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# *A Clasical Approach to Higher-Derivative Gravity**

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Abstract Two classical routes towards higher-derivative gravity theory are described. The first one is a geometrical route, starting from first principles. The second route is a formal one. and is based on a recent theorem by Castagnino *et al.* [J.Math.Phys. 28 *1854* (1987)). A cosmological solution of the higher-derivative field equations is exhibited. which in a classical framework singles out this gravitation theory.

## 1. INTRODUCTION

In spite of the fact that all available evidence from experiments in macrophysics attests to the validity of Einstein's general theory of relativity as a description of the gravitational interaction. it is highly desirable. for the sake of unity and consistency of physics, that we can quantize gravity. Certainly, some unification between (essentially) microphysics (quantum mechanics) and macrophysics (general relativity) must be part of nature's design.

At present the $R + R^2$ theory of gravity has been suggested as a possible solution to the infinities plaguing the quantization of general relativity[1-5]. Its action for gravitation is given by

$$I = \int d^4x\sqrt{-g}\left[\frac{R}{2k} - \frac{\Lambda}{k} + \alpha R^2 + \beta R_{\mu\nu}R^{\mu\nu}\right] \tag{1.1}$$

where $\alpha$ and $\beta$ are dimensionless coupling constants (in natural units). and $k$ and $\Lambda$ are the Einstein and cosmological constants. respectively. For the quantum field theorist this fourth-order theory has the great advantage of being renormalizable by power counting[2], whereas. as it is well known, classical general relativity is clearly perturbatively nonrenormalizable by power counting in four dimensions[6,7]. Recent

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work[1] has shown that the presence of a ghost responsible for a pseudononunitarity of the theory. which was considered its Achilles's heel. is no more a vulnerable point of it. The reason is that the ghost is unstable. Consequently, the quantum interest concerning these quadratic Lagrangian theories is well suited.

Here we want to focus our attention on the classical features of such higher-derivative theory. The main purpose of this investigation is to show. from a classical viewpoint. that the aforementioned theory may be considered as a possible generalization of general relativity with the extra advantage that it can predict some results not expected to be found in standard general relativity. In particular, we discuss the possibility of regarding this theory as a kind of therapy to certain chronic pathologies of general relativity.

We begin by describing two classical routes towards this higher-derivative gravity theory in Sec.2. The first one is a ***geometrical route*** that. in a sense, starts frorn first principles. In other words, we build up the theory taking as a prototype Einstein's gravity theory. The second route is a formal one, and is based on a very recent theorern by Castagnino and al.[8]. In Sec. 3 we exhibit a completely causal vacuum solution of the Godel type concerning the higher-derivative field equations. This very peculiar and rare result is the first known exact vacuum solution of the fourth-order gravity theory that is not a solution of the corresponding Einstein's equations.

## 2. TWO CLASSICAL ROUTES TOWARDS HIGHER-DERIVATIVE GRAVITY

Suppose we want to construct a geometrical theory of gravitation, via a principle of least action, that is, frorn a staternent that some functional of the dynamical variables, the action, is stationary with respect to small variations of these variables. A possible way to achieve this, and here we appeal to Einstein's theory as a paradigm, is to start from a ***purely gravitational action*** of the form

$$I = \int d^4x \sqrt{-g} G \tag{2.1}$$

Here G is a scalar that depends on geometry alone or. in other words, is a functions of $g_{\mu\nu}$ and its derivatives, but it otherwise arbitrary. For the sake of generality of the theory, we require that the above action be invariant under arbitrary (continuous) coordinate transformations (general covariance), whose infinitesimal form is written as follows

$$\bar{x}^{\mu} = x^{\mu} + \xi^{\mu} \tag{2.2}$$

Under this transformation

$$\delta I = -2 \int d^4x \sqrt{-g}\, \xi_{\mu} G^{\mu\nu}{}_{;\nu} \tag{2.3}$$

where

$$G_{\mu\nu} := \frac{1}{\sqrt{-g}} \left\{ \frac{\partial(\sqrt{-g}G)}{\partial g^{\mu\nu}} - \partial_{\alpha} \frac{\partial(\sqrt{-g}G)}{\partial \partial_{\alpha} g^{\mu\nu}} + \partial_{\alpha}\partial_{\beta} \frac{\partial(\sqrt{-g}G)}{\partial \partial_{\alpha}\partial_{\beta} g^{\mu\nu}} - \ldots \right\} \tag{2.4}$$

Equating 6 1 to zero and taking into account that the $\xi^{\mu}$ are arbitrary. we conclude that

$$G^{\mu\nu}{}_{;\nu} = 0 \tag{2.5}$$

Thus, mathematically. the *contracted Bianchi identities* are a consequence of the fact that the action integral eq.(2.1) is invariant under general (continuous) coordinate transformations. It is worth noticing. in passing. that this result was obtained regardless of any particularization of the form of G. On the other hand. if we appeal to Noether's powerful second theorem[9,10]. we get the same conclusion in a trivial way. In fact. this theorem asserts that the invariance of the action under local gauge transformations implies the so called *generalized Bianchi identities.* a name just borowed from general relativity. for which they are identical to the contracted Bianchi identities. The existence of the identities eq.(2.5) reveals the singular nature of the Lagrangian density of the theory, i.e., it indicates the presence of constraints in the theory.

At this point it is reasonable to turn our attention to the problem concerning the determination of the scalar G. Certainly, the simplest choice for this scalar should be G = R. which leads to Einstein's gravitational theory. It is obvious that this is only one of the possible options for this scalar. while a multitude of invariants regarding curved space-time remains at our disposal. We restrict our analysis to those in variants that are quadratic in the curvature tensor it its ordinary contractions, namely

$$R^2, \quad R_{\mu\nu} R^{\mu\nu}, \quad R^{\alpha\beta\gamma\delta} R_{\alpha\beta\gamma\delta} \tag{2.6}$$

In this case the corresponding *field equatione* are given by

$$R^2 \to_{(1)} G_{\mu\nu} = -\frac{1}{2} R^2 g_{\mu\nu} + 2RR_{\mu\nu} + 2R_{;\mu\nu} - 2g_{\mu\nu} \Box R = 0 \tag{2.7}$$

$$\begin{aligned} R^{\mu\nu} R_{\mu\nu} \to_{(2)} G_{\mu\nu} = & R_{;\mu\nu} + 2R_{\mu\theta\alpha\nu} R^{\theta\alpha} - \Box R_{\mu\nu} \\ & - \frac{1}{2} g_{\mu\nu} \Box R - \frac{1}{2} R_{\rho\theta} R^{\rho\theta} g_{\mu\nu} = 0 \end{aligned} \tag{2.8}$$

$$\begin{aligned} R^{\alpha\beta\gamma\delta} R_{\alpha\beta\gamma\delta} \to_{(3)} G_{\mu\nu} = & -\frac{1}{2} R_{\alpha\beta\gamma\delta} R^{\alpha\beta\gamma\delta} g_{\mu\nu} + 2R_{\mu\alpha\beta\gamma} R_{\nu}{}^{\alpha\beta\gamma} \\ & + 2R_{\mu\alpha\nu\beta}{}^{;\beta\alpha} + 2R_{\mu\alpha\nu\beta}{}^{;\alpha\beta} = 0 \end{aligned} \tag{2.9}$$

In reality these theories are not independent due to the Bach-Lanczos[11,12] identity

$$\delta \int \sqrt{-g} d^4x (R_{\alpha\beta\gamma\delta} R^{\alpha\beta\gamma\delta} - 4R_{\alpha\beta} R^{\alpha\beta} + R^2) \equiv 0 \tag{2.10}$$

which is usually known as Gauss-Bonnet theorem. Consequently only two of the theories under consideration are independent. As usual. we adopt simplicity as our criterion of judgement, which leads us to choose the theories generated by $R^2$ and $R_{\alpha\beta} R^{\alpha\beta}$, respectively. to analyse.

We remark that any vacuum solution of Einstein's equations is also a solution of the *vacuum equations* concerning our gravity theories. This result is trivially verified by inspection. The reciprocal is not true in general, because Einstein's equations are second order whereas the equations regarding the alternative theories we are considering are fourth-order.

In order to introduce the sources into the theory we appeal again to Einstein's gravity theory, which leads us to take the sources proportional to the energy-momentum tensor $T^{\mu\nu}$. It follows then that our *gravity theories* may be written formally as

$$G_{\mu\nu} = -kT_{\mu\nu} \tag{2.11}$$

wherein k is a constant with a suitable dimension. whose numerical value will be taken equal to the usual Einstein's constant, in order to have Einstein's general theory of relativity as a member of the above set of *gravitational theories.* We restrict our study to theories of second and fourth order respectively. In the first case, $[k]= L^2$. whereas in the second one k is dimensionless.

Otherwise, the *generalized Bianchi identities* imply that the covariant divergence of $T^{\mu\nu}$ is null. In other words. the gauge invariance conducts us locally to the *conservation law*

$$T^{\mu\nu}{}_{;\nu} = 0 \tag{2.12}$$

Undoubtedly. any candidate for a gravitational theory must be compatible with Newton's law of gravitation in the nonrelativistic limit. So. in order to test our theories. we look at its behaviour in the light of the weak field approximation. Following the conventional method[13], we write the metric tensor as

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu} \tag{2.13}$$

where

$$\eta_{\mu\nu} = (1,-1,-1,-1)$$

$$|h_{\mu\nu}| << 1$$

and we retain in our field equations only the terms which are linear in $h_{\mu\nu}$ or the derivatives of $h_{\mu\nu}$. In this approximation our field equations assume the form

$$_{(i)}G^{(L)}_{\mu\nu} = -kT_{\mu\nu}(i = 1,2) \tag{2.14}$$

$$
{}_{(1)}G^{(L)}_{\mu\nu} = \Box(\partial_\mu\partial_\nu - \eta_{\mu\nu}\Box)h - 2\eta^{\lambda\rho}\eta^{\alpha\beta}(\partial_\mu\partial_\nu - \eta_{\mu\nu}\Box)\bar{h}_{\lambda\alpha,\beta\rho} \tag{2.15}
$$

$$
{}_{(2)}G^{(L)}_{\mu\nu} = \frac{1}{2}\Box\left[\left(\partial_\mu\partial_\nu - \frac{1}{2}\eta_{\mu\nu}\Box\right)h - \Box h_{\mu\nu}\right] - \eta^{\lambda\rho}\eta^{\alpha\beta}\bar{h}_{\lambda\alpha,\beta\rho\mu\nu} + \frac{1}{2}\eta^{\lambda\rho}\Box\left(\bar{h}_{\lambda\mu,\nu\rho} + \bar{h}_{\lambda\nu,\mu\rho} + \eta_{\mu\nu}\eta^{\omega\sigma}\bar{h}_{\lambda\omega,\sigma\rho}\right) \tag{2.16}
$$

where

$$
\bar{h}_{\mu\nu} = h_{\mu\nu} - \frac{1}{2}\eta_{\mu\nu}h, \; h = \eta^{\mu\nu}h_{\mu\nu} \tag{2.17}
$$

$$
\Box = \partial^\mu\partial_\mu \tag{2.18}
$$

The **comma** denotes **partial** derivative. and the indices are raised and lowered with the Minkowski metric $\eta_{\mu\nu}$. The symbol (L) stands for linear approximation.

On the other hand. it is not difficult to show that

$$
{}_{(i)}G^{(L)}{}_{\mu\nu}{}^{,\nu} = 0 \tag{2.19}
$$

Thus, the linearized equations **imply** that

$$
T_{\mu\nu}{}^{,\nu} = 0 \tag{2.20}
$$

which is the conservation law of energy and momentum is special relativity.

Now the linearized gravitational **field** equations are invariant under the gauge transformation

$$
h_{\mu\nu} \rightarrow h'_{\mu\nu} = h_{\mu\nu} - \partial_\mu\Lambda_\nu - \partial_\nu\Lambda_\mu \tag{2.21}
$$

where $\Lambda_\nu$ are four **small** but otherwise arbitrary functions. Also, **eq.(2.21) allows** us to put the harmonic condition

$$
\bar{h}_{\mu\nu}{}^{,\nu} = 0 \tag{2.22}
$$

which we assume henceforth. The linearized eqs. (2.15) and (2.16) are then given, respectively. by

$$\Box(\partial_\mu\partial_\nu \quad \eta_{\nu\mu}\Box)h = -kT_{\mu\nu} \tag{2.23}$$

$$\frac{1}{2}\Box\left[\left(\partial_\mu\partial_\nu - \frac{1}{2}\eta_{\mu\nu}\Box\right)h - \Box h_{\mu\nu}\right] - kT_{\mu\nu} \tag{2.24}$$

Contracting these equations. we get respectively

$$3\Box\Box h = kT \tag{3.25}$$

$$\Box\Box h = kT \tag{2.26}$$

Let us consider the gravitational field of a point particle located at the origin, for which $T^{\mu\nu}$ is gíven by

$$T^{\mu\nu} = \delta^\mu_0\delta^\nu_o M\delta^3(\vec{x}) \tag{2.27}$$

From eqs. (2.25) and (2.26) we get in the nonrelativistic limit that

$$h = -\frac{GMr}{3} \tag{2.28}$$

where $r = |\vec{x}|, k = 8\pi G$, and G is the gravitational constant. In deriving this result we have used the fact the Green function for $V^4$ is $-r/8\pi$. From eq.(2.28) we obtain $\nabla^2 h_{,12} \neq 0$, while from eq.(2.23) we get $\nabla^2 h_{,12} = 0$. Therefore the systern eq.(2.23) has no solution at all for a point mass at rest at the origin.

If we proceed in a similar way with respect to eq.(2.24) we arrive at the following result

$$h_{00} = -\frac{3}{2}rGM \tag{2.29}$$

Obviously this result is physically unacceptable, since it provides us a gravitational potential proportional. rather than inversely proportional. to the distance. Thus we conclude that gravity theories generated by the scalars $R^2$ and $R^{\mu\nu}R_{\mu\nu}$ are not compatible with Newton's law of gravitation in the nonrelativistic limit.

Now. as it is well known, the gravitational theory of Einstein conducts. in the nonrelativistic limit, to Newton's law. Indeed. in this case, the linearized equations in the gauge (2.22) assume the form

$$\Box \bar{h}_{\mu\nu} = -2kT_{\mu\nu} \tag{2.30}$$

and reduce, in the nonrelativistic limit and for a point particle, to

$$\nabla^2 \bar{h}_{00} = 2kM\delta^3(\vec{x})$$

which implies that the potential is given by

$$\Phi \equiv \frac{\bar{h}_{00}}{2} = -\frac{2MG}{r}$$

It follows then that. in order to maintain the connection with Newton's law in the nonrelativistic limit. we ought to modify Einstein's theory in such a manner that the higher-derivative terms introduced into the theory are negligible at macroscopic distances. Formally, a way to achieve this is through the replacement of the special relativistic operator $\Box$ by

$$\Box(1 + d^2\Box) \tag{2.31}$$

where d is a constant with the dimension of length. In the nonrelativistic limit this operator reduces to

$$\nabla^2(-1 + d^2\nabla^2) \tag{2.32}$$

and its Green function is given by

$$\frac{1 - e^{-r/d}}{4\pi r} \tag{2.33}$$

which shows us that at distances $r >> d$ Newton's law is not changed.

Taking into account the previous considerations, we take the Lagrangian density corresponding to our gravitational theory as

$$L = \sqrt{-g}\left[\frac{\gamma R}{k} + \alpha R^2 + \beta R_{\mu\nu}R^{\mu\nu}\right] + \sqrt{-g}L_m \tag{2.23}$$

where $\alpha, \beta, \gamma$ are çonstants and $L_m$ is the Lagrangian matter density. In this case the field equations are

$$G_{\mu\nu} = -\frac{1}{2} T_{\mu\nu} \tag{2.35}$$

$$\begin{aligned} G_{\mu\nu} = & \frac{\gamma}{k}\Big(R_{\mu\nu} - \frac{1}{2} R G_{\mu\nu}\Big) \\ & + \alpha(-\frac{1}{2} R^2 g_{\mu\nu} + 2RR_{\mu\nu} + 2R_{;\mu\nu} - 2g_{\mu\nu}\Box R) \\ & + \beta\Big(R_{;\mu\nu} + 2R_{\mu\theta\rho\nu} R^{\theta\rho} - \Box R_{\mu\nu} - \frac{1}{2} g_{\mu\nu}\Box R \\ & - \frac{1}{2} R_{\rho\theta} R^{\rho\theta} g_{\mu\nu}\Big) \end{aligned}$$

$$\delta \int \sqrt{-g} L_m d^4x \equiv \frac{1}{2} \int \sqrt{-g} T_{\mu\nu} \delta g^{\mu\nu} \tag{2.36}$$

In the weak field approximation and in the gauge eq.(2.22) these equations assume the form

$$\frac{\gamma}{2k}\Box\bar{h}_{\mu\nu} + \alpha\Box\Big(\partial_\mu\partial_\nu - \eta_{\mu\nu}\Box\Big)h + \frac{\beta}{2}\Box\Big[\Big(\partial_\mu\partial_\nu - \frac{1}{2}\eta_{\mu\nu}\Box\Big)h - \Box h_{\mu\nu}\Big] = -\frac{1}{2} T_{\mu\nu} \tag{2.37}$$

In the nonrelativistic limit we get

$$\Phi \equiv h_{00} = \frac{GM}{\gamma}\Big[-\frac{1}{r} + \frac{4}{3}\frac{e^{-rm_2}}{r} - \frac{1}{3}\frac{e^{-rm_0}}{r}\Big] \tag{2.38}$$

$$h = \frac{2GM}{\gamma}\Big(\frac{1}{r} - \frac{e^{-rm_0}}{r}\Big) \tag{2.39}$$

$$m_0^2 \equiv \frac{\gamma}{2k(3\alpha + \beta)} \tag{2.40}$$

$$m_2^2 \equiv \frac{-\gamma}{k\beta} \tag{2.41}$$

Comparison at infinity with the Newtonian result $\Phi = -2GM/r$ shows that the correct physical value of $\gamma$ is 1/2. Thus eq. (2.38) may be rnade to approach the Newtonian limit $1/r$ as closely as we wish. by ensuring that $m_2$ and $m_0$ are large enough.

Of course we are assurning that the pararneters $m_0, m_2$ are positive. which in its turn implies that $\alpha, \beta$ are not arbitarry. but must satisfy the relations

$$3\alpha + \beta > 0 \tag{2.42}$$

$$\beta < 0 \tag{2.43}$$

What signification may we attribute to these constrainst? The answer is straightforward if we note that the higher-derivative theory contains two mass scales. associated with the spin-0 and spin-2 particles present in the lienarized theory. They are given respectively. by[6]

$$m_0^2 = \frac{1}{4k(3\alpha + \beta)} \tag{2.44}$$

$$m_2^2 = -\frac{1}{2k\beta} \tag{2.45}$$

So, nontachyonic spin-0 and spin-2 particle require $(3\alpha + \beta)$ to be positive and $\beta$ to be negative. respectively. It is worth noticing that the spin-2 particle has significance even in the nonlinear sector of the theory[14].

For sirnplicity. we have not considered the ***cosmological constant.*** which would only contribute with a negligible modification of Newton's law for noncosmological distances. without affecting our main conclusions.

In summary we may say that from an entirely classical point of view. higher-derivative gravity may be thought of as a generalization of Einstein's general relativity. since it respects the geometrical nature of gravity as well as its gauge symmetry. i.e. its invariance under general coordinate transformations. The theory is also in asymptotic agreement with Newton's law in the nonrelativistic limit in the case whsn the parameters $\alpha, \beta$ obey suitable relations. These constraints on the parameters admit an interesting interpretation froni the quantum field theory viewpoint. In a sense, they establish a connection between macrophysics and mycrophysics.

Our aim now is to find. in a rigorous way. the quadratic Lagrangian density corresponding to the action (1.1). To accomplish this we get benefit from a very recent theorem by Castagnino et al.[8]. According to it. if L is a Lagrangian density of the form

$$L = L(g_{\mu\nu}; g_{\mu\nu,\rho}; g_{\mu\nu,\rho\sigma}; A_\mu; A_{\mu;\nu}; \Phi; \Phi_{,\nu})$$

which satisfies suitable hypotheses. then gauge invariance of the associated Euler-Lagrange equations implies gauge invariance of the Lagrangian and it becomes

$$\begin{aligned} L = {} & b_1\sqrt{-g}\Phi^2 + b_2\sqrt{-g}\Phi^4 + b_3\sqrt{-g}g^{\mu\nu}\Phi_{,\mu}\Phi_{,\nu} \\ & b_4\sqrt{-g}F_{\mu\nu}F^{\mu\nu} + b_5\sqrt{-g}R^2 + b_6\sqrt{-g}R_{\alpha\beta\gamma\delta}R^{\alpha\beta\gamma\delta} \\ & + b_7\sqrt{-g}R_{\mu\nu}R^{\mu\nu} \end{aligned} \tag{2.46}$$

wherein $A_\mu$ is the electromagnetic potential. $\Phi$ is a scalar field, and $F_{\mu\nu} = A_{\mu,\nu} - A_{\nu,\mu}$.

The pure gravitational terms of the Lagrangian of eq.(2.46). with the identification of $b_1\Phi^2$ with $(2k)^{-1}$ and $b_2\Phi^4$ with $-\Lambda/k$. are those of the gravitational action (1.1). Taking now. $b_5 = \alpha, b_7 = \beta, b_3 = b_4 = 0$. and making use of the Bach-Lanczos identity, which is given by eq.(2.10). we arrive immediately at the action (1.1).

## 3. NEW CLASSICAL FEATURES OF HIGHER-DERIVATIVE GRAVITY

The previous analysis. even though it cannot be considered as exhaustive. indicates that from a classical viewpoint higher-derivative gravity may be considered as a possible generalization of general relativity. It seems natural then to investigate the novel consequences that can be extracted from this higher-derivative theory.

As is well known, one of the most intringuing problems in general relativity emerges when we analyses the so-called Gödel-type universes, that is, models that are defined by the line element[15]

$$ds^2 = [dt + H(r)d\Phi]^2 - D^2(r)d\Phi^2 - dr^2 - dz^2 \qquad (3.1)$$

which in general admit closed timelike curves. The Gödel model [27] is undoubtedly the best known example of a cosmological solution of Einstein's field equations in which causality may be violated. It was Gödel himself who first drew attention to the fact that in his space-time one could possibly *travel to the past, or otherwise influence the past*, breaking therefore the relation of cause and effect. Perhaps this appealing idea justifies in part the recent surge of interest concerning the research on Gödel-type universes[15-26].

Recently, Rebouças and Tiomno[15] have demonstrated that the necessary and sufficient conditions for a Gödel-type metric to be space-time homogeneous are

$$\frac{H'}{D} = constant := 2\Omega, \quad \frac{D''}{D} = const := m^2 \qquad (3.2)$$

They have also shown that only in case

$$m^2 \geq 4\Omega^2$$

there is no breakdown of causality of Gödel-type. They have restricted their study to the section $t = z = const$ (cylindrical coordinates) of the Gödel-type space-time manifolds. In other words, they have only examined the breakdown of causality of the type that occurs in Gödel universe. Otherwise, it is not difficult to show, from their work, that vaccum solutions of the Gödel-type related to space-time homogeneous models, are not allowed in the context of general relativity. The following interesting question can now be posed: are there vacuum solutions concerning the homogenous Gödel-type models in the higher-derivative gravity framework?

In order to answer this questions we write the field eq.(2.35) related to the homogeneous Gödel-type models eq.(3.2). In the present case we have no sources and we assume the presence of a cosmological constant $\Lambda$. The resulting equations are the following

$$-200\Omega^4(\alpha+3\beta)-2m^4(2\alpha+\beta)+24m^2\Omega^2(\alpha+\beta)+\frac{1}{k}(-3\Omega^2+m^2)+\frac{\Lambda}{k}=0 \quad (3.3)$$

$$12\Omega^4(\alpha+3\beta)-2m^4(2\alpha-\beta)+16m^2\Omega^2(\alpha+8)+\frac{1}{k}(-\Omega^2)-\frac{\Lambda}{k}=0 \quad (3.4)$$

$$4\Omega^4(\alpha+3\beta)+2m^4(2\alpha+\beta)-8m^2\Omega^2(\alpha+\beta)+\frac{1}{k}(\Omega^2-m^2)-\frac{\Lambda}{k}=0 \quad (3.5)$$

We draw the reader's attention to the **fact** that these equations can be **easily worked** out from **eqs.(3.9)-(3.11)** with $T_{\mu\nu}=0$ and $\Lambda \neq 0$. of Ref. 18.

The solution of the above equations is given by

$$\Omega^2=\frac{m^2}{4}=\frac{1}{8(3\alpha+\beta)k}=-\frac{2}{3}\Lambda \quad (3.6)$$

**It follows** then from eqs. (3.1). (3.2) and (3.6) that

$$ds^2=dt^2+\frac{2}{\Omega}\,sinh^2(\Omega r)d\Phi dt-dr^2-dz^2-\frac{1}{\Omega^2}\,sinh^2(\Omega r)d\Phi^2 \quad (3.6)$$

We have thus succeeded in finding a vacuum solution (counting the **cosmological** constant as vacuum) of the **Gödel type** in the framework of **fourth-order** gravity. **It** is **worth** mentioning that this solution has already been found by us **in** a previous **paper**[18]. **It** comes in here as an illustration which makes clear that the higher-derivative theory admits a vacuum solution that is not a vacuum solutian **of** the corresponding Einstein's equations, although a solution. On the other hand this solution is also interesting because it links Newton's constant $\alpha,\beta$ and the **value** of the cosmological constant, establishing a bridge between miêrophysics and macrophysics.

We are ready now to focus our attention on the probiem of the **existence** of causal anomalies (in the form of cloased timelike curves) in our solution. The **presence** of closed timelike curves of the **Gödel-type**, that is. the circles defined by t, $r = constant$, depends on the behaviour of the function

$$f(r) = D^2(r) - H^2(r) \qquad (3.8)$$

In fact. if f(r) becomes negative for a certain range of values of $r(r_1 < r < r_2$, say). Gödel circles are closed timelike curves. . In the specific situation we are analysing. $f(r)$ is given by

$$f(r) = \frac{1}{\Omega^2} sinh^2(\Omega r) \qquad (3.9)$$

So. we can guarantee that there is no violation of causality of the Gödel-type (circles) in our model. Of course. we can not assure that all possible curves are causal by looking over the causality of $r, t, z$ = constant curves. On the other hand. following an ingenious prcedure proposed by Calvão et al[25]. which in a sense, has already been used by Penrose[28]. Maitra[29]. and Ozsváth and Schücking[30] among others, it is easy to show that our solution is completely causal. The aforementioned procedure is valid as far as the space-time manifolds are homeomorphic to $R^4$. which can then be covered by a single coordinate patch. Certainly this is not too strong a constraint, since many important space-times have the same underlying manifold, $R^4$ [31].

Could it be that the solution we have found is just flat space or some other simple space? One can demonstrate that this space-time has a seven parameter maximal group of motions $(G_7)$ while the remaining homogeneous Godel-tye metrics have a $G_5$ [19]. Otherwise. it is a well established fact that solutions with a $G_7$ of motions are very rare[32].

## 4. FINAL COMMENTS

We have shown, through an specific example. the potentialities of higher-derivative gravity in treating an involved problem such as the causal anomalies (in the form of closed timelike curves) of the space-time homogeneous Godel-type universes. In particular, this theory has bequeathed to us a very peculiar and rare result: a completely causal vacuum solution of the Godel type. The problem concerning the existence of other causal solutions is still ah open question. unless we introduce artificial constraints between the parameters a and $\beta$. Investigations concerning this subject are already in progress and we intend to publish them elsewhere.

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## Resumo

Duas rotas clássicas são apresentadas para a teoria de gravição com derivadas de ordem mais alta. A primeira é uma rota geomdtrica. que parte de princípios primeiros. A segunda rota é formal, e se baseia num recente teorema de Castagnino *et al.* [J. Math. Phys. *28*, 1854 (1987)]. Uma solução cosmológica das equações de campo da teoria com derivadas de ordem mais alta é exibida, a qual num contexto clássico evidencia esta teoria de gravitção.