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# A two - gauge - field induced $CP^n$ - model

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Abstract The duplication of gauge potentials in association with a single local symmetry group may trigger the generation of a $CP^n$ - model whenever $(n+1)$ complex scalars are coupled in a suitable way to the pair of gauge potentials.

The possibility of extending the gauge principle, through the introduction of a family of gauge potentials in association with a single simple and compact gauge group, has been considered and discussed under several points of view. Dynamical considerations lead to the conclusion that the different potentials actually describe independent physicai degrees of freedom and a number of quantum features of this sort of extended models are being investigated[1].

In the case one contemplates the inclusion of more potential fields transforming under a common simple gauge group according to

$$A_\mu \longrightarrow U\ A_\mu U^{-1} + i/gU(\partial U^{-1}) \tag{1a}$$

$$B_\mu \longrightarrow U\ B_\mu U^{-1} + i/gU(\partial U^{-1}) \tag{1b}$$

$$F_\mu \longrightarrow U\ F_\mu U^{-1} + i/gU(\partial U^{-1})\ , \tag{1c}$$

the Kaluza-Klein approach, relying basically on the mechanism of spontaneous compactification allows one to trace the origin of the potentials $A_\mu$, $B_\mu$ and $F_\mu$ back to the vielbein, spin-connection and Yang-Mills field of a higher-dimensional gravity-matter coupled theory spontaneously compactified on an internal space with torsion[2].

The appearance of more than one gauge potential within a common simple group would also be a natural fact in super-Yang-Mills theories if one did not have to face the fact of eliminating from the theory an undesirable spin-$3/2$ field (not the gravitino). In a superspace formulation, this problem becomes very clear. In gauge covariantising the space-time and supersymmetric covariant derivatives, different connection superfields are introduced, which then accommodate independent gauge potentials amongst their physical components. However, since higher-spin fields also appear, they have to be eliminated by suitable constraints on the field-strength superfields. In so doing, it happens, as a side effect, that the different vector potentials are related to one another and do not carry independent degrees of freedom. But, it appears that, by softly breaking supersymmetry, it is possible to relax the conventional superspace constraints and eliminate the higher spins without necessarily relating the different gauge potentials[3].

In this letter, it is our main effort to present another possible formal motivation (and maybe advantage) for considering the study of the extended models we have discussed so far; it is in connection with the well-known bosonic $CP^n$-non-linear $\sigma$-model[4,5]. For that, let us consider a $U(1)$-gauge group to which one associates two gauge potentials transforming according to

$$A_\mu(x) \longrightarrow A_\mu(x) + \partial_\mu \alpha(x) \tag{2a}$$

$$B_\mu(x) \longrightarrow B_\mu(x) + \partial_\mu \alpha(x) \tag{2b}$$

Assume also the existence of $(n+1)$ complex fields, $\varphi_i(x)$, all possessing the value q = 1 for the $U(1)$-charge:

$$\varphi_i(x) \longrightarrow \varphi_i(x) = e^{iq\alpha(x)}\varphi_i(x) \ , \tag{3}$$

and fulfilling the constraint

$$\sum_{i=0}^{n+1} \varphi_i^*(x)\varphi_i(x) = 1 \ , \tag{4}$$

which defines the well-known projective complex manifold $CP^n$. In other words: in considering a $CP^n$ non-linear a-model, the complex unit vector $(\varphi_1(x); \ldots ; \varphi_{n+1}(x))$ denotes one of the unit length representatives of the equivalente classes of vectors parametrising the projective space $CP^n$.

A possible gauge invariant Lagrangian for gauge potential fields $A$, and $B$, and the $(n+1)$ scalars $\varphi_i(x)$ is given by

$$\begin{aligned}\mathcal{L} = \left[D_\mu(A)\varphi_i^*\right]\left[D^\mu(A)\varphi_i\right] + \left[D_\mu(B)\varphi_i^*\right]\left[D^\mu(B)\varphi_i\right] \\ - 1/4A_{\mu\nu}A^{\mu\nu} + 1/2m^2(A_\mu - B_\mu)^2 \ ,\end{aligned} \tag{5}$$

where

$$D_\mu(A)\varphi_i = \partial_\mu\varphi_i + igqA_\mu\varphi_i \tag{6a}$$

$$D_\mu(B)\varphi_i = \partial_\mu\varphi_i + igqB_\mu\varphi_i \tag{6b}$$

and

$$A_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu \tag{6c}$$

Notice that the potential field $B_\mu(x)$ is not a dynamical degree of freedom: it works as an auxiliary field to give the dynamical potential $A_\mu(x)$ a gauge-invariant mass m. This can indeed be readily confirmed by a glance at the mamentum-space propagator for the latter:

$$< T(A_\mu(-k)A_\nu(k)) >= \frac{-i}{k^2+m^2}\left(\eta_{\mu\nu} - \frac{k_\mu k_\nu}{k^2}\right) + \frac{i\alpha}{k^2-\alpha m^2}\frac{k_\mu k_\nu}{k^2} \ , \tag{7}$$

where the **gauge-fixing** term $1/(2)(\partial_\mu A^\mu)^2$ has been adjoined to the Lagrangian (5). The fact that the dynamical **spin-1** component of ***A,*** carries a tachyonic mass does not cause problems in our case, since, as will be shown, the field ***A, (x)*** is not to be interpreted as elementary, but rather as a composite field.

Since the gauge potential $B_\mu(x)$ appears only algebraically in (5), it is reasonable to eliminate it in favour of its classical equation of motion and analyse the effective theory for ***A,*** **and** the scalars $\varphi_i$. The $B_\mu$**-field** equation is

$$m^2 A_\mu + (2g^2 q^2 \varphi_i^* \varphi_i - m^2) B_\mu - igq(\varphi^* \overleftrightarrow{\partial}_\mu \varphi_i) = 0 \tag{8}$$

**From** the **constraint** on the scalars, the bilinear term ***B,*** drops out from ***(B)*** **and** one gets

$$A, = i\frac{gq}{m^2}(o;\overleftrightarrow{\partial}_\mu \varphi_i \;, \tag{9}$$

that is, the classical dynamical fixes ***A,*** as a composite Abelian gauge field.

Since the inclusion of more than one potential field enlarges the possibilities of writing down gauge-invariant terms, let us contemplate the case where both potentials appear as background-gauge fields whereas the scalars propagate with a **dynamics** fixed through the Lagrangian

$$\begin{aligned} \mathcal{L} = & \left[D_\mu(A)\varphi_i^*\right]\left[D^\mu(A)\varphi^i\right] + \left[D_\mu(B)\varphi_i^*\right]\left[D^\mu(B)\varphi_i\right] \\ & + \left[D_\mu(A)\varphi_i^*\right]\left[D^\mu(B)\varphi_i\right] - 1/2m^2(A_\mu - B_\mu)^2 \end{aligned} \tag{10}$$

Now, both ***A,*** and $B_\mu$ can be **eliminated** through their respective classical **equa**tions of motion which are:

$$-igq\varphi_i^* \partial_\mu \varphi_i - igq\varphi_i^* \partial_\mu \varphi_i + g^2 q^2 \varphi_i^* (2A_\mu + B_\mu) - m^2(A_\mu - B_\mu) = 0 \tag{11a}$$

and

$$-igq\varphi_i^* \partial_\mu \varphi_i + igq\varphi_i^* (\partial_\mu \varphi_i^*)\varphi_i + g^2 q^2 \varphi_i^* \varphi_i (A, + 2B_\mu) + m^2 (A, - B\mu) = 0 \tag{11b}$$

Subtracting now these two equations of motion yields

$$-igq\partial_\mu(\varphi_i^*\varphi_i) + g^2q^2\varphi_i^*\varphi_i(A_\mu - B_\mu) - 2m^2(A_\mu - B_\mu) = 0 \qquad (12)$$

which gives

$$\partial_\mu(\varphi_i^*\varphi_i) = 0\ , \qquad (13a)$$

and

$$(g^2q^2\varphi_i^*\varphi_i - 2m^2)(A_\mu - B_\mu) = 0 \qquad (13b)$$

So, a solution which satisfies both equations is

$$\varphi_i^*\varphi_i = \frac{2m^2}{g^2q^2} \qquad (14)$$

and this is nothing but the constraint defining a $CP^n$-non-linear model, up to a simple normalization of the fields.

The remarkable fact now is that the constraint on the propagating scalars is determined through the dynamics of the classical potentials $A$, and $B_\mu$ and need not be introduced by hand. It would be interesting however to have a clear geometrical understanding of how the gauge background, which dominates the space-time through the configurations of the gauge potentials, dictates the nori-linear constraint on the propagating scalars. This shall be matter of further investigations. For the moment, we would only like to stress the marriage between gauge theories and non-linear a-models in the same number of space-time dimensions. Usually, the association exists between two dimensional non-linear a-models and four dimensional Yang-Mills theories. In our case, by duplicating the number of gauge potentials in a given group, we do not need to go to D = 4: the association already appears in D = 2.

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**Resumo**

A duplicação de potenciais de gauge em associação a um único grupo de simetria local pode induzir a geração de um modelo-$CP^n$ no caso em que $(n+1)$ escalares complexos estejam acoplados, de modo conveniente, ao par de potenciais vetoriais.