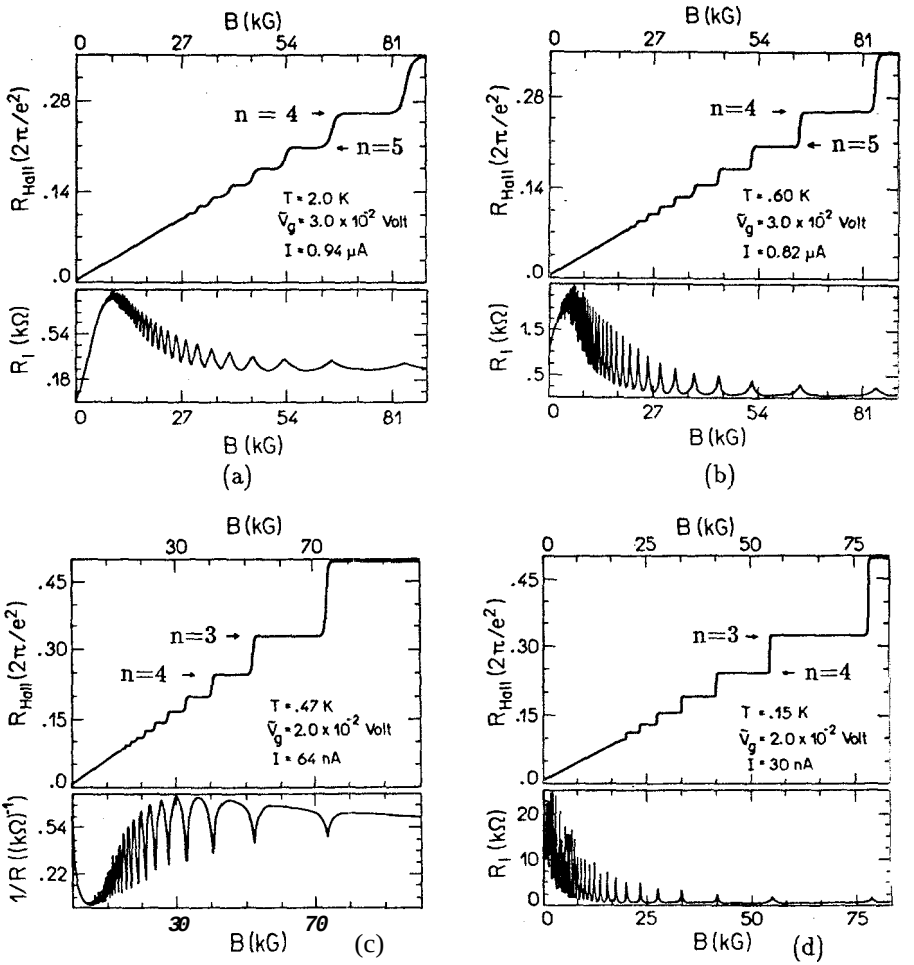

Fig.4 – Hall resistance and longitudinal resistance of the standard device as functions of the magnetic field, for $M = 0.10M$, and (a) $\tilde{V}_g = 30\text{mVolt}$, $T = 2.0^0\text{K}$ and $I = 0.94\mu A$; (b) $\tilde{V}_g = 30\text{mVolt}$, $T = 0.60^0\text{K}$ and $I = 0.82\mu A$; (c) Hall resistance and longitudinal conductance, for $\tilde{V}_g = 20\text{mVolt}$, $T = 0.47^0\text{K}$ and $I = 64\text{nA}$; and (d) $\tilde{V}_g = 20\text{mVolt}$, $T = 0.15^0\text{K}$ and $I = 30\text{nA}$.

(h) Ohmic behavior

The authors of Reference **(5)** succeded in observing the system in Ohmic regime at very low temperature.

In this regard, we report that, in some calculations, at very low temperature

and current, we have also observed the Hall voltage ohmic behavior. However we have not seen the ohmic behavior of longitudinal voltage. In that region, longitudinal voltage seems to grow with the square root of current.

In some other calculations, we observed the formation of dips at the center of longitudinal voltage peaks. We interpret such an effect as being due to the thermal competition of the first excited state, where the average electric flux has a sign opposite to that of the ground state electric flux.

(i) Mechanism of dissipation

The existence of a longitudinal voltage means that dissipation must occur in the system.

On the other hand the orbital is in a definite quantum state of well defined energy.

Then, we are led to conclude that dissipation must occur outside the orbitals, in the "ele ctron bath".

One might say that, because of electron number fluctuations, electrons alternately pass from the orbital to the "bath" and vice-versa. Since there is a tendency of accelerating the electron inside the orbitals, which is due to the externa1 source perturbation, it then happens that the bath supplies the orbitals with low energy electrons, and receive them back with higher speed. Or equivalently one can say that electrons belonging to the bath are scattered by the orbital to higher energies.

It is just the cooling in the bath of those electrons coming back with higher energy that is irreversible, and causes the dissipation effect. But, in spite of that, the orbital quantum state energy is well defined.

(j) Specific heat

To conclude the analysis about the normal Hall effect, we will show some results referring to the system's specific heat. Figure 5 shows examples of specific heat calculations for Hamiltonian H_{charge}, at two temperature values $T_a = eB_{\max}/15M$ and $T_b = eB_{\max}/100M$, where $B_{\max} = 100$ kG.

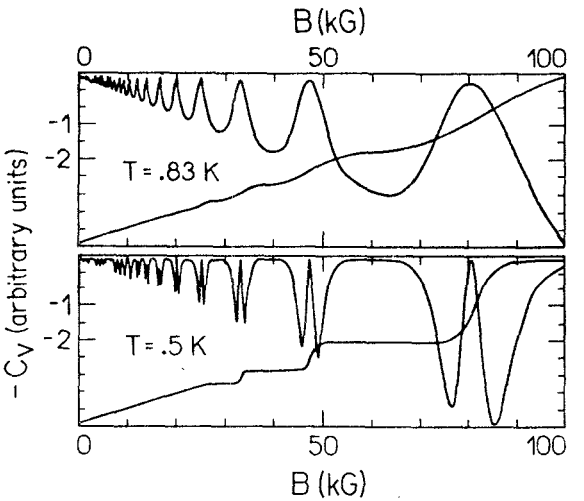

Fig.5 – Specific heat of the electron gas computed with Hamiltonian H_{charge}, as function of the magnetic field, for M = M. The figure also shows the system's Hall resistance.

The energy spacing between successive levels of Hamiltonian H„ is somewhat larger than the gap between levels of H_{charge}; and then, in the considered temperature region, the contribution of the former Hamiltonian to the specific heat is negligible in comparison with that of H_{charge}; so that, results of Figure 5 are already a good approximation for the system's total specific heat.

If the electron effective mass is $\bar{M}$ = M, then temperatures T_a and T_b are respectively $.83^0$K and $.5^0$K. If however $\bar{M} = 0.10M$, then those temperatures increase by a factor ten.

Our result at temperature T_a compares reasonably well with electronic specific heat phenomenological calculations by Gornik et al.[8], which have been used to interpret experimental data of temperature variation, in a device heated by electric

field pulses, in the temperature region from 1.0^0K to 5.0^0K. The calculations of those authors are in turn based upon a method by Zawadzki and Lassing[22], which introduces a phenomenological density of states around each Landau level.

11. The anomalous effect

Let us again take a system of surface S, charged with total charge eN. At very high magnetic field, the ãystem will have more Landau orbitals than electrons, because the ratio BS/N becomes larger than the magnitude of a magnetic quantum $flw 2\pi/e$.

But since magnetic flux is quantized at integer multiples of $2\pi/e$, there occurs the formation of larger orbitals, each one of them with p quanta of magnetic flux. And, in that case, one gets Hall resistance plateaus of the type:

$$R_{\mathrm{Hall}}^{p,n} = \frac{2\pi}{e^2}\,\frac{p}{n} \tag{57}$$

which characterizes the so-called anomalous effect.

Our procedure here will be similar to the one used in the normal effect case, with an important difference in the Hamiltonian dependence on the magnetic flux/momentum.

(i) First, let us see how the global conditions determine a classical value for the ratio p/n (regarding p and n as continuous variables). If N is the fixed total number of carriers, $K = eBS/2\pi p$ the number of orbitals, and n = N/K the number of electrons per orbital, then *classically* we have the relation

$$\frac{p}{n} = \left[\frac{eS}{2\pi N}\right] B = \gamma B \tag{58}$$

with $7 = eS/2\pi n$.

(ii) The orbital energy associated to the interaction with the magnetic field is

$$\varepsilon^0(n) = \frac{1}{2}\,\frac{eB}{M}\,n\,, \tag{59}$$

because now, in the orbital ground state, all electrons will be at the lowest energy level.

(ii) Since only the first few energy levels contribute to the phenomenon we consider here, the orbital is an essentially quantum mechanical system. It works in the extreme quantum limit, in opposition to the semiclassical regime of other phenomena. So it is difficult to visualize the form of an orbital.

In spite of that, and in order to simplify this preliminary discussion of the fractional effect, in Figure 6 we outline a picutre of a sector in the ensemble of orbitals. The figure is based on the paper's theory, specially on the form of the Landau wave function, but also on classical intuition.

In Figure 6, the dotted lines define in the device's plane the space regions belonging to the n^{th} orbital, to the $(n+1)^{th}$ one, and so on. Line AB shows where the n^{th} orbital Landau wave function (the even ones) have their maxima. Line CD does the same for $(n+1)^{th}$ orbital. The circular lines are only to recall the classical view of electrons rotating because of magnetic force. The arrow shows the direction of current flow.

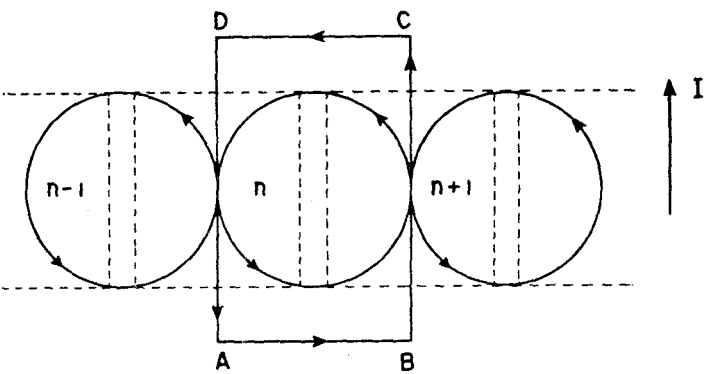

Fig.6 – Sketch of a sector of the orbitals' ensemble. The dotted lines determine the domains of different orbitals (see the text for details).

In order to construct the n^{th} orbital dynamics, we take line CD in the neighbouring orbital as a line of reference for the vector potential, making first $\int_C^B \hat{A} \cdot d\vec{\ell} = 0$. We also ascribe to the n^{th} orbital the rnagnetic flux $\hat{\phi}_{Bn} = \oint_{ABCD} \vec{A} \cdot d\vec{\ell} = \int_A^B \vec{A} \cdot$ dt?(assuming $\int_B^C \vec{A} \cdot d\vec{\ell}$ is compensated by $-\int_A^C \vec{A} \cdot d\vec{\ell}$).

If we then reintroduce the operator $\int_C^B \vec{A}\cdot d\vec{\ell}$, its average value will contribute to the external sources of the $\mathbf{n}^{\text{th}}$ orbital dynamics.

(iv) Now we make the extreme simplification of supposing that ϕ_B may be simultaneously the orbital magnetic flux and the orbital momentum, as we explain:

(a) Consider that the system is initially uncoupled to electric sources (being in a ϕ_B eigenstate, $\frac{2\pi}{e}$ p). In this case, if the system is in its ground state, we interpret ϕ_B as the orbital magnetic flux; and

(b) However, if that same system is then coupled to a very weak electric source, we then interpret the difference $\delta\phi_B = \phi_B - \frac{2\pi}{e}$ p as a avariable proportional to the orbital momentum: $p_l = \frac{e}{L}\,\delta\phi_B$. And p_l will thus be related to the current produced by the electric source.

(v) In order to show an example of calculation for the anomalous effect, we solved the phenomenological Hamiltonian:

$$\frac{eB}{M}\left\{\frac{1}{2}\hat{n} + \frac{e^2}{4\pi}\hat{n}^{-1}\hat{\phi}_B^2 - \gamma B e\hat{\phi}_B + W\left[\pi\gamma^2 B^2 - \frac{1}{2}\right]\hat{n}^2\right\}, \qquad (60)$$

coupled to external sources of the type H_l^1 and H_l^2. In Eq.(60) W is a phenomenological constant.

The reasons for constructing such phenomenological Hamiltonian are:

(a) It has a term like that of Eq.(59).

(b) In the classical limit it respects relation *(58)*.

(c) At any given value of n, by expanding the Hamiltonian in ϕ_B around its classical minimum we get: $\varepsilon(n) + \frac{1}{2Mn}\tilde{p}^2 + \cdots$, where $\tilde{p}_l = \frac{e}{L}(\phi_B - \phi_B^0)$ and ϕ_B^0 is the value of ϕ_B that minimizes the classical energy. So that the second term in that expansion can be interpreted as the kinetic energy of the orbital electronic cluster.

(vi) Figure 7 shows the result of a calculation with Hamiltonian (60), made with W = 0.70, at very low temperature (T = $eB/40M$). In this calculation, the electric source was kept fixed, for every value of B. There one sees fractional plateaus of the type $\frac{3}{2}$, 2 and 3. One notices also the plateau $p/n = 2$. As far as we

know, there is no report on the observation in experiment of anomalous plateaus p/n, with an even p and odd n.

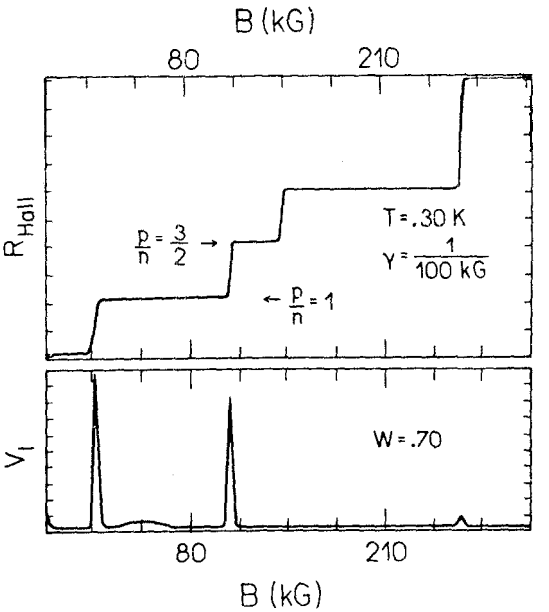

Fig.7 – Example of anomalous effect calculation: Hall resistance and longitudinal voltage, as functions of the magnetic field.

In conclusion, we say that the theory presented here provides a good treatment for the quantized Hall effect of the normal type, leading to a quite complete picture of the phenomenon although the descreption of the anomalous effect is still phenomenological and tentative.

The theoretical method introduced in Reference (I) and in this paper, which emphasizes a few modes of quantum flux as the relevant variables, showed to be the natural framework for treating the quantized Hall effect.

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Resumo

O artigo apresenta uma teoria do efeito Hall quantizado baseada na quantização de fluxo. Devido à ação do intenso campo magnético, o dispostivo é subdividido num número muito grande de orbitais semelhantes, aproximadamente quadrados. O orbital é o sistema dinâmico elementar. É uma região de correlação, onde poucos elétrons correlacionados interagem com o mesmo quantum de fluxo magnético. Sobre cada uma dessas regiões de correlação atua uma voltagem de gate efetiva, proporcional à voltagem de gate do dispositivo global, e cuja variação faz alterar o número de elétrons de cada orbital. Mas, como esse número é quantizado, sua variação se faz aos saltos, produzindo a seqüência de plateaus do efeito normal. O acoplamento do fluxo elétrico do orbital a uma fonte elétrica externa é que provoca a passagem de corrente. O fluxo elétrico também se acopla ao operador que descreve flutuações do número de elétrons do orbital. Estuda-se o comportamento de um dispositivo padrão a temperatura finita. Obtém-se a voltagem Hall e a resistência longitudinal como funções das variáveis voltagem de gate, campo magnético, temperatura, e da corrente do dispositivo. Calcula-se também o calor específico como função do campo magnético. Os resultados comparam favoravelmente como o que se observa nas experiências sobre o efeito Hall quantizado. Apresenta-se também uma análise preliminar a respeito do efeito anômalo.