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# Pion-Cloud Corrections to the Relativistic S + V Harmonic Potential Model

B.E. PALLADINO* and P. LEAL FERREIRA

*Instituto de Física Teórica, Universidade Estadual Paulista, Rua Pamplona 145, São Paulo, 01405, SP, Brasil*

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Abstract Pionic corrections to the mass spectrum of low-lying $S$-wave baryons are incorporated in a relativistic independent quark model with equally mixed borentz scalar and vector harmonic potentials. The introduction of a pion-cloud interacting linearly with a quark core stems from the requirement of chiral symmetry in the $(u, d) SU(2)$ sector of the model. A different method for calculatíng the pion-cloud corrections is presented which provides an alternative fitting to that of Barik and Dash and leads to a smaller average accuracy of the mass spectrum. Our quantitative analysis is extented to some other static baryonic properties, such as strong coupling constants, in good agreement with data.

## 1. INTRODUCTION

In the last few years. relativistic quark potential models[1] have been improved aiming at a more accurate description of hadronic properties. In this direction one-gluon exchange corrections and also center-of-mass corrections have been made and applied to the mass spectrum of low-lying S-wave baryons and their static properties such as magnetic moments, charge radii and coupling constants[2].

An important feature of quantum chromodynamics. namely chiral symmetry of the massless $(u, d) SU(2)$ sector has also been incorporated[3]. In those models, the quark axial current is not conserved due to the presence of the Lorentz scalar component of the confining potential. In order to restore chiral symmetry. an elementary pion field is introduced. interacting with the quarks of the bare *core* in a linear way. As a consequence, pion exchange and self-energy effects give rise to additional contributions to the physical mass spectrum of baryons. In the same spirit of the cloudy-bag mode1[4] these pionic effects are assumed to be small (or moderate) and treated perturbatively in lowest order in the quark-pion coupling constant.

In this work we have adopted. for simplicity, the relativistic *S + V potential model,* in which the independent quarks obey Dirac equation with an equally mixed

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scalar and vector confining potential of harmonic type[5]. The method used here to calculate pion energy corrections is formally similar to that applied to calculate one-gluon-exchange (OGE) corrections[6] and takes into account the partial conservation of the axial current (PCAC) for the case of massive pions. Our method differs from that of Barik and Dash[7] who made use of the same perturbative treatment employed in the cloudy-bag model for the calculation of pion corrections[8]. In ref.(7) the parameters of the $S+V$ harmonic model were fixed in order to adjust the quark-core previously. However, our approach provided an alternative fitting of the model. obtained in a more coherent way, since the presence of the pion-cloud influences directly the behaviour of the quark-core. Our improved fitting of the baryon mass spectrum leads to an average accuracy of circa 6 MeV. Finally, by using the parameters of our fitting. we extended our quantitative analysis by calculating other physical quantities such as the axial and strong coupling constants which were found to be in good agreement with data.

This work is organized as follows. In section 2, we briefly recall the basic formulation of the model. In section 3, one-pion corrections are calculated whereas section 4 is devoted to the presentation of the results. Our main conclusions are presented in section 5.

## 2. BASIC FRAMEWORK

In the S+V potential model each quark in the hadron obeys a Dirac equation

$$\left[\vec{\alpha}\cdot\vec{p}+\beta m_i+\frac{1}{2}(1+\beta)V(r)\right]\Psi_i(\vec{r})=E_i\Psi_i(\vec{r}) \tag{1}$$

where i is the flavour index of the quark.

We will adopt here the radial potential $V(r)$ in the harmonic form

$$V(r)=V_0+\frac{1}{2}Kr^2 \tag{2}$$

which allows a simple analytical treatment of all our calculations.

We note that the linear potential gives nearly the same results as the harmonic potential, both for the mass spectrum and the static properties. As we shall see. the same result is found when pionic corrections are applied.

The S-wave solutions of eq.(1) are of the form

$$\Psi_i(r) = N_i \begin{pmatrix} \varphi_i(r)\chi \\ \frac{\vec{\sigma}\cdot\vec{p}}{E_i+m_i}\varphi_i(r)\chi \end{pmatrix} \tag{3}$$

where $\chi$ is a **Pauli** spinor and $\varphi(r)$ is the norrnalized eigenfunction of the radial equation

$$p^2\varphi_i(r) = (E_i + m_i)[E_i - m_i - V(r)]\varphi_i(r) \tag{4}$$

having the form

$$\varphi_i(r) = \left(\frac{1}{\pi R_i^2}\right)^{\frac{3}{4}} e^{r^2/2R_i^2} \tag{5}$$

with the definitions

$$R_i = \left(\frac{2}{x_i K}\right)^{\frac{1}{4}} \quad x_i = E_i + m_i \tag{6}$$

The norrnalization constant is given by

$$N_i^2 = \left[1 + \frac{3}{2}\frac{1}{(x_i R_i)^2}\right]^{-1} \tag{7}$$

and the S-wave single-quark energies are

$$E_i = m_i + V_0 + \frac{3}{x_i R_i^2} \tag{8}$$

The Lagrangian density that corresponds to the Dirac **eq.(1)** is

$$L_i(x) = \bar{\Psi}_i(z)\left[\frac{i}{2}\gamma^\mu \overleftrightarrow{\partial}_\mu - \frac{1}{2}(1+\beta)V(r) - m_i\right]\Psi_i(z) \tag{9}$$

One can note that under a global infinitesimal chiral transformation in the $(u,$ d)-flavour sector, the axial current of quarks is not conserved due to the presente of the scalar $m_i + \frac{1}{2}V(r)$ in **eq.(1)**.

**In** order to recover the chiral syrnrnetry in the model we proceed in the usual manner. introducing an elernentary pion **field** $\phi(x)$ of **small** and finite mass $m_\pi =$ *140* **MeV** with *a* quark-pion interaction Lagrangian density given by

$$\mathcal{L}_I^\pi = -\frac{i}{f_\pi}G(r)\bar{\Psi}_i(x)\gamma^5(\vec{\tau}_i \cdot \vec{\phi})\Psi_i(x) \tag{10}$$

where

$$G(r) = m_i + \frac{1}{2}V(r) \tag{11}$$

and $f_\pi = 93$ MeV is the phenomenological pion-decay constant.

This yields the usual PCAC relation

$$\partial_\mu A^\mu(x) = -f_\pi m_\pi^2 \vec{\phi}(x) \tag{12}$$

Consequently. the pion coupling to the non-strange quarks would give rise to pionic energy corrections to the physical masses of the baryons.

In our model, the baryon masses are expressed as

$$M_B = \left[(E_0 + E_1)^2 - < P^2 > \right]^{\frac{1}{2}} + \Delta E_\pi \tag{13}$$

where $E_0$ is the sum of the single-quark energies. $E_1$ is the total OGE correction, given as a sum of *gluon electric* and *gluon magnetic* energy parts, $< P^2 >$ is the c.m. correction term evaluated in our earlier work[2] and $\Delta E_\pi$ stands for the pion-cloud energy correction to be discussed in the next section. We have

$$E_0 = \sum_i E_i$$

$$< P^2 > = \sum_i \frac{1}{R_i^2}\left(\frac{5}{2} - N_i^2\right)$$

and

$$E_1 = \Delta E_E + \Delta E_M$$

with

$$\Delta E_E = \frac{1}{8\pi}\sum_a \sum_{i,j} \int \left\langle B \right| \frac{\rho_i^a(\vec{r})\rho_j^a(\vec{r}')}{|\vec{r} - \vec{r}'|} \left| B \right\rangle d^3\vec{r} d^3\vec{r}' \tag{14}$$

$$\Delta E_M = -\frac{1}{8\pi}\sum_a \sum_{i,j} \int \left\langle B \right| \frac{\vec{j}_i^a(\vec{r})\vec{j}_j^a(\vec{r})}{|\vec{r} - \vec{r}'|} \left| B \right\rangle d^3\vec{r} d^3\vec{r}' \tag{14}$$

where a denotes the colour index and $i$, $j$ are the quark flavour indices. $\rho_i^a$ and $\vec{j}_i^a$ are the charge-densities and colour-currents of the quark $i$. The expressions

(14) can be analytically calculated and written as function of the parameters of the model. For further details, see **ref.(2).** Now. we would like to outline our procedure to calculate the pionic energy corrections.

## 3. PION-CLOUD CORRECTIONS

**In** this section the energy corrections due to the pion-quark coupling are calculated. As mentioned **before,** our approach differs from that of **Barik** and Dash[7] and provides an alternative analysis of the pionic effects involved. We **shall** proceed to our exposition in a step-by-step way.

**If** one wishes to work on the assurnption of massless pions. that is, in the chiral **limit, it** is possible to write the energy corrections $AE,$ associated with the quark-pion coupling in a similar way as in the magnetic one-gluon-exchange energy, namely

$$\Delta E_\pi = -\frac{1}{8\pi}\sum_{i,j}\int \left\langle B\right|\frac{\vec{J}_i^{\,5}(\vec{r})\cdot\vec{J}_j^{\,5}(\vec{r}')}{|\vec{r}-\vec{r}'|}\left|B\right\rangle d^3\vec{r}d^3\vec{r}' \qquad (15)$$

where $\vec{J}_i^5$ is the spatial **part** of the axial **current** taken, in an approxirnate way, as

$$J_j^5(\vec{r}) = -ig_{q\pi}\bar{\Psi}_i(\vec{r})\gamma_5\vec{\tau}_j\Psi_i(\vec{r}) \qquad (16)$$

where the $\vec{\tau}_j$ are expressed in terms of **Pauli** isospin matrices associated to the j-th quark and $g_{q\pi}$ represents an effective quark-pion coupling strength.

Making use of the model wave-function. given by **eq.(3),** one **finds**

$$J_i^5(\vec{r}) = -2g_{q\pi}\frac{N_i^2}{x_i}\varphi_i(r)\varphi_i'(r)\vec{\tau}_i\vec{\sigma}_i\cdot\hat{r} \qquad (17)$$

By substituting this expression in **eq.(15)** and performing the integration. we find

$$AE, = -\alpha_\pi\frac{1}{3\sqrt{2\pi}}\frac{1}{\&}\frac{N_0^4}{(x_0R_0)^2}\left\langle B\right|\sum_{i,J}\vec{\sigma}_i\cdot\vec{\sigma}_j\vec{\tau}_i\cdot\vec{\tau}_j\left|B\right\rangle \qquad (18)$$

where the subscript zero indicates a light quark *(u* or *d)* with $m_u = m_d \neq m_s$ and $a, = g_{q\pi}^2/4\pi$.

We remark that a similar result was obtained by Hatsuda[10] using a variational approach. However. he worked with a confining potential of the *linear* type, giving rise to a different numerical constant. He finds a factor $1/\sqrt{3\pi}$ instead of $1/3\sqrt{2\pi}$ in eq. (18) and this gives rise to a slight change in the fitted value of $\alpha_\pi$. Apart from this. the expression obtained by Hatsuda coincides with our eq.(18). This can be understood as an indication that the harmonic potential gives nearly the same results as the linear potential.

For pions of finite mass $m_\pi$, a similar calculation can be performed but now one has to use in eq.(14) the Green function appropriate to the massive case

$$\Delta E_\pi = -\frac{1}{8\pi}\sum_{i,j}\int \left\langle B\right|\frac{\vec{J}_i^{\,5}(\vec{r})\cdot\vec{J}_j^{\,5}(\vec{r}\,')}{|\vec{r}-\vec{r}\,'|}e^{-m_\pi|\vec{r}-\vec{r}\,'|}\left|B\right\rangle d^3\vec{r}d^3\vec{r}\,' \tag{19}$$

By use of

$$\frac{e^{-m_\pi|\vec{r}-\vec{r}\,'|}}{|\vec{r}-\vec{r}\,'|} = \frac{1}{2\pi^2}\int d^3k\frac{e^{i\vec{k}\cdot(\vec{r}-\vec{r}\,')}}{\vec{k}^2+m_\pi^2}$$

where $\vec{k}$ is the momentum of the interchanged pion, one gets

$$\Delta E_\pi = \alpha_\pi\left\langle B\right|\sum_{i,j}(\vec{\sigma}_i\cdot\vec{\sigma}_j)(\vec{\tau}_i\cdot\vec{\tau}_j)I_{ij}^\pi\left|B\right\rangle \tag{20}$$

with

$$I_{ij}^\pi = -\frac{1}{3\pi}\frac{N_i^2N_j^2}{x_ix_j}\int_0^\infty \frac{k^4}{k^2+m_\pi^2}e^{-k^2(R_i^2+R_j^2)/4}dk$$

The integration in eq.(20) gives

$$\Delta E_\pi = -\alpha_\pi\frac{m_\pi^3}{6\pi}\frac{N_0^4}{x_0^2}\left[\pi e^z+\frac{\sqrt{\pi}}{2}z^{-\frac{3}{2}}(1-2z\Phi(1,\frac{1}{2};z))\right]$$
$$\times\left\langle B\right|\sum_{i,j}(\vec{\sigma}_i\cdot\vec{\sigma}_j)(\vec{\tau}_i\cdot\vec{\tau}_j)\left|B\right\rangle \tag{21}$$

where $z=\frac{1}{2}m_\pi R_0^2$ and $\Phi(a,b;z)$ are hypergeometric Kummer functions.

One can also show that in the limit $m_\pi\to 0$, eq.(21) goes to eq.(18), as expected. Both expressions were found by assuming a direct pion-quark coupling which gives rise to a dependence on $\alpha_\pi$ in $\Delta E_\pi$. This is an alternative approach to that of Barik and Dash, ref. (7), where they have, as we shall show, a parametrization for the model by means of the nucleon-pion coupling.

Finally, a similar but more complete calculation can be performed this time by using in eq.(19) the axial current expressed as

$$\vec{J}_j^{5}(\vec{r}) = \frac{-i}{f_\pi} G(r) \bar{\Psi}_I(\vec{r}) \gamma_5 \vec{\tau}_j \Psi_i(\vec{r}) \tag{22}$$

whit $G(r)$ given by eq.(11). This expression for the axial current is a consequence of the interaction Lagrangian density eq.(10), introduced in order to restore the chíral ínvariance. By íntegration, one finds (see Appendíx A)

$$\Delta E_\pi = -\frac{1}{12\pi}\frac{1}{f_\pi^2} m_\pi^2 (m_0 R_0 + \frac{1}{2} V_0 R_0 + \frac{5}{8} K R_0^3)^2 \frac{N_0^4}{(x_0 R_0)^2} C_{ij} I_\pi \tag{23}$$

where

$$C_{ij} = \left\langle B \right| \sum_{i,j} (\vec{\sigma}_i \cdot \vec{\sigma}_j)(\vec{\tau}_i \cdot \vec{\tau}_j) \left| B \right\rangle$$

and

$$I_\pi = \frac{1}{\pi m_\pi^2} \int_0^\infty \frac{k^4}{w_k^2} (1 - A R_0^2 k^2)^2 e^{-k^2 R_0^2/2} dk \tag{24}$$

with

$$w_k^2 = \vec{k}^2 + m_\pi^2$$

One sees that the presence of the façtor $G(r)$ in eq. (22) results in the appearance in eq.(24) of a form-factor $u(k)$ given by

$$u(k) = (1 - A R_0^2 k^2) e^{-k^2 R_0^2/4} \tag{25}$$

where

$$A = \frac{E_0 - m_0}{2(5E_0 + 7m_0)}$$

We note that this result coincides with that obtained in ref.(7). Furthermore, keeping only the lowest order term in $k$, in the form factor in eq.(24), we recover exgression (20), showing the similar behaviour of both expressions.

Recalling the Goldberger-Treiman reiation

$$\sqrt{4\pi}\frac{f_{NN\pi}}{m_\pi} = \frac{g_A}{2f_\pi} \tag{26}$$

and the expression for the axial constant in the rnodel

$$g_A = \frac{5}{9}(4N_0^2 - 1) = \frac{5}{3}\frac{2N_0^2}{x_0}\left[m_0 + \frac{1}{2}V_0 + \frac{5}{8}KR_0^2\right] \tag{27}$$

one can also rewrite eq.(23) as

$$\Delta E_\pi = -\frac{1}{3}f_{NN\pi}^2 \cdot \frac{9}{25}C_{ij}I_\pi \tag{28}$$

In this way one finds the same nucleon-pion coupling pararnetrization for the model. as was obtained by Barik and Dash[7].

From the computational view point, we note that eq.(18), obtained in the approximation of massless pions, reprsduces accurately. by adjusting the value of $a_\pi$. the results obtained rnaking use of eq.(28). In fact. eq.(18) with $\alpha_\pi =$ 0.4614 allows us to reproduce the fit of ref.(7) based on eq.(28), showing that the massless pion approximation is a good one. An alternative and improved fitting to that of ref.(7) will be discussed in the next section.

In Appendix B, we give some useful expressions for the integrals appearing in the calculation of the different baryonic properties in the S + V harmonic model, and also details of the numerical calculations of $I_\pi$. Now. we shall present our results and main conclusions.

## 4. RESULTS

We shall present now our rnain results. Firstly, the fitting of baryonic spectrurn will be discussed. As mentioned before, we have tested the validity of the massless pion approximation (the chiral limit) by showing that it is possible to reproduce, with good accuracy. the fitting of ref.(7) by means of our eq.(18) with a value $a_\pi = 0.461$.

We note that in ref.(7) the parameters of the model were fixed by previously adjusting the quark-core. In the present work, an alternative fitting was made trying to avoid this procedure since the description of the core may be rnodified by the presence of the pion-cloud. Our new fitting has shown good results which we report here now.

In our previous fitting[2], the pararneters were: $m_0 = 27.8$ MeV, $m_s = 229.1$ MeV, $V_0 = 110.3$ MeV, $k = 21.4 10^6 MeV^3$ and $a_s = 0.82$. By including the pion-cloud energy corrections we obtained. in the present fitting. the values: $m_0 = 7$ MeV, $m_s = 176$ MeV, $V_0 = 130$ MeV, $K = 23.10^6 MeV^3$, $a_s = 0.66$ and $a_\pi = 0.527$. These results show that the inclusion of the pionic corrections leads to a decrease of the quark masses and of the quark-gluon coupling constant $a_s$.

We note that the value of $a_\pi = 0.527$ was obtained in the chiral limit, by rneans of eq.(18), representing an effective quark-pion coupling. An estirnation of the strenght of this effective coupling in the model could be done making use of the relation[10]

$$g_{8\pi} = \frac{1}{f_\pi} \frac{\int G(r)\bar{\psi}(\vec{r})\gamma^5\psi(\vec{r})e^{i\vec{k}\cdot\vec{r}}d\vec{r}}{\int \bar{\psi}(\vec{r})\gamma^5\psi(\vec{r})e^{i\vec{k}\cdot\vec{r}}d\vec{r}} \tag{29}$$

calculated with the quark wave-function of the rnodel. eq.(3. One obtains

$$g_{q\pi} = \frac{1}{f_\pi}\left[m_0 + \frac{1}{2}V_0 + \frac{5}{8}KR_0^2\right] \tag{30}$$

This equation shows a direct relation of the coupling strength in function of the parameters of the rnodel. With $f_\pi = 93$ MeV, eq. (30) gives $a_\pi = 0.573$. in a reasonable agreement with the value 0.527 found in our fitting. Notice that eqs. (29) and (30) correspond to an evaluation taken in the chiral limit, since these equations do not depend on the pion rnass.

Our results for the low-lying S-wave baryons are displayed in table 1. They have an average accuracy $< \delta M > \simeq 6.5$ MeV, while that of ref.(7) has $< \delta M > \simeq 10$ MeV. However. in the fitting of ref.(7) they used the experimental quantity $f_{NN\pi}^2 = 0.08$ as input, while in the present work we calculate it. In our fitting we do not use any quantities as input: instead we varied the paaameters and looked for the best fit of the baryonic spectrurn.

We also wish to call attention that Barik and Dash[7] have for the constituent quark rnasses the values $m_u = 78.5$ MeV and $m_s = 315.5$ MeV while for $V_0$ they get the negative value of -137.5 MeV, so that by defining the quantities

$$m'_q = m_q + \frac{V_0}{2}$$

$$E'_q = E_q - \frac{V_0}{2}$$

the values $m'_u = 10$ MeV and $m'_s = 247$ MeV were obtained, which they interpret as current quark masses. In our work, we have the opposite. We used a positive $V_0$ and adjusted the model parameters $m_u$ and $m_s$. Then values close to the QCD expectations naturally emerged from our fitting. namely $m_u = 7$ MeV and $m_s = 176$ MeV.

In order to examine the accuracy of our fitting, we extended our analysis to some static baryonic properties. such as strong coupling constants and electromagnetic properties of the nucleon.

Following ref.(11). we calculated the pseudoscalar coupling constant $G_{NN\pi}$ and the pseudovector coupling constant $f_{NN\pi}$. Our results are shown in table 2. in comparison with those of ref.(11) and the experimental data. One sees a better agreement with experiment in the present fitting.

**Table 1** - Masses of the low-lying S-wave baryons and their energy corrections (in MeV). The parameters of our fitting are: $m_0 = 7$ MeV. $m_s = 176$ MeV. $V_0 = 130$ MeV. $K = 23.10^6 MeV^3$. $a_s = 0.66$ and $a_q = 0.527$.

| Baryons | $E_0$ | $\Delta E_M$ | $\Delta E_E$ | $< P^2 >^{\frac{1}{2}}$ | $\Delta E_\pi$ | $M$ | $M_{exp}$ |
|---|---|---|---|---|---|---|---|
| $N$ | 1689.26 | -422.04 | 0 | 656.08 | -149.965 | 934.20 | 938 |
| Λ | 1784.41 | -400.87 | 4.875 | 671.53 | -94.715 | 1120.49 | 1116 |
| Σ | 1784.41 | -385.12 | 4.875 | 671.53 | -52.62 | 1180.56 | 1193 |
| Ξ | 1879.55 | -371.00 | 4.875 | 686.64 | -23.68 | 1325.02 | 1318 |
| Δ | 1689.26 | -211.02 | 0 | 656.08 | -86.82 | 1237.85 | 1232 |
| Σ* | 1784.41 | -197.73 | 4.875 | 671.53 | -52.62 | 1390.33 | 1385 |
| Ξ* | 1879.55 | -183.61 | 4.875 | 686.64 | -23.68 | 1532.37 | 1530 |
| Ω | 1974.69 | -168.68 | 0 | 701.41 | 0 | 1664.24 | 1672 |

**Table 2** - Strong coupling constants.

| Coupling constants | | Ref.(11) | Present fitting | Experiment[15] |
|---|---|---|---|---|
| Pseudoscalar | $\frac{G^2_{NN\pi}}{4\pi}$ | 13.027 | 14.459 | 14.1 |
| Unrenormalized pseudovector | $f_{NN_\pi}$ | 0.269 | 0.283 | 0.283 |
| Renormalized pseudovector | $f^{ren}_{NN\pi}$ | 0.23 | 0.247 | 0.283 |

We note that the calculated value for the unrenormalized pseudovector coupling constant $f_{NN\pi}$ using our parameters coincides with the experimental value. namely $F_{NN\pi} = 0.283$. For the renormalized constant we have a slightly smaller value: $f^{ren}_{NN\pi} = 0.247$.

We also calculated the proton and neutron charge radii and magnetic moments. These calculations include both the effects of the center-of-mass and pion-cloud corrections. Details can be found in ref.(11). Our results are given in table 3, in comparison with ref.(11), the cloudy-bag mode1[4] and experimental data. Finally. we remark that the center-of-mass corrections were included in our calculation of the axial-vector coupling constant $g_A$. which is given by

Table 3 - Electromagnetic static properties of the nucleon

| | Ref.11 | Present fitting | Cloudy-bag model Ref.(4) | Experiment[15] |
|---|---|---|---|---|
| $< r^2 >_p^{1/2}$ | 0.79 fm | 0.804 fm | 0.83 fm | 0.85 fm |
| $< r^2 >_n^{1/2}$ | -0.344 fm | -0.301 fm | -0.36 fm | -0.341 fm |
| $\mu_p$ | 2.730 | 2.7462 | 2.6 | 2.7928 |
| $\mu_n$ | -1.975 | -1.9573 | -2.01 | -1.913 |
| $g_A$ | 1.182 | 1.235 | 1.33 | 1.255 |

$$g_a = g_A^0 \left[1 + \frac{1}{3} \frac{< p^2 >}{M_p^2}\right]$$

where $g_A^0$ is given by eq.(27). One sees that in the above equation the c.m. correction term has a positive sign[13], which increasas the value of $g_A$. However, in ref.(11), Barik and Dash applied a negative carrection. so that they calculated a smaller value for $g_A$ than the correct one.

## 5. CONCLUSIONS

The present work is based on the assumption that a dominant role is played by the binding of the individual quarks in the relativistic $S + V$ harmonic potential which represents. in a phenomenological way. the confining non-perturbative regime of the quark-gluon interactions. It is also assumed that. after corrections to eliminate the spurious center-of-mass motion are made, residual quark-gluon and pion-quark interactions give rise to relatively small effects which can be evaluated perturbatively. We note that this important aspect of our model is also shared by the cioudy-bag mode1[4].

We have shown that the present model when implement by introducing pions. regarded as Nambu-Goldstone bosons[14] and interacting with quarks in a linear way. yields results in reasonable agreement with data.

Our fitting of the low-lying S-wave baryon spectrum improves slightly that of Barik and Dash[7] and gives comparable results for the baryon static properties.

We wish to make clear that the pion-energy correction, eq.(23). with exception of the phenomenological pion-decay constant $f_\pi = 93$ MeV. is completely determined by the pararneters of the model. Since it has been calculated out of the chiral limit. it depends on the pion mass too.

However. we adopted by simplicity a fitting procedure based on the parametrization in terms of $a_\pi$, 'regarded as an effective quark-pion coupling. as introduced in Section 3.

Perhaps the main need for improvement of our work is concerned with the necessity of quark recoil corrections in relation with the behaviour of the electromagnetic form-factors of the nucleon. for not too small momentum transfers[11].

Finally. we remark that renormalization effects on the coupling constant $f_{NN_\pi}$. as calculated in ref.(11). turn out to be small. This results is similar to that occurring in ref.(4) and seems to support our basic assumption referred to at the beginning of this section that pionic effects are small or moderate and consequently. amenable to a a perturbative treatment.

## APPENDIX A

Here, we will present the calculation of the pionic self-energy corrections given by eq.(23). Starting with the axial current. eq.(22). and using the model wave-function, eq.(3). one finds

$$J_i^5(\vec{r}) = -\frac{1}{f_\pi} G(r) 2 \frac{N_i^2}{x_i} \varphi_i(r) \varphi_i'(r) \vec{\tau}_i \vec{\sigma}_i \cdot \vec{r} \tag{A.1}$$

Consequently. the energy correction $\Delta E_\pi$ will be given by

$$\Delta E_\pi = -\frac{1}{8\pi} \sum_{i,j} \int \left\langle B \middle| \frac{\vec{J}_i^5(\vec{r}) \cdot \vec{J}_j^5(\vec{r}')}{|\vec{r}-\vec{r}'|} e^{-m_\pi |\vec{r}-\vec{r}'|} \middle| B \right\rangle d^3\vec{r} d^3\vec{r}' =$$

$$= -\frac{1}{f_\pi^2} \frac{16}{3} \sum_{i,j} \frac{N_i^2 N_j^2}{x_i x_j} \langle B | \vec{\sigma}_i \cdot \vec{\sigma}_j \vec{\tau}_i \cdot \vec{\tau}_j I_{rad} | B \rangle \tag{A.2}$$

where

$$I_{rad} = \int \frac{k^2}{k^2 + m_\pi^2} I_k^2$$

with

$$I_k = \int_0^\infty G(r)\varphi_i(r)\varphi_i'(r) j_1(kr) r^2 dr \qquad (A.3)$$

Using the radial function of the model given by **eq.(5)** and the **Bessel** function $j_1(kr)$, one finds

$$I_{rad} = \frac{1}{64\pi^2}\left(m_0 + \frac{1}{2}V_0 + \frac{5}{8}KR_0^2\right)^2 \int_0^\infty \frac{k^4}{k^2 + m_\pi^2}(1 - AR_0^2k^2)^2 e^{-k^2R_0^2/2} dk \qquad (A.4)$$

where we see that the form-factor naturally emerges from the **presence** of $G(r)$ in eqs. (A.I)-(A.3).

Now, defining

$$I_\pi = \frac{1}{\pi m_\pi^2}\int_0^\infty \frac{k^4}{k^2 + m_\pi^2}(1 - AR_0^2k^2)^2 e^{-k^2R_0^2/2} dk \qquad (A.5)$$

which can be evaluated numerically in the model, and substituting **eqs.** (A.4) and (A.5) into eq. **(A.2)**, one gets the result

$$\Delta E_\pi = -\frac{1}{12\pi}\frac{1}{f_\pi^2} m_\pi^2 \left(m_0R_0 + \frac{1}{2}V_0R_0 + \frac{5}{8}KR_0^3\right)^2 \frac{N_0^4}{(x_0R_0)^2} C_{ij} I_\pi \qquad (A.6)$$

where, except for the phenomenological pion-decay constant $f_\pi = 93$ **MeV**, the pion-correction $AE$, is completely determined by the parameters of the model.

## APPENDIX B

**In** the calculation of different baryonic properties, integrais similar to $I$,. **eq.(24)**, are found. **In** this appendix we **will** discuss them.

As we have shown, in the calculation of the pionic self-energies of baryons we have[7]

$$I, = \frac{1}{\pi m_\pi^2}\int_0^\infty \frac{k^4}{w_k^2} u^2(k) dk \qquad (B.1)$$

A similar integral. found **in ref.(11)**, namely

$$I_{\pi_1} = \frac{1}{\pi m_\pi^2}\int_0^\infty \frac{k^4}{w_k^3} u^2(k) dk \qquad (B.2)$$

appears (i) in the calculation of the electric form factors of the nucleon and consequently in the charge radii of the proton and the neutron. (ii) in the renormalization of the nucleon state due to the $q-\pi$ coupling, (iii) in the core contribution to the magnetic form factors of the nucleon, due to the renormalization effects and consequently contributes to the quark-core part of the magnetic moments.

In the calculation of the magnetic moments of baryons. one also finds[11,12]

$$I_{\pi_2} = \frac{1}{\pi m_\pi^2}\int_0^\infty \frac{k^4}{w_k^4} u^2(k)dk \qquad (B.3)$$

which appears in the rnagnetic form factors of the nucleon (pion-cloud contribution).

We can write all these expressions together, in the form

$$I_{\pi_i} = \frac{1}{\pi m_\pi^2}\int_0^\infty \frac{k^4}{w_k^{i+2}} u^2(k)dk \qquad (B.4)$$

where the case of $i=0$ corresponds to $I_,$. For $i=2$ an additional multiplying factor of $M_p$, the proton mass. was defined in ref.(11) but here we do not use it. Defining $x=k^2/m_\pi^2$, we can rewrite eq.(B.4) as

$$I_{\pi_i} = \frac{(m_\pi)^{1-i}}{2\pi}\int_0^\infty \frac{x^{3/2}}{(x+1)^{1+\frac{i}{2}}}(1-2Azx)^2 e^{-zx}dx \qquad (B.5)$$

where

$$z = \frac{1}{2}m_\pi^2 R_0^2 \simeq 0.121$$

in the present fitting, so that the quantity $2Az$ is $\simeq 0.02$ in the form factor in eq.(B.5).

The integral eq.(B.5) can be solved using

$$I_{x_i}(n) = \int_0^\infty \frac{x^{n-\frac{1}{2}}}{(x-1)^{1+\frac{i}{2}}} e^{-zx}dx = \Gamma\left(n+\frac{1}{2}\right)\Psi\left(n+\frac{1}{2}, n+\frac{1}{2}-\frac{i}{2}; z\right) \quad (B.6)$$

where the $\Psi(\alpha,\gamma;z)$ are the Kummer hypergeometric confluent functions of the second kind and are related to the degenerate hypergeometric functions $\Phi(\alpha,7;z)$ by[9]

$$\Psi(\alpha,\gamma;z) = \frac{\Gamma(1-\gamma)}{\Gamma(\alpha-\gamma+1)}\Phi(\alpha,\gamma;z) + \frac{\Gamma(\gamma-1)}{\Gamma(\alpha)} z^{1-\gamma}\Phi(\alpha-\gamma+1, 2-\gamma; z) \quad (B.7)$$

where, in particular

$$\Phi(0,\gamma;z) = 1$$
$$\Phi(\alpha,\alpha;z) = e^z$$
$$\Phi(1,\gamma;z) = 1 + \frac{z}{\gamma}\Phi(1,\gamma+1;z) \qquad (B.8)$$

With eqs. (B.6)-(B.8) we can write a general result in a simple form

$$I_{,,,.} = \frac{(m_\pi)^{1-i}}{2\pi}\left[I_{x_i}(2) - 4AzI_{x_i}(3) + 4A^2 z_\pi^2 I_{x_i}(4)\right) \qquad (B.9)$$

The first term, namely $I_{x_i}(2)$, with $i = 0$. corresponds to the expression inside the brackets in eq. (21). Although the other terms in eq.(B.9) are not smaller than $I_x(2)$ and also contribute significatively to the result. the behaviour of $I,$ is given by the function $I_x(2)$, because we have

$$I_x(2) = \pi e^z + \frac{\sqrt{\pi}}{2}z^{\frac{-3}{2}} - \sqrt{\pi}z^{\frac{-1}{2}}\Phi(1,\frac{1}{2};z)$$
$$I_x(3) = \frac{3\sqrt{\pi}}{4}z^{\frac{-5}{2}} - I_x(2)$$
$$I_x(4) = \frac{15\sqrt{\pi}}{8}z^{\frac{-7}{2}} - \frac{3\sqrt{\pi}}{4}z^{\frac{-5}{2}} + I_x(2) \qquad (B.10)$$

which gives

$$I_{,,} = \frac{m_\pi}{2\pi}\left[(1+2Az)^2 I_x(2) - 3\sqrt{\pi}Az^{\frac{-3}{2}}(1 - \frac{5}{2}A + Az)\right] \qquad (B.11)$$

This result enters in the pionic self-energy equations, eq.(23) and eq.(28). With the parameters of our fitting, we have $z = 0.121$ and $\Phi(1,\frac{1}{2};z) = 1.265$, giving the result $I, = 213.61$ MeV.

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## Resumo

Correções piônicas ao espectro de massas dos bárions fundamentais em onda S são incorporadas a um modelo relativista de quarks independentes. com uma mistura em partes iguais de potenciais harmônicos que se transformam como um escalar e um vetor de Lorentz. A introdução de uma nuvem de píons que interage linearmente com um caroço de quarks origina-se do requisito de simetria quiral no setor não estranho do modelo. Apresenta-se um método diferente para calcular as correções piônicas. que conduz a um ajuste alternativo àquele de Barik e Dash com um desvio médio menor. Estendemos nossa análise quantitativa a outras propriedades estáticas dos báiions, tais como constantes de interação forte, em bom acordo com experiência.