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Two-loop effective potential for Wess-Zumino model using superfields

R.P. dos Santos and P.P. Srivastava

Centro Brasileiro de Pesquisas Físicas, Rua Xavier Sigaud 150, Rio de Janeiro, 22290, RJ, Brasil

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Abstract For the case of several interacting chiral superfields the propagators for the unconstrained superfield potentials in the 'shifted' theory, where the supersymmetry is explicitly broken, are derived in a compact form. They are used to compute the one-loop effective potential in the general case, while a superfield calculation of the renormalized effective potential to two loops for the Wess-Zumino madel is performed.

1. Introduction

For the globally supersymmetric theories[1] the superfield' formulation is very economical to calculate quantum corrections. The cancellation of (higher) divergentes associated with the bosonic loops against those of the fermionic loops is automatically taken care of through the superpropagators. The number of supergraphs required to be considered is greatly reduced compared to that needed in the component formulation. The non-renormalization theorems may be shown directly. The superfield path integral formulation is a powerful calculational tool, for example, through the background field method[3], of calculating the effective potential, and the possibility of performing a change of variables to derive Ward identities apart from its compactness. The superfield formulation has now been developed suficiently[3,4] so as to allow manangeable calculations to higher order loops.

We derive in Sec.2 for the case of several interacting chiral superfields the superpropagators of the unconstrained superpotentials in the presence of a classical background[5,6]. In Sec.3 we discuss the superfield tadpole[6] and bubble methods for calculating the effective potential using the shifted theory propagators. The expressions for the one-loop effective potentials are derived and a procedure for the two-loop case as well as for renormalization is indicated. In Sec.4 we discuss in detail the superfield calculation of effective potential up to two loops for the case of a single chiral superfield. The computation is performed in a modified minimal subtraction scheme as well as in a scheme where the renormalization constants are functions of the background field[7] in order to avoid, for sufficiently large values of the physical (scalar) field, the kinetic terms with the wrong sign.

2. 'Shifted' theory propagators

The chiral superfields ,Φ_i, $i = 1,2,\ldots,n$, satisfy the differential constraints $\bar{D}\Phi_i = 0$, $D\bar{\Phi}_i = 0$ and it seems difficult to formulate the functional integral over Φ and $\bar{\Phi}$. We may, bowever, analogous to the case of e.m. field, introduce the *unconstrained superfield potentials*[5,8] S and $\bar{S}$ such that

$$\Phi_i = -1/4\,\bar{D}^2 S_i \quad\text{and}\quad \bar{\Phi}_i = -1/4\,D^2 \bar{S}_i \tag{2.1}$$

This introduces in the theory an additional invariance under the Abelian gauge transformations: $S \to S + \bar{D}\bar{F}$, $\bar{S} \to \bar{S} + DF$. We may take care of it by adding to the action the following ghost-free gauge-fixing term[10]

$$I_{\mathrm{G.F.}} = \alpha^{-1}\int d^8 z\, \bar{S}_i (1 - P_1)\Box S_i \tag{2.2}$$

The functional integral may then be formulated easily over S and S[10]. The perturbation theory performed with S, $\bar{S}$ propagators rather than Φ, $\bar{\Phi}$ ones involves integrals over full superspace as is evident from

$$I_{\mathrm{int}} = \frac{1}{3} g_{ijk} \int d^8 z\, S_i \Big(-1/4\,\bar{D}^2 S_j ig\Big)\Big(-1/4\,\bar{D}^2 S_k ig\Big) + \mathrm{c.c.} \tag{2.3}$$

The Feynman rules for the vertices may be read from eq.(2.3) on applying Wick's theorem in the conventional way.

The background field method for calculating the effective action requires the splitting of each superfield into a classical background piece plus a quantum one. In our context we perform the shifts $\Phi_i \to \Phi_i + C_i$, where C_i are background chiral superfields, $\bar{D}C_i = 0$. In the case of n interacting chiral superfields with the action

$$\int d^8 z\, \bar{\Phi}_i Z_{ij} \Phi_j + \left[\int d^6 s\, W(\Phi) + c.c.ig \right] \tag{2.4}$$

where $W(\Phi)$ is the renormalizable superpotential

$$W(\Phi) = \lambda_i \Phi_i + \frac{1}{2} m_{ij} \Phi_i \Phi_j + \frac{1}{3} g_{ijk} \Phi_i \Phi_j \Phi_k \tag{2.5}$$

the 'shifted' theory contains the following terms :

$$\begin{aligned} I_o = & \int d^8 z \left[\bar{S}_i P_1 \Box Z_{ij} S_j - 1/8\, C_{ij} S_i \bar{D}^2 S_j - 1/8\, \bar{C}_{ij} \bar{S}_i D^2 S_j \right] \\ & + \alpha^{-1} \int d^8 z\, \bar{S}_i \left(1 - P_1\right) \Box Z_{ij} S_j + \int d^8 z \left(J_i S_i + \bar{J}_i \bar{S}_i \right) \end{aligned} \tag{2.6}$$

$$\begin{aligned} I_{int} = & \frac{1}{3} g_{\ell jk} \int d^8 z\, \Phi_\ell S_j \left(- 1/4\, \bar{D}^2 S_k \right) \\ & + \int d^6 s \left[\lambda_i \Phi_i + m_{ij} C_i \Phi_j + g_{ijk} C_i C_j \Phi_k + \int d^2 \theta\, \bar{C}_i Z_{ij} \Phi_j \right] \\ & + c.c. \end{aligned} \tag{2.7}$$

$$I_b = \int d^8 z\, \bar{C}_i Z_{ij} C_j + \left[\int d^6 s\, W(C) + c.c. \right] \tag{2.8}$$

Here I_o is the free action to which we have added the gauge-fixing and external source term, whereas I_b is the background action. We define the matrix C $= \left(C_{ij}\right) = \left(m_{ij}\right) + 2\left(g_{ijk} C_k\right)$ and have intmduced for later use the renormaliration constant Z_{ij} which are the elements of a positive definite hermitian matrix.

The free 'shifted' theory effective propagators[1'] may be derived straightforwardly and we find[5,6]

$$\Delta^{\bar{S}\bar{S}} = \frac{\left(\bar{Z}^{-1}C\right)}{4\Box}\bar{D}^2\Delta^{S\bar{S}} \tag{2.9}$$

$$\Delta^{S\bar{S}} = i\left[\alpha\frac{\left(P_2+P_T\right)}{\Box} + \left\{\Box - Z^{-1}\bar{C}P_1\bar{Z}^{-1}C\right\}^{-1} fP_1\right]\delta^8(z-z') \tag{2.10}$$

with analogous expressions for Δ^{SS} and $\Delta^{\bar{S}S}$. The term independent of the gauge-fixing parameter in eq.(2.10) may be expressed for the constant background $C_k = a_k + f_k\theta^2$ in a compact form by making explicit the poles and θ, $\bar{\theta}$ dependence[6] which renders the superspace integrations to be performed easily. We find $(Z=1)$

$$\begin{aligned}\Delta^{S\bar{S}} &= iP_1\left(\Box - \bar{M}M\right)^{-1}\delta^8(z-z')\\ &+ ie^{i(\theta\sigma\partial\,\bar{\theta} + \theta'\sigma\partial\bar{\theta}')}\left[A\bar{\theta}^2\theta'^2 + B + C\bar{\theta}^2 + E\theta'^2\right]\delta^4(x-x')\\ &= iP_i\left\{P_1\left(\Box - \bar{M}M\right)^{-1} + \left[A\bar{\theta}^2P_1\theta^2 + \frac{B}{16\Box}D^2e^{-2i\theta\sigma\partial\,\bar{\theta}}D^2\right.\right.\\ &\left.\left. + G\bar{\theta}^2P_1 + EP_1\theta^2\right]\Box\right\}\delta^8(z-z')\end{aligned} \tag{2.11}$$

where

$$M = \left(M_{ij}\right) = \left(m_{ij} + 2g_{ijk}a_k\right),$$

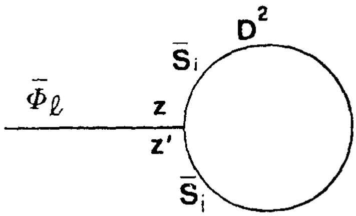
$$f = 2\left(g_{ijk}f_k\right)$$

and

$$C = M + f\theta^2$$

The expressions for A, B, G, E are given in Appendix A. The second term in eq.(2.11) arises solely from the explicit supersymmetry breaking terms in eq.(2.6)

Fig.1 - One-loop tadpole supergraph,



and clearly vanishes when $f_i = 0$. For the case of a single superfield ($\lambda = 0$)

$$A = \frac{Z^{-3}|f'|^2}{(\Box - Z^{-2}|a'|^2[(Z^{-2}|a'|^2)^2 - Z^{-2}|f'|^2]} \quad \Box^2 B = Z^{-2}|a'|^2 A$$

$$G = \bar{E} = \frac{Z^{-3}\bar{f}' a'}{\left[\left(\Box - Z^{-2}|a'|^2\right)^2 - Z^{-2}|f'|^2\right]} \tag{2.12}$$

where $a' = m + 2ga$, $f' = 2gj$.

3. Superfield method for Susy effective potential

The effective scalar potential may be easily computed using the explicit expressions of the 'shifted' theory superfield propagators given in eqs.(2.9) and (2.11). In the superfield tadpole method[5], for example, we are required to compute the tadpole supergraphs for the 'shifted' theory to the desired number of loops. The number of such supergraphs is greatly reduced compared to those encountered in a calculation using the component fields. Moreover, the well-known compensation in a S. S. theory of the higher divergences of boson loops with those arising from fermion loops is already taken care of through the effective superfield propagators derived above. The superfield tadpole method allows us to read off directly the partial derivatives[12] of the effective potential with regard to all the scalar fields present in the theory.

The one-loop correction V_1 to the effective potential for the action in eq.(2.4), to give an illustration, requires the evaluation of a single tadpole supergraph for the 'shifted' theory (see fig. 1) and we find

$$i\Gamma_1^{(1)} = i\frac{1}{3}3\int d^2\theta\left[g_{ijk}\tilde{\bar{\Phi}}_i(0,\theta,\bar{\theta})\left(\Delta^{\bar{\Phi}\bar{\Phi}}\right)_{jk}\right]_{x=x'}$$
$$= i\int d^2\theta\tilde{\bar{\Phi}}_i \mathrm{Tr} g_i \Delta^{\bar{\Phi}\bar{\Phi}} \tag{3.1}$$

where $(g_\ell)_{ij} = g_{\ell ij}$, $\Phi(0,6,\bar{\theta}) = A(0) + \sqrt{2}\theta\Psi(0) + \theta^2 F(0)$ and a tilde denotes the Fourier transform. Performing the θ integration we read off from the coefficients of $A(0)$ and $F(0)$ the following partial derivatives[5]

$$\frac{\partial V_1}{\partial \bar{f}_\ell} = -\frac{i}{2}\mathrm{tr}\frac{\partial(\bar{H}H)}{\partial \bar{f}_\ell}(I_n - \bar{H}H)^{-1}$$
$$= \frac{1}{2}\mathrm{tr}\left(\frac{\partial X^2}{\partial \bar{f}_\ell}\right)\left(k^2 I_{2n} + X^2\right)^{-1} \tag{3.2}$$

$$\frac{\partial V_1}{\partial \bar{a}_\ell} = -\frac{1}{2}\mathrm{tr}\left[\frac{\partial(M\bar{M})}{\partial \bar{a}_\ell}\left(k^2 + M\bar{M}\right)^{-1}\left(I_n - H\bar{H}\right)^{-1}\right.$$
$$\left. + \frac{\partial(\bar{M}M)}{\partial \bar{a}_\ell}\left(k^2 + \bar{M}M\right)^{-1}\left(I_n - \bar{H}H\right)^{-1}\right]$$
$$= \frac{1}{2}\mathrm{tr}\left(\frac{\partial Y^2}{\partial \bar{a}_\ell}\right)\left[\left(k^2 + X^2\right)^{-1} - \left(k^2 + Y^2\right)^{-1}\right] \tag{3.3}$$

and

$$V_1 = \frac{1}{2}\mathrm{tr}\ln\left[I_n - H\bar{H}\right]$$
$$= \frac{1}{2}\mathrm{tr}\left[\ln\left(k^2 I_{2n} + X^2\right) - \ln\left(k^2 I_{2n} + Y^2\right)\right] \tag{3.4}$$

which was also derived by other methods[13]. Here

$$H = f\left(k^2 + \bar{M}M\right)^{-1}$$
$$\mathrm{tr} \equiv -i\int \frac{d^4k}{(2\pi)^4}\mathrm{Tr},$$
$$\frac{\partial V}{\partial f_\ell} \equiv \frac{\partial V}{F_c}\Big|_{\substack{A_c = a \\ F_c = f}} \text{ etc.,}$$

where c indicates classical field and we define $2n \times 2n$ matrices X^2 , Y^2 by

$$X^2 = \begin{pmatrix} M\bar{M} & f \\ \bar{f} & \bar{M}M \end{pmatrix} \qquad Y^2 = \begin{pmatrix} M\bar{M} & 0 \\ 0 & \bar{M}M \end{pmatrix} \tag{3.5}$$

The logarithmic divergence in eqs. (3.2) or (3.4) may be handled employing dimensional regularization[14]. Since the coefficient of the divergent integral in eq.(3.4) is $\bar{f}f$ which arises in the kinetic term, we need to perform only a wave function renormalization.

The superfield vacuum bubble method, on the other hand, directly gives rise to the effective potential and may sometimes be convenient. For example, at the zero-loop we obtain from eq.(2.8)

$$\begin{aligned} -iV_o &= i\Big[\int d^4\theta\, \bar{C}_i Z_{ij} C_j + \int d^2\theta\, W(C) + \int d^2\bar{\theta}\, \bar{W}(\bar{C})\Big] \\ &= i\Big[\bar{f}_i Z_{ij} f_j + f_i \frac{\partial W}{\partial a_i} + \bar{f}_i \frac{\partial \bar{W}}{\partial \bar{a}_i}\Big] \end{aligned} \tag{3.6}$$

The two-loop contribution similarly requires the computation of the following vacuum bubble supergraphs for the 'shifted' theory (see fig.2). In the superfield tadpole method we simply have to attach, in all possible ways, an external Φ (or $\bar{\Phi}$) leg to these diagrams.

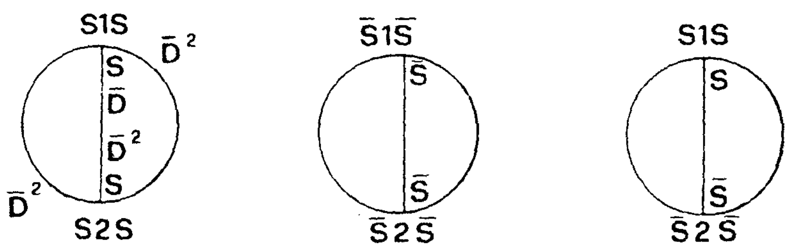


Fig.2 - Two-loop vacuum bubbles supergraphs.

The renormalized effective potential may be obtained by constructing the counterterms recursively starting from the action given in eq.(2.4) where it is understood to be written in terms of the renormalized quantities (the suffix R being supressed for convenience). The renormalization constant matrix Z is expanded[15] in powers of $\hbar$

$$Z = I + \hbar Z_1 + \hbar^2 Z_2 + \dots \tag{3.7}$$

and $Z_1, Z_2, \dots$ are determined by requiring that the divergences cancel to the order of loops being considered.

The procedure adopted here and in the previous section is clearly adapted for the case when the gauge superfields are also present. However, we obtain quadratic terms of the type ΦV in the free action of the 'shifted' theory. In the presence of the explicitly broken supersymmetry we have not been able to find[16] a suitable gauge-fixing condition which may remove such terms and consequently diagonalize the superfield propagators.

4. Two-loopf effective potential for the Wess-Zumino model

In the case of a single chiral superfield we obtain at the tree level from eq.(3.6)

$$V_0 = -\left[Z|f|^2 + (ma + ga^2)f + (m\bar{a} + g\bar{a}^2)\bar{f} \right] \tag{4.1}$$

Writing

$$V_0 = V_0^{(0)} + V_0^{(1)} + V_0^{(2)} + \dots \tag{4.2}$$

we find

$$V_0^{(0)} = V_{0R} = -\left[|f|^2 + (ma + ga^2)f + (m\bar{a} + g\bar{a}^2)\bar{f} \right] \tag{4.3}$$

$$V_0^{(1)} = -\hbar Z_1 |f|^2 \tag{4.4}$$

$$V_0^{(2)} = -\hbar^2 Z_2 |f|^2 \tag{4.5}$$

where $V_0^{(0)}$ is the (regular) zero-loop contribution to the effective potential while $V_0^{(1)}$, $V_0^{(2)}$, ... act as counterterrns for the cancellation of the divergent terms in the higher loop contributions.

We find from eq.(3.2) at one-loop

$$\frac{\partial V_1}{\partial \bar{f}'} = \frac{1}{2}\mathrm{tr}Z^{-2}\frac{f'}{\left[\left(k^2 + Z^{-2}|a'|^2\right)^2 - Z^{-2}|f'|^2\right]}$$
$$= -\frac{i}{2Z^2}\int \frac{d^4k}{(2\pi)^4}\frac{f'}{\left[\left(k^2 + |a'|^2\right)^2 - Z^2|f'|^2\right]} \tag{4.6}$$

where we make the change of variable $k \to Zk$. On performing dimensional regularization we obtain

$$\frac{\partial V_1}{\partial \bar{f}'} = \frac{1}{64\pi^2}\frac{1}{Z^2}f'\Big\{-\left(\frac{4}{\epsilon} + 2 - 2\gamma\right)$$
$$+ \frac{1}{Z|f'|}\Big[\left(|a'|^2 + Z|f'|\right)\ln\frac{|a'|^2 + Z|f'|}{\mu^2}$$
$$- \left(|a'|^2 - Z|f'|\right)\ln\frac{|a'|^2 Z|f'|}{\mu^2}\Big]\Big\} \tag{4.7}$$

Writing

$$V_1 = V_1^{(1)} + V_1^{(2)} + \cdots$$

we find from eq.(4.7)

$$\frac{\partial V_1^{(1)}}{\partial \bar{f}'} = \frac{1}{64\pi^2}\Big[-\left(\frac{4}{\epsilon} + 2 - 2\gamma\right)$$
$$+ \frac{1}{|f'|}\left(a_+^2 \ln\frac{a_+^2}{\mu^2} - a_-^2 \ln\frac{a_-^2}{\mu^2}\right)\Big]f' \tag{4.9}$$

$$\frac{\partial V_1^{(2)}}{\partial \bar{f}'} = \frac{1}{64\pi^2}Z_1\Big[\left(\frac{2}{\epsilon} + \frac{1}{2} - \gamma\right) + \frac{1}{4}\left(\ln\frac{a_+^2}{\mu^2} + \ln\frac{a_-^2}{\mu^2}\right)$$
$$- \frac{3}{4}\frac{1}{|f'vert}\left(a_+^2 \ln\frac{\tilde{a}_\pm^2}{\mu^2} - a_-^2 \ln\frac{a_=^2}{\mu^2}\right)\Big]f' \tag{4.10}$$

where $a_\pm^2 = b^2 \pm |f'|$ and we set $b^2 = |a'|^2$. The expression for the partial derivative $\partial V_1/\partial\bar{a}'$ is also found easily and we find, after integrating the partial differential equations

$$V_1^{(1)} = \frac{1}{64\pi^2}\Big[-\Big(\frac{4}{\epsilon}+3-2\gamma\Big)|f'|^2 + a_+^4 \ln\frac{a_+^2}{\mu^2} + a_-^4 \ln\frac{a_-^2}{\mu^2} - 2b^4 \ln\frac{b^2}{\mu^2}\Big] \tag{4.11}$$

$$V_1^{(2)} = \frac{Z_1}{16\pi^2}\Big[2\Big(\frac{1}{\epsilon}+\frac{1}{2}\Big)|f'|^2 - a_+^4 \ln\frac{a_+^2}{\mu^2} - a_-^4 \ln\frac{a_-^2}{\mu^2} + 2b^4 \ln\frac{b^2}{\mu^2} + \frac{1}{2}|f'|\Big(a_+^2 \ln\frac{a_+^2}{\mu^2} - a_-^2 \ln\frac{a_-^2}{\mu^2}\Big)\Big] \tag{4.12}$$

They are seen to vanish for $f \to 0$. In order to remove the divergence to first order, e.g. in $\left(V_0^{(1)} + V_1^{(1)}\right)$ we may choose for Z_1 the value

$$Z_1 = -\frac{g^2}{16\pi^2}\Big(\frac{4}{\epsilon}+3-2\gamma\Big) \tag{4.13}$$

The regularized one-loop contribution to the effective potential is then given by

$$V_{,,} = \frac{1}{64\pi^2}\Big[a_+^4 \ln\frac{a_+^2}{\mu^2} + a_-^4 \ln\frac{a_-^2}{\mu^2} - 2b^4 \ln\frac{b^2}{\mu^2}\Big] \tag{4.14}$$

For the corrections up to two loops we may set $Z = 1$ in the computation of the two-loop vacuum bubles (fig.2) (or tadpoles). It is also convenient to integrate by parts over the full superspace integrals and write the contributions in the form

$$\int d^6 s_1\, d^6 s_2 \left(\bar{D}_1^2 \bar{D}_2^2 \Delta^{ss}\right)^3 \tag{4.15}$$

$$\int d^6 \bar{s}_1\, d^6 \bar{s}_2 \left(D_1^2 D_2^2 \Delta^{\bar{s}\bar{s}}\right)^3 \tag{4.16}$$

$$2\int d^6 s_1\, d^6 \bar{s}_2 \left(\bar{D}_1^2 D_2^2 \Delta^{s\bar{s}}\right)^3 \tag{4.17}$$

We find for the divergent terms

$$V_2 = \frac{4g^2}{(16\pi^2)^2}\Big\{\frac{1}{\epsilon^2}|f'|^2 + \frac{1}{\epsilon}\Big[\Big(\frac{3}{2}-\gamma\Big)|f'|^2 - a_+^4 \ln\frac{a_+^2}{\mu^2} - a_-^4 \ln\frac{a_-^2}{\mu^2}\Big\} + 2b^4 \ln\frac{b^2}{\mu^2} + \frac{1}{2}|f'|\Big(a_+^2 \ln\frac{a_+^2}{\mu^2} - a_-^2 \ln\frac{a_-^2}{\mu^2}\Big)\Big] + \cdots\Big\} \tag{4.18}$$

The divergent terms $1/\epsilon$ as well as $1/\epsilon^2$ are removed up to second order, e.g. in $V_0^{(2)} + V_1^{(2)} + V_2$ if along with eq.(4.13) we choose for Z_2 the value

$$Z_2 = -\left(\frac{4g^2}{16\pi^2}\right)^2 \left[\frac{1}{\epsilon^2} + \frac{1-\gamma}{\epsilon} - \frac{1+\gamma}{4} - \frac{\pi^2}{24}\right] \tag{4.19}$$

where $\gamma = 0.5772$ is the Euler-Mascheroni constant.

The regularized two-loop contribution to the effective potential is obtained to be

$$\begin{aligned} V_{2R} = \frac{4g^2}{16\pi^2}^2 \Big\{ & \frac{1}{2}\left(a_+^2 \ln\frac{a_+^2}{\mu^2} + a_-^2 \ln\frac{a_-^2}{\mu^2} - 2b^4 \ln\frac{b^2}{\mu^2}\right)^2 \\ & - r\frac{3-2\gamma}{2}\left(a_+^4 \ln\frac{a_+^2}{\mu^2} + a_-^4 \ln\frac{a_-^2}{\mu^2} - 2b^4 \ln\frac{b^4}{\mu^2}\right) \\ & - \frac{3-2\gamma}{2} b^2 \left(a_+^2 \ln\frac{a_+^2}{\mu^2} + a_-^2 \ln\frac{a_-^2}{\mu^2} - 2b^4 \ln\frac{b^2}{\mu^2}\right) \\ & - \frac{3}{4}\left(a'^2 f' + a'^{*2} f'^* - 2b^4\right) J(1) \\ & - \frac{1}{8}\left(-2b^2 + a'^2\sqrt{\frac{f'}{f'^*}} + a'^{*2}\sqrt{\frac{f'^*}{f'}}\right) a_+^2\, J(a_+^2/a_-^2) \\ & - \frac{1}{8}\left(-2b^2 - a'^2\sqrt{\frac{f'}{f'^*}} - a'^{*2}\sqrt{\frac{f'^*}{f'}}\right) a_-^2\, J(a_-^2/a_-^2) \\ & - \left(a_-^2 - a'^2\sqrt{\frac{f'}{f'^*}} - a'^{*2}\sqrt{\frac{f'^*}{f'}}\right) a_+^2\, J(a_+^2/b^2) \\ & - \left(a_+^2 + a'^2\sqrt{\frac{f'}{f'^*}} + a'^{*2}\sqrt{\frac{f'^*}{f'}}\right) a_-^2\, J(a_-^2/b^2) \Big\} \end{aligned} \tag{4.20}$$

and V," $= V_{0R} + V_{1R} + V_{2R}$ gives the regularized effective potential up to two loops in the modified minimal subtraction scheme specified by eqs.(4.13) and (4.19) where the function J is defined in Appendix B.

Each of V_{0R}, V_{1R} and V_{2R} and their partial derivatives vanish in the limit f $\rightarrow$ 0 in agreement with the non-renormalization theorem when the supersymmetry is not broken at the tree level. The tree level renormalization constraints

$$\left.\frac{\partial^2 V^{eff}}{\partial a \partial f}\right|_{a=f=0} = m$$
$$-\frac{1}{2}\left.\frac{\partial^3 V^{eff}}{\partial a^2 \partial f}\right|_{a=f=0} = g \qquad (4.21)$$

are also unaffected by the radiative corrections showing that the superpotential is not renormalized. However, the wave function renormalization results in

$$\left.\frac{\partial^2 V_R^{eff}}{\partial f^* \partial f}\right|_{f=0} = 1 - \left(\frac{g^2}{16\pi^2}\right)\left[3 + 2\ln\frac{b^2}{\mu^2}\right] + \frac{1}{2}\left(\frac{g^2}{16\pi^2}\right)\left[3 - 2\gamma - J(1) - J'(1) + \frac{3-2\gamma}{2}\ln\frac{b^2}{\mu^2}\right] \qquad (4.22)$$

We may also calculate the β-functions. In the context of dimensional regularization the canonical dimension of the superfield @ is [@] = (n – 2)/2 while $[g] = (4-n)/2 = \epsilon/2$. Introducing the dimensionless renormalized coupling constant, $\tilde{g} = g\mu^{-\epsilon/2}$ we have $g_0 = \mu^{\epsilon/2}\tilde{g}(\mu)Z^{-3/2}$ where g_0 is the bare coupling constant. Hence[17] to two loops[18]

$$\beta = \mu\left.\frac{\partial\tilde{g}}{\partial\mu}\right|_{g_0}$$
$$_{\epsilon\to 0} = g\left[\frac{3}{2}\left(\frac{g^2}{4\pi^2}\right) - \frac{3}{2}\left(\frac{g^2}{4\pi^2}\right)^2\right] \qquad (4.23)$$

It is worth remarking on the ensuing renormalization constraint (eq.(4.22). Its left hand side arises in the kinetic energy term $\bar{\Phi}\Phi$ and a negative value for it, say, for sufficiently large b^2 may lead to the wrong sign for the kinetic energy terms of the physical fields. Since[7] in supersymmetric theories in general we have no independent coupling constant renormalization we cannot rectify the situation. In our case we may, alternatively, determine Z_1 and Z_2 by imposing the following constraint:

$$\left.\frac{\partial^2 V^{eff}}{\partial f^* \partial f}\right|_{f=0} = 1 \qquad (4.24)$$

The renormalization can be successfully performed and we obtain at the background field value we wish to consider

$$
\begin{aligned}
Z_1 &= -\frac{g^2}{16\pi^2}\left(\frac{4}{\epsilon} - 2\gamma - 2\ln\frac{b^2}{\mu^2}\right) \\
Z_2 &= -\left(\frac{4g^2}{16\pi^2}\right)^2\left[\frac{1}{\epsilon^2} - \left(\frac{1}{2} + \gamma + \ln\frac{b^2}{\mu^2}\right)\frac{1}{\epsilon} + 2\ln\frac{b^2}{\mu^2}\right. \\
&\left. - \frac{1}{2}\ln^2\frac{b^2}{\mu^2} - \frac{1}{2}J(1) - \frac{1}{4}J'(1) + \frac{1}{8}J''(1) + \frac{5}{4} - \frac{\pi^2}{24}\right]
\end{aligned}
\tag{4.25}
$$

which depend on μ as well as the background field constant b^2. For the effective potential we find now

$$
\begin{aligned}
V'_{1R} &= V_{1R} - \frac{1}{64\pi^2}\left(3 + 2\ln\frac{b^2}{\mu^2}\right)|f'|^2 \\
V'_{2R} &= V_{2R} + \frac{g^2}{(16\pi^2)^2}\left[\left(3 - \gamma + \frac{1}{8}J''(1) - \frac{1}{4}J'(1) - \frac{1}{2}J(1)\right.\right. \\
&+ \frac{5-\gamma}{2}\ln\frac{b^2}{\mu^2} + \frac{1}{2}\ln^2\frac{b^2}{\mu^2}\Big)|f'|^2 - \frac{1}{2}\left(\frac{3}{2} + \ln\frac{b^2}{\mu^2}\right)\Big(a_+^4\ln\frac{a_+^2}{\mu^2} \\
&\left.\left. + a_-^4\ln\frac{a_-^2}{\mu^2} - 2b^4\ln\frac{b^2}{\mu^2}\right) - \frac{1}{2}|f'|\left(a_+^2\ln\frac{a_+^2}{\mu^2} - a_-^2\ln\frac{a_-^2}{\mu^2}\right)\right]
\end{aligned}
\tag{4.26}
$$

The effective potential as a function of the physical (classical) field is obtained by eliminating the auxiliary field f using its corrected equation of motion $\partial V^{eff}/\partial f = 0$ since it does acquire radiative corrections. The dependence of the real and imaginary part of f as functions of the physical field a as well as the effective potential to zero loop are plotted in figs.3(a),(b),(c) and (d) where $X = |a'|/m$, $Y = |f'|/m^2$ and $G \equiv g$ and to one and two loops in the Amati-Chou scheme in figs.4(a), (b). (c) where $X = |a'|/m$, $Y = |f'|/m^2$ and $G \equiv g$. We observe that the radiative corrections do not alter the tree level minima (eq.(4.3)), the potential remains positive definite, and supersymmetry is not broken. Around the origin $X \approx 0$ (or $a \approx -m/2g$) there is a region where $Y > X^2$ (or $a_-^2 < 0$) and the effective potential becomes complex for these values. In the minimal subtraction scheme, figs.5(a),(b) we find, however, also a multivalent effective potential at two loops.

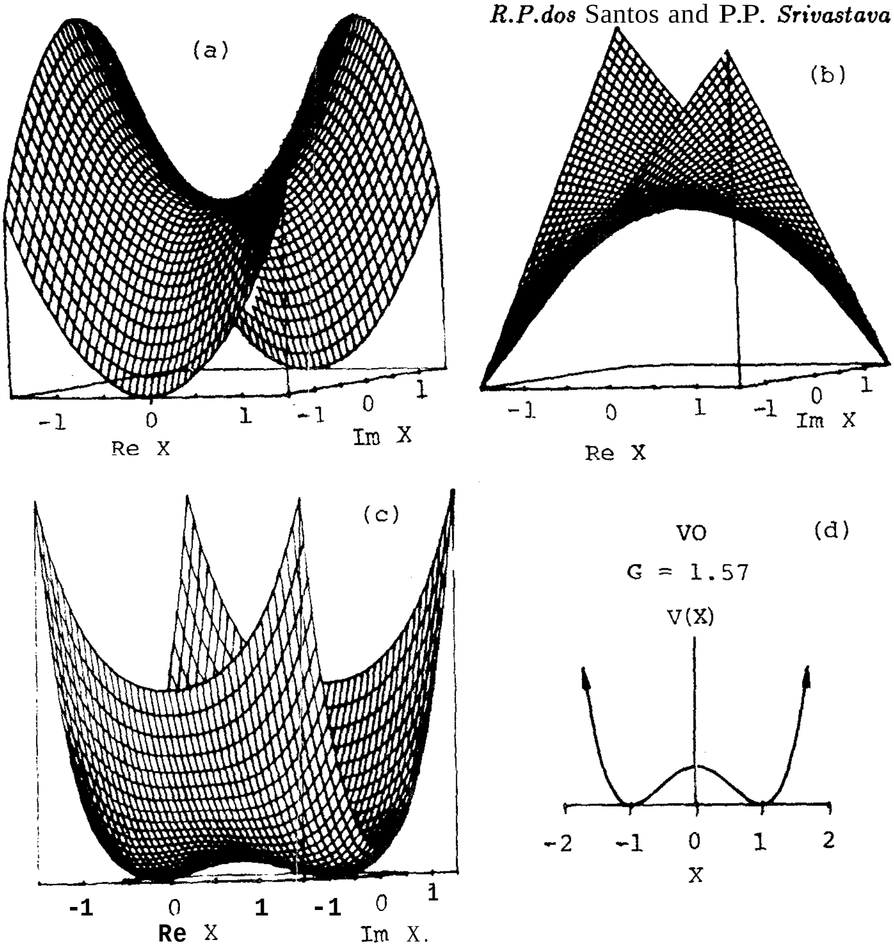


Fig.3 - (a) - Real part of f (Y); (b) - Imaginas. part of f (Y); (c) - Effective Potential to zero loop (V0); (d) - Effective Potential V0 for real $X = |a'|/m$.

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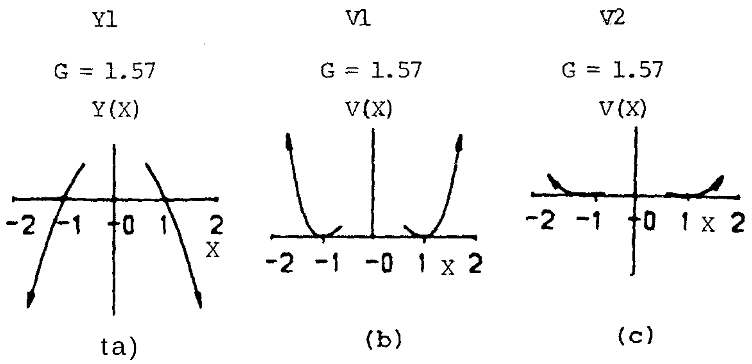

Fig.4 - (a) - Auxiliary field f (Y1); (b) - Effective Potential to one-loop (V1); (c) - Effective Potential to two-loop (V2).

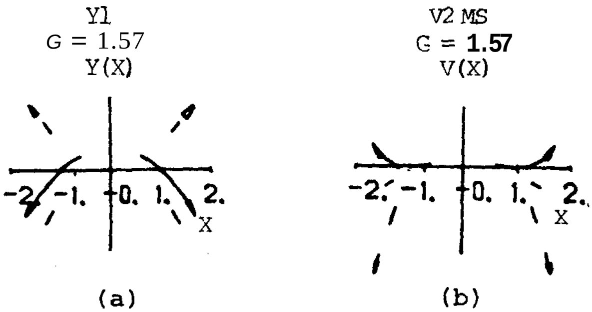

Fig.5 - (a) - Auxiliary field f (Y2); (b) - Effective Potential to two-loop (V2MS) in MS scheme.

Appendix A

The expressions for A, B, G and E in (2.11) are[6] $(\partial_t \to ik_t)$

$$A = (k^2 + \bar{M}M)^{-1} - [k^2 + \bar{M}M - \bar{f}(k^2 + M\bar{M})^{-1} f]^{-1},$$

$$k^2 B = \bar{M}(k^2 + M\bar{M})^{-1} f G,$$

$$k^2 G = [\bar{f}(k^2 + M\bar{M})^{-1} f - k^2 - \bar{M}M]^{-1} \bar{f} M (k^2 + \bar{M}M)^{-1},$$

$$k^2 E = (k^2 + \bar{M}M)^{-1}\bar{M} f [\bar{f}(k^2 + M\bar{M})^{-1} f - k^2 - \bar{M}M]^{-1}$$

Appendix B

We collect here some useful integrals including the definitions of the function $J(x)$:

$$\int \frac{d^{4-\epsilon}p}{(2\pi)^4}\frac{1}{(p^2+m^2)} = -i\frac{m^2}{16\pi^2}\Big[\frac{2}{\epsilon}+1-\gamma-\ln(m^2/\mu^2)+O(\epsilon)\Big] \qquad (B.1)$$

$$\int \frac{d^{4-\epsilon}p}{(2\pi)^4}\frac{1}{(p^2+m_1^2)(p^2+m_2^2)} = i\frac{1}{16\pi^2}\Big[\frac{2}{\epsilon}+1-\gamma - \frac{m_1^2\ln(m_1^2/\mu^2)-m_2^2\ln(m_2^2/\mu^2)}{m_1^2-m_2^2}+O(\epsilon)\Big] \qquad (B.2)$$

$$\int \frac{d^{4-\epsilon}p}{(2\pi)^4}\frac{d^{4-\epsilon}q}{(2\pi)^4}\frac{1}{(p^2+m_1^2)((p+q)^2+m_1^2)(q^2+m_2^2)} = -\frac{1}{(16\pi^2)^2}$$

$$\Big\{\frac{2}{\epsilon^2}(2m_1^2+m_2^2)+\frac{1}{\epsilon}\Big[(3-2\gamma)(2m_1^2+m_2^2)$$

$$-2\Big(2m_1^2\ln(m_1^2/\mu^2)+m_2^2\ln(m_2^2/\mu^2)\Big)\Big]+\Big[2m_1^2\ln^2(m_1^2/\mu^2)$$

$$+m_2^2\ln^2(m_2^2/\mu^2)\Big]-(3-2\gamma)\Big[2m_1^2\ln(m_1^2/\mu^2)$$

$$+m_2^2\ln(m_2^2/\mu^2)\Big]+\Big(\frac{7}{2}-3\gamma+\gamma^2+\frac{\pi^2}{12}\Big)(2m_1^2+m_2^2)$$

$$+m_2^2 J(m_2^2/m_1^2)\Big\} \qquad (B.3)$$

$$J(x) \overset{(x\leq 4)}{=} -4+2\ln(x)-\frac{1}{2}\ln^2(x)$$

$$-\frac{\sqrt{\pi}}{2}\ln(x)\sum_{j=1}\frac{\Gamma(j)}{\Gamma(j+3/2)}\Big(\frac{x}{4}\Big)^j$$

$$- \frac{\sqrt{\pi}}{2} \sum_{j=1} \frac{\Gamma(j)}{\Gamma(j+3/2)} \left(\frac{x}{4}\right)^j \Big[\Psi(j)$$

$$- \Psi(j+3/2) - 2\ln 2\Big]$$

$$\stackrel{(x \geq 4)}{=} \frac{\pi^2}{6} - \left(2 + \frac{\pi^2}{3}\right)\frac{1}{x} + \frac{2}{x}\ln(x) \sum_{j=1} \frac{\Gamma(j)}{\Gamma(j+2)} x^{-j}$$

$$+ \frac{2}{x} \sum_{j=1} \frac{\Gamma(j)}{\Gamma(j+2)} \Big[\Psi(j+2) - \Psi(j)\Big] x^{-j}$$

$$+ \frac{1}{x} \ln^2(x) \sum_{j=1} \sum_{k=1} \frac{\Gamma(j+k-1)\Gamma(j+k)}{\Gamma(j)\Gamma(k) j! k!} \Big[2\Psi(j+k-1)$$

$$+ 2\Psi(j+k) - \Psi(j+1) - \Psi(k+1) - \Psi(j) - \Psi(k)\Big] x^{-j-k}$$

$$- \sum_{j=1} \sum_{k=1} \frac{\Gamma(j+k-1)\Gamma(j+k)}{\Gamma(j)\Gamma(k) j! k!} \Big\{ \Big[2\Psi(j+k-1) + \Psi(j+k)\Big] \times$$

$$Big[\Psi(j+k-1) - \Psi(j+k) - \Psi(k) - \Psi(k+1) - \Psi(j) - \Psi(j+1)\Big]$$

$$+ \Big[\Psi(j) + \Psi(j+1)\Big]\Big[\Psi(k) + \Psi(k+1)\Big] - \Psi'(j+k-1)$$

$$+ \Psi'(j+k)\Big\} x^{-j-k} \qquad (B.4)$$

The last integral was derived in the following way. Combining the two denominators in q using the conventional Feynman parametrization, integrating in q and in the angular part of p, we obtain

$$I = -\frac{(\mu^2)^{4-2n}}{(16\pi^2)^n} \frac{\Gamma(3-2n)}{2-n} (m_1^2)^{n-2} (m_2^2)^{n-1}$$
$$= \int_0^1 dx\, F(2-n, n; 3-n; 1 - x(1-x) m_1^2/m_2^2)$$

For $m_2^2/m_1^2 < 4$ we can expand the hypergeometric function in a power series of the argument and integrate each term in x obtaining, after the limit $n \to 2-\epsilon/2$, the expression above for the integral and for $J(x)$ with $x < 4$.

For the other expression for $J(x)$ we start combining the denominators in p, obtaining

$$I = -\frac{(\mu^2)^{4-2n}}{(16\pi^2)^n}\frac{\Gamma(2-n)}{\Gamma(n)}\int_0^1 dx\, x^{n-2}(1-x)^{n-2}\left(\frac{x(1-x)}{m_1^2 x + m_2^2(1-x)}\right)^{2-n}\times$$
$$\frac{1}{(m_1^2)^{1-n}}\frac{\Gamma(n)\Gamma(3-2n)}{\Gamma(3-n)}F\left(2-n, n; 3-n; 1-\frac{x(1-x)m_1^2}{m_1^2 x + m_2^2(1-x)}\right)$$

Again expanding the Iiypergeometric in series and integrating in x for $m_2^2/m_1^2 > 4$ and performing the limit $n \to 2-\epsilon/2$ we obtain the same expression above for the integral with the other one for $J(x)$.

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$$dg_0 = \frac{\partial g_0}{\partial \tilde{g}}\Big|_\mu d\tilde{g} + \frac{\partial g_0}{\partial \mu}\Big|_{\tilde{g}} d\mu$$

The running coupling constant at 1-loop is given by

$$g^2 = g_0^2 \left[1 - 6\frac{g_0^2}{16\pi^2} \ln \frac{\mu^2}{m^2}\right]^{-1}$$

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Resumo

Para o caso de vários supercampos quirais em interação, derivam-se, numa forma compacta, os propagadores para os supercampos potenciais não-vinculados na teoria 'transladada', onde a, supersimetria é explicitamente quebrada . Eles são utilizados para se calcular o potencial efetivo a um loop no caso geral, enquanto que um cálculo do potencial efetivo renormalizado a dois loops para o modelo de Wess-Zumino é realizado.