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A cosmic lattice as the substratum of quantum fields

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Abstract A cosmology inspired structure for phase space is introduced, which leads to finitization and lattice-like discretization of position and momentum eigenvalues in a preferred, cosmic frame. Lorentz invariance is broken at very high energies, inaccessible at present. The divergent perturbation terms in quantum electrodynamics become finite and small: this could become a requirement leading to model restrictions in other perturbative gauge theories. So the very success of the usual renormalization procedures is simply explained by their finitization, and is viewed as indicating the reality of the lattice.

1. The boundless but finite cosmic lattice

The idea of assigning a fundamental length to physical space has a long history. A large part of the literature can be traced from the work of Gudder and Naroditsky[1]. This notion arises from dissatisfaction with the way renormalization operations are performed in quantum field theory - both the non-rigorous and the too rigorous methods. A lattice structure for space or spacetime is an obvious remedy for the worst problem, ultraviolet divergences, but people have been discouraged by the prospect of Lorentz and rotational non-invariance. But recently some work has appeared which faces just this possibility. Thus Nielsen and collaborators have developed both a quantum electrodynamics (QED) and a Yang-Mills theory that violate Lorentz invariance[2], and Zee[3] has gone as far as suggesting experimental verification of such a breakdown. Some time ago Wheeler[4] pointed out a possible breakdown of the spacetime concept itself on the scale of Planck's length.

In this paper I introduce upper limits for both measurable length and measurable momentum, as a means of reaching a kind of lattice structure for phase space. The conclusion will be that the perturbative series of QED works so well because its terms become, in a quasi-invariant sense to be defined, convergent to small numbers. The idea has a cosrnological inspiration: a Friedman-Robertson-Walker (FRW) model with flat space sections[5] has the local metric $ds^2 = c^2 dt^2 - b(t)^2 dx^2$. Its global spatial geometry is taken to be that of a flat torus T^3, which is an identification space isometric to E^3/Γ, where E^3 is Euclidean space and Γ is the group generated by finite translations in the three directions. For the development and motivations of this idea in cosmology see the list of refs. 6, in particular Ellis and Schreiber's recent work.

The expansion factor in this metric is $b(t) = (t/t_0)^{2/3}$, where $t_0 \sim 10^{10}$ yr is the age of the universe. Since $(db/dt)(t_0) \sim 10^{-18}$ sec^{-1}, let us put $b(t) = 1$ in ds^2 for the discussion of laboratory physics (adiabatic approximation). We are left with a locally Minkowskian spacetime, whose T^3 spatial sections may be obtained from a cube of side L, upon the identification ("gluing") of opposite faces. Therefore $\sqrt{3}$ L will be a maximum distance in space, and I take $L = c/H$, where $H \approx 75$ km/sec/Mpc is Hubble's constant.

Now I make the crucial assumption that momentum space (relative to the above cosmic frame) has the same T^3 topology as configuration space, so that the corresponding phase space is the product manifold $T^3 \times T^3$. Besides providing an upper limit for momentum, this sort of duality appears to the author as a more reasonable way of assuring discretization of position than just drawing from crystal structure analogy, with its "neo-ether" connotation. The flat torus for momentum is obtained by identifying opposite faces of a cube of side $P = 2\pi\hbar/a$, where $a = (G\hbar/c^3)^{1/2} = 1.61 \times 10^{-33}$ cm is Planck's length. Incidentally, a cutoff for momentum is quite reasonable, for otherwise the energy of a single virtual particle can exceed the total mass of the observable universe. It is also in agreement with Wheeler"s idea cited above: if spacetime breaks down at Planck's length, so must the validity of momenta greater than P. (Of course the $T^3 \times T^3$ topology postulate may come to be seen as a phenomenological assumption, if and when future developments lead to a more general theoretical framework, where phase space is seen as an approximation suitable for 1988 physics - much like the idea of orbit became an approximation suitable for classical motion.)

In quantum mechanics we are used to box quantization with periodic boundary conditions imposed for convenience. But in the above defined cosmic tori these conditions are just the natural ones. So we get eigenvalues $x^k = n^k a$ and $p^k = n^k(\pi\hbar/Na)$, $k = 1,2,3$, $-N < n^k = \text{integer} \leq N = L/2a \approx 3.8 \times 10^{60}$. Space thus becomes a very large box, which is both finite and boundless, with discrete eigenvalues for particle position and momentum. This suggests that we call it a 'cosmic lattice' (CL), but note that is an abstract, not a granulated, crystal-like lattice.

2. Lorentz quasi-invariance

What about Lorentz invariance? First, the preferred status of the CL frame should not cause much surprise, It is the home frame of our cosmos, similar to the comoving system of Einstein-de Sitter's cosmology, the $2.7^0 K$ radiation providing its concrete realization (except for spatial orientation; see Gott[6] for an explanation on how the apparent *isotropy* of cosmic observations is preserved despite the loss of global invariance under rotations). Second, let us define the composition of 4-momenta p_1^μ and p_2^μ in the CL system. Setting $\hbar = c = 1$, if $|p_1^k + p_2^k| \leq \pi/a$, then energy-momenta are composed as usually: $p^\mu = p_1^\mu + p_2^\mu$. If $|p_1^k + p_2^k| > \pi/a$, then we add or subtract $2\pi/a$, so as to obtain p^k in the allowed range. This p^k ($\equiv p_1^k + p_2^k \mod 2\pi/a$) is defined to be the resultant. With $s = (p_1^0 + p_2^0)^2 - (\mathbf{p}_1 + \mathbf{p}_2)^2$, the resultant energy is $p^0 = (s + \mathbf{p}^2)^{1/2} < p_1^0 + p_2^0$. Therefore energy-momentum is only conserved incollisions if $\mathbf{p}_1 + \mathbf{p}_2$ is a CL momentum eigenvalue. But this is hardly a constraint, since laboratory momenta are far from our limit, $\sqrt{3}\pi/a \sim 10^{20}$ GeV/c.

Thirdly, Lorentz transformations (including rotations) are performed as usually, but the limits of p^k in an arbitrary frame are derived from those in the CL frame. Thus if $\vec{\beta} = (v,0,0)$, $v > 0$, with no rotation, then

$$p^1_{max} = \gamma[\pi/a - v(m^2 + \pi^2/a^2)^{1/2}] \approx \pi/2a\gamma,$$

and

$$p^{1}_{min} = \gamma[-\pi/a - v(m^{a} + \pi^2/a^2)^{1/2}] \approx 2\gamma\pi/a,$$

for large 7. The limits for p^2, p^3 remain unaltered. If we now rotate this frame, the physics will be invariant if all relevant momenta are smaller than $\pi/2a\gamma$, although the limits on each component could be different from each other (and awkwardly expressed). Again, in laboratory situations we do not have to worry about these limits. Summarizing, Lorentz and rotational invariance are preserved if we restrict ourselves to laboratory energies and to a theoretical range of frames - say, $\gamma < 10^{10}$ with respect to the CL, which guaranteees invariance up to $\sim 10^9$ GeV in the moving frame. I shall refer to this restricted meaning as Lorentz *quasi-invariance.*

3. Finite renormalization

The practice of renormalization in QED has been so strongly associated with the removal of infinities that the fundamental meaning of the former became blurred. See for example Schweber's[7] warning against this tendency. kctually the aim of renormalization is to combine some unobservable parameters of a basic model into observable ones, so that the renormalized model is expressed in terms of the latter. The advantage of this is obvious when one considers that the purpose of theoretical models is to represent experimental facts. Consider mass renormalization in QED: we write $m = m_0 - \delta m$, and say that m is the experimental mass of the electron. 'But the underlying formalism suggests that we also interpret m_0 and δm anyway, as bare mass and the effect of virtual photons always surrounding the electron respectively. It seems to the author that if m_0 and δm can be made finite, so much the better: their interpretation is reinforced, and we may even think of making them observable in another context - as when we tried to assign mass differences in isospin multiplets to electro-magnetic interactions[S]. (See, however,ref.[9].)

Therefore my program is not to abandon renormalization, but rather to make it step-by-step finite[10]. I will essentially follow the stablished formalism with a few adaptations: (a) integrals are in principle replaced by sums over the CL, but in practice the latter are approximated by integrals that formally resemble the original ones but are now finite and small (and justifiably so); (b) the calculations are preferably performed in the CL, which is the natural system in this context, just like a Sun-centered system is natural for planetary astronomy (this naturalness can of course be formalized); (c) restricted Lorentz transformations, as discussed in Sec. 2, are seen to hold for the results of calculations.

Let us examine some problems of perturbative field theory in terms of the above ideas. The great successes of the usual formalism suggests that we try to keep its analytical basis, rather than for example switching to difference equations[1,11]. This is physically reasonable, since the scale of Planck's length is so much finer than those of currently observable processes. Therefore I will here assume minimum departures from established analytical expressions. For comparison with standard results I will rely on Itzykson and Zuber's textbook[12], henceforth referred to as (IZn), n = page number.

The infrared catastrophes will be transformed in finite contributions (since the minimum energy of a massless particle is π/Na, not zero), and if these are still too Iarge they may be dealt with as usually, e.g. as in (12334). Consider now charge renormalization in QED. The notation below is adapted from (IZ319ff). The one-loop contribution to vacuum polarization, after use of Feynman's trick $(ab)^{-1} = \int_0^1 dz[az + b(1-z)]^{-2}$, is[13]

$$\bar{\omega}_{\rho\nu}(\mathrm{k}) = -4\mathrm{e}^2 \int^{1} dz \Big\{ (z - z^2) \sum_{q \in CL} \int_{-\infty}^{\infty} \frac{dq_0}{2\pi} \frac{(g_{\rho\nu}k^2 - 2k_\rho k_\nu)(z - z^2) - g_{\rho\nu}(q^2/2 - m^2)}{[q^2 + k^2(z - z)^2 - m^2 + i\epsilon]^2} \Big\} \tag{1}$$

This expression is well defined, so it can safely be simplified by gauge invariance, which leads to

$$\bar{\omega}_{\rho\nu}(k) = -i(g_{\rho\nu}k^2 - k_\rho k_\nu)\bar{\omega}(k^2) \tag{2}$$

with

$$\bar{\omega}(k^2) = 2e^2 \int_0^1 dz(z-z)^2 \sum_q [q^2 + m^2 + k^2(z-z^2)]^{-3/2}$$

Eq.(2) is Lorentz quasi-invariant, as defined above. Although calculated in the CL system, $\bar{\omega}(k^2)$ is an invariant - like, say, the contribution of vibrational energy to the mass of a crystal. Approximating[14] the sum over the cosmic box by an integral over a ball of radius π/a, and neglecting positive powers of ma, I obtained, for $k^2 < 4m^2$,

$$\bar{\omega}(k^2) = (\alpha/3\pi)[\log(2\pi/ma)^2 - 2 + k^2/5m^2 \ldots].$$

Hence $Z_3 = [1+\bar{\omega}(0)]^{-1} = ,925$ and $e = .962\ e_o$. The Uehling term is the same as in (12237). Similarly, to the same order I got

$$m/m_0 = 1 + (3\alpha/4\pi)[\log(2\pi/ma)^2 - 1/3] = 1.185,$$

and, in Feynman's gauge,

$$Z_2^{-1} - 1 = (\alpha/4\pi)[\log(2\pi/ma)^2 + 2\log(\pi/Nma)^2 + 3],$$

hence $Z_2 = 1.160$.

If we compare the above results with their counterparts in (12325, 334, 335), we see that the logarithmic terms in the former can be obtained from those in the latter if we replace A by $2\pi/a$, μ by π/Na. This is as expected (at least to order of magnitude), since a cutoff prescription is one of several regularization procedures that produce the same final results in ordinary renormalization (12374). The author hopes to derive similar results for higher orders in quantum electrodynamics, and for other gauge theories of fundamental processes. The important immediate consequence of the achieved finitization is that the perturbative series, which are normaily understood in a context of formal procedures, to "extract sensible results from apparently ill-defined expressions" (IZ318), become legitimized as ordinary convergent series. Theories satisfying this condition could be called *perturbatively* renormalizable, and this property might be a further guide for model building.

(So, for example, the perturbative treatment of $\lambda\varphi^4$ models might be deemed **un**-acceptable, because of quadratic **divergence** in mass renormalization.) As bonuses, the calculations **become less** difficult - the **"naive prescription"** (12374) of cutting off large momenta becomes the natural one - and the meaning of the **renormalized** lagrangia. gets a numerical foundation - compare **(IZ345-346).** Interpreted in this light, the fact that renormalization theory has **been** so successful **can** be invoked as an argument for the physicality of the CL or some related conception. It remains to be seen whether this notion **will** be tested, for example in proton decay **as** suggested by **Zee**[3].

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10. One of the discoverers of QED renormalization has written that the rules for eliminating the infinities are like "sweeping the dirt under the rug"[8], p. 137. These rules can perhaps be justified by highly sophisticated mathematics. Here instead I am suggesting a mathematically simple and physically sensible way to avoid the appearsnce of infinities.
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13. This trick implies the translation $q \rightarrow q - \mathbf{k}(1 - z)$ in the virtual momenta. Hence the sum will no longer be strictly symmetrical in the CL. But since $|\mathbf{k}| << \pi/a$ we disregard this asymmetry.
14. A better approximation would be to integrate over a cube of side $2\pi/a$, but for this preliminary work I adopted the simpler integration over a ball.

Resumo

Introduz-se uma estrutura para o espaço de fase, baseada na cosmologia relativística, que leva à finitização e discretização dos autovalores de momentum e posição num sistema de referência preferencial. A invariância de Lorentz é quebrada para energias muito altas, presentemente inacessíveis. Os termos de perturbação divergentes em eletrodinâmica quântica tornam-se finitos e pequenos; isto poderia tornar-se uma condição que restringiria os modelos em outras teorias de gauge perturbativas. Assim o próprio sucesso do procedimento usual de renormalização é explicado pela sua finitização, e é visto como indicando a realidade da rede cósmica.