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Constraints in an Extended U(1)-Model

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Abstract Two types of constraints regulated by gauge invariance are identified for systems involving more than one potential field transforming under a single group. They are the constraints of the symmetry and of the circumstances. An extended Gauss law and the Hamiltonian covariance are also studied.

1. INTRODUCTION

Different aspects should be required in order to justify the possibility of including more than one potential field in the same gauge group. It has already been observed from arguments such as counting the number of degrees of freedom. Kaluza-Klein compactification[1] and supersymmetry with no conventional constraints[2] that there are enough elements to allow the existence of gauge theories which include N-potential fields transforming as

$$\begin{aligned} A_\mu &\longrightarrow \Lambda A_\mu \Lambda^{-1} + i/g\Lambda\partial_\mu\Lambda^{-1} \\ B_\mu &\longrightarrow \Lambda B_\mu \Lambda^{-1} + i/g\Lambda\partial_\mu\Lambda^{-1} \\ &\vdots \longrightarrow \vdots \\ N_\mu &\longrightarrow \Lambda N_\mu \Lambda^{-1} + i/g\Lambda\partial_\mu\Lambda^{-1} \end{aligned} \tag{1}$$

where

$$\Lambda = e^{i\omega^a t_a}$$

Eq.(1) means that the group parameters $\omega^a(x)$ specify transformations which act on fields with different quantum numbers.

In this work our motivation is to analyse such an extended gauge model in the Lagrangian and Hamiltonian formalisms, for they can provide important tests of consistency. As it is well-know, any Hamiltonian formalism will not have its Lorentz covariance manifested due to the fact that it is described in terms of just one of the components of the energy momentum tensor. Thus. it is worth understanding the general properties of the theory and, in particular, testing if the inclusion of different potential fields transforming as eq.(1) can be supported by the covariance of this extended model.

Considering that a momentum-independent field reparametrization does not affect the S-matrix elements, we prefer to rewrite the original field eq.(1) as

$$\begin{aligned} D_\mu &= A_\mu + B_\mu + \ldots + N_\mu \\ X_{\mu 1} &= A_\mu - B_\mu \\ &\vdots \\ X_{\mu<N-1>} &= A_\mu - N_\mu \end{aligned} \tag{2}$$

The fields defined through eq.(2) are called constructor fields. Although our concern is with the physical description, we will repeatedly make use of the constructor-field sector which easily intermediates the interplay between usual gauge theories and the kind of models studied in the text.

The most general $U(1)$-gauge invariant Lagrangian obtained from eq.(2) is

$$\mathcal{L} = \mathcal{L}_{Kin.} + \mathcal{L}_{Int.} + \mathcal{L}_{Mass} \tag{3}$$

with

$$\mathcal{L}_{Kin.} = \mathcal{L}_{Sym.} + \mathcal{L}_{Ant.}$$

where

$$\begin{aligned} \mathcal{L}_{Ant.} = {} & a[(\partial_\mu D_\nu)^2 - \partial_\mu D_\nu \partial^\nu D^\nu] + b_\alpha[(\partial_\mu D_\nu - \partial_\nu D_\mu)\partial^\mu X^{\alpha v}] \\ & + C_{(\alpha\beta)}[\partial_\mu X^\alpha_\nu(\partial^\mu X^{\nu\beta} - \partial^\nu X^{\mu\beta})] \end{aligned}$$

$$\mathcal{L}_{Sym.} = D_{(\alpha\beta)}\partial_\mu X^\alpha_\nu \partial^\mu X^{\nu\beta} + E_{(\alpha\beta)}\partial_\mu X^\alpha_\nu \partial^\nu X^{\beta\mu} + F_{(\alpha\beta)}(\partial\cdot X)^\alpha(\partial\cdot X)^\beta$$

$$\mathcal{L}_{Int.} = a_{\alpha\beta\gamma}(\partial_\mu X_\nu{}^\alpha)X^{\mu\beta}X^{\nu\gamma} + b_{\alpha\beta\gamma}(\partial\cdot X)^\alpha X_\nu^\beta X^{\nu\gamma} + C_{\alpha\beta\gamma\delta}X_\mu{}^\alpha X_\nu{}^\beta X^{\mu\gamma}X^{\nu\delta}$$

and

$$\mathcal{L}_{Mass} = 1/2 M^2_{\alpha\beta} X_\mu{}^\alpha \cdot X^{\beta\mu}$$

The parameters a, b $\ldots, C_{\alpha\beta\gamma\delta}$ are referred to as free coefficients of the theory. However. in perturbation theory a physical particle is **defined** as the pole of the complete and renormalized two-point Green's function. Therefore. from the fact that **eq.(3) naturally** yields rnixed propagators. it happens that the physical fields. G^I_μ, are not trivially read off it. They are obtained as linear cornbinations of the constructor fields

$$\begin{pmatrix} G^1_\mu \\ G^2_\mu \\ \vdots \\ G^N_\mu \end{pmatrix} U = \begin{pmatrix} D_\mu \\ X^1_m u \\ \vdots \\ X^{(N-1)}_\mu \end{pmatrix} \tag{4}$$

with

$$U = \begin{pmatrix} u_{10} & u_{11} & \cdots & u_{1(N-1)} \\ u_{20} & u_{21} & \cdots & u_{2(N-1)} \\ \vdots & & & \\ u_{N0} & u_{N1} & \cdots & u_{N(N-1)} \end{pmatrix} \tag{5}$$

U must be orthogonal. This rneans that **eq.(4)** preserves the **number** of **independent** fields given by **eq.(1)**. The inverse matrix elements **will** be denoted by $\bar{u}_{IJ}$.

Thus. the theory can be focused in terrns of two differents sets of variables; the former, described by the constructor fields

$$set\ I \quad (D, X_\alpha) \equiv [D_\mu, X_{\mu 1}, \ldots, X_{\mu N-1}] \tag{6}$$

and the latter, based on the physical fields

$$set\ II \quad (G_I) \equiv [G_{\mu 1}, G_{\mu 2}, \ldots, G_{\mu N}] \tag{7}$$

which is built up by N independent vectors with well-defined physical masses. Eqs. (6) and (7) are connected by the relationship

$$G_{\mu I}=u_{I0}D_{\mu}+u_{I\alpha}X_{\mu}{}^{\alpha} \tag{8}$$

Note from eqs.(5) and (8) the presence of new variables. the parameters u_{IJ}.

This work is organized as follows. In section 2, the Lagrangian and Hamiltonian formulations are studied for the Abelian $U(1)$-case and a discussion on the concept of symmetry constraints is presented. The degrees of freedom are examined and counted in section 3. Then. in order to control the dynamical variables, the notion of circumstances for the constraints becomes necessary. The 4 section is devoted to the study of the Hamiltonian covariance. Finally, in section 5, as a physical consequence from the constraint considerations. an extended Gauss law is analysed from the Noether theorem. Our conclusions are stated in section 6. An appendix follows where the meaning of the free coefficients is better discussed.

2. LAGRANGIAN AND HAMILTONIAN FORMALISMS

The different field parametrizations that the theory provides to analyse the same system yield the existence of Lagrangians with different functional dependentes on these respective fields

$$\mathcal{L}(D;X_{\alpha})=\mathcal{L}'(G_I(D;X_{\alpha})) \tag{9}$$

and vice-versa. Although reference systems eqs.(6) and (7) are connected by simple field transformations, eq.(8), it is propitious and necessary to include some discussion about the fact that they describe the same physics. Take for example their respective equations of motion and consider the action functional corresponding to eq.(6)

$$S=\int dx^4\mathcal{L}[D_{\mu}(x),\partial_{\nu}D_{\mu}(x);X_{\nu}^{\alpha}(x),\partial_{\nu}X_{\mu}^{\alpha}(x)] \tag{10}$$

Considering the minimum action principle. we get the following correspondence

$$\partial_{\mu}\frac{\partial\mathcal{L}}{\partial(\partial_{\mu}D_{\nu})}-\frac{\partial\mathcal{L}}{\partial D_{\nu}}=u_0^I\Big[\partial_{\mu}\frac{\partial\mathcal{L}'}{\partial(\partial_{\mu}G_{\nu}^I)}-\frac{\partial\mathcal{L}'}{\partial G_{\nu}^I}\Big]=0 \tag{11}$$

and

$$\partial_\mu \frac{\partial \mathcal{L}}{\partial(\partial_\mu X_\alpha)} - \frac{\partial \mathcal{L}}{\partial_\mu X^\alpha} = u^I_\alpha \left[\partial_\mu \frac{\partial \mathcal{L}'}{\partial(\partial_\mu G^I_\nu)} - \frac{\partial \mathcal{L}'}{\partial G^I_\nu}\right] = 0 \tag{12}$$

Eqs.(11) and (12) exhibit the same content for both sectors. since reparametrizations do not alter on-shell information: if the reference system eq. (7) describes on-shell physical fields, then the constructor field D_r and X^α_μ in eq.(6) will also be on-shell. Thus. one can argue that there is an initial consistency for choosing the physical set. eq.(7), to organize the Lagrangian

$$\mathcal{L}'(G_I) = \mathcal{L}'_{Ant.} + \mathcal{L}'_{Sym.} + \mathcal{L}'_{Mass} + \mathcal{L}'_{Int.} \tag{13}$$

where

$$\begin{aligned}\mathcal{L}'_{Ant.} = {} & a\bar{u}_{0I}\bar{u}_{0J}\left[\partial_\mu G^I_\nu \partial^\mu G^{\nu J} - \partial_\mu G^I_\nu \partial^\nu G^{J\mu}\right] \\ & + b_\alpha \bar{u}_{0I}\bar{u}_{\alpha J}\left[\partial_\mu G^I_\nu - \partial_\nu G^I_\mu\right]\partial^\mu G^{J\nu} \\ & + C_{(\alpha\beta)}\bar{u}^\alpha_I \bar{u}^\beta_J\left[\partial_\mu G^I_\nu(\partial^\mu G^{\nu J} - \partial^\nu G^{\mu J})\right]\end{aligned} \tag{14}$$

$$\begin{aligned}\mathcal{L}'_{Sym.} = {} & D_{(\alpha\beta)}\bar{u}^\alpha_I \bar{u}^\beta_J \partial_\mu G^I_\nu \partial^\mu G^{\nu J} + E_{(\alpha\beta)}\bar{u}^\alpha_I \bar{u}^\beta_J \partial_\mu G^I_\nu \partial^\nu G^{J\mu} \\ & + F_{(\alpha\beta)}\bar{u}^\alpha_I \bar{u}^\beta_J \partial_\mu G^{\mu I}\partial_\nu G^{\nu J}\end{aligned} \tag{15}$$

and

$$\mathcal{L}'_{Mass} = 1/2\bar{u}^\alpha_I \bar{u}^\beta_J M^2_{\alpha\beta} G^I_\mu G^{\mu J} \tag{16}$$

The corresponding canonically conjugate momenta are given by

$$\begin{aligned}\Pi^\rho_{G_K} = {} & \bar{u}_{0K}\left[2a\bar{u}_{0I}(\partial^0 G^I_\rho - \partial_\rho G^{I0}) + b_\alpha \bar{u}^\alpha_J(\partial^0 G^J_\rho - \partial^{0J}_\rho)\right] \\ & + \bar{u}^a lpha_K\left[b_\alpha \bar{u}_{0I}(\partial^0 G^I_\rho - \partial_\rho G^{0I}) + C_{(\alpha\beta)}\bar{u}^\beta_J(\partial^0 G^J_\rho - \partial_\rho G^{0J})\right]\end{aligned}$$

$$+ \bar{u}^{\alpha}_{K}\bar{u}^{\beta}_{J}\Big[D_{(\alpha\beta)}\partial^0 G^J_\rho + E_{(\alpha\beta)}\partial_\rho G^J + 2F_{(\alpha\beta)}\partial_\nu G^{\nu J}\delta_{0\rho}\Big]$$

$$+ a_{\alpha\beta\gamma}\bar{u}^{\alpha}_{K}\bar{u}^{\beta}_{J}\bar{u}^{\gamma}_{L}G^{0J}G^{\rho L} + b_{\alpha\beta\gamma}\bar{u}^{\alpha}_{K}\bar{u}^{\beta}_{J}\bar{u}^{\gamma}_{L}\delta^\rho G^J_\nu G^{\nu L} \tag{18}$$

Observe that eq.(18) shows that the system works with all fields contributing to the dynamics of a particular field. These contributions are obtained from the kinetic and interaction terms. In this aspect, eq. (18) has a similarity with the case of scalar QED. However. in the latter. only compensating fields appear in the momenta of the scalar fields.

The next purpose to understand such an extended model regards its consistency with the following topics: Lorentz covariance. equal-time commutation relations without conflicting covariance and a Hamiltonian formulation that satisfies covariance.

Lorentz covariance of the system. described by the Lagrangian density eq.(13). is immediatly verified by the facts that the equations of motion can be derived from an action principle and that eq.(13) is a Lorentz invariant functional of the fields and their gradients.

The second consistency topic can be verified now. Assuming that the original fields satisfy the fundamental Poisson brackets. we get the following relation for the constructor fields

$$\left\{\Pi^\mu_D(x), D_\nu(y)\right\}_{x^0=y^0} = -\delta^\mu_\nu \delta^3(\vec{x}-\vec{y})$$

$$\left\{\Pi^\mu_{x_\alpha}(x), X^\nu_\beta(y)\right\}_{x^0=y^0} = -\delta_{\alpha\beta}\delta^{\mu\nu}\delta^3(\vec{x}-\vec{y}) \tag{19}$$

with all other commutation relations vanishing. And then. let us express the conjugate momenta of the constructor fields in terms of the conjugate momenta of the physical fields

$$\Pi^\mu_D = u_{I0}\Pi^\mu_{G_I} \tag{20}$$

$$\Pi^\mu_{x_\alpha} = u_{I\alpha}\Pi^\mu_{G_I} \tag{21}$$

Replacing eqs. (20) and (21) in eq.(19). and considering from eq.(18) that the canonical momentum of the time component is not generally zero. one notices that, for the physical fields. the usual mismatch between the right- and left-hand sides of the equal time commutation relations is bypassed. Nevertheless. it is possible that the quanta associated to each longitudinal part of the physical fields risk creating states with negative norm. thus violating unitarity. The third purpose, a covariant Hamiltonian formulation. is left to section 4.

Since the theory has survived initial tests of consistency, let us follows the Dirac method quantization[9]. In order to calculate the canonical Hamiltonian density, $\mathcal{H}_c$, for the theory it is advantageous to introduce the matrix notation

$$\Pi_G^\mu = \begin{pmatrix} \Pi_{G_1}^\mu \\ \Pi_{G_2}^\mu \\ \vdots \\ \Pi_{G_N} \end{pmatrix}; \qquad G = \begin{pmatrix} G_1^\mu \\ G_2^\mu \\ \vdots \\ G_N^\mu \end{pmatrix}$$

$$b = \begin{pmatrix} b_1 \\ b_2 \\ \vdots \\ b_N \end{pmatrix}; \qquad \beta = bb^t - 4a(C+D) \tag{24}$$

where Π_G^μ displays the N conjugated physical canonical momenta in a column vector. and C, D, E and F correspond to symmetric n x n matrices depending on the free coefficients $C_{\alpha\beta}, F_{\alpha\beta}, E_{\alpha\beta}$. and $F_{\alpha\beta}$ respectively. Then

$$\begin{aligned}\mathcal{H}_c = {} & a\Pi_G^{it}U'\beta^{-1}U'\Pi_G^i - \Pi_G^{it}U'\beta^{-1}bU_{I0}\Pi_G^i - \Pi_G^{it}U'\beta^{-1}(bb^t - 4a(C-E))U'^d\partial_i G^0 \\ & + \Pi_G^{it}[1/4a(-1 + b^t\beta^{-1}b)U_{I0}U_{I0}]\Pi_G^i - \Pi_G^{ot}U_{I0}U_{I0}\partial_i G^0 \\ & + 2\Pi_G^{it}U_{I0}b^t\beta^{-1}(E+D)\bar{U}'\partial_i G^0 + 1/4\Pi_G^{0t}U'(D+E+F)^{-1}U'\Pi_G^0 \\ & + \partial_i G^{0t}\bar{U}'^t\eta\bar{U}'\partial_i G^0 + \partial_i G^{it}U'^t F(D+E+F)^{-1}FU'\partial_i G^i \\ & + \Pi_G^{0t}U'(D+E+F)^{-1}F\bar{U}'\partial_i G^i + \vartheta_0 + \mathcal{H}_I\end{aligned}$$

where

$$\mathcal{H}_I = \bar{u}_I^{\alpha}\bar{u}_J^{\beta}\bar{u}_K^{\gamma}\{ - G^{0I}\big[(a_{\alpha\beta\gamma} + b_{\alpha\beta\gamma})G^{0J}G^{0K} + b_{\alpha\beta\gamma}G_i^J G^{iK}\big]$$

$$- a_{\alpha\beta\gamma}\big[G^{i\alpha}G^{0\beta}G^{i\gamma} + \partial_i G^{0I}G^{iJ}G^{Kj}\big] + b_{\alpha\beta\gamma}\partial_i G_I^i G_{\nu J}G_K^{\nu}\}$$

$$+ C_{\alpha\beta\gamma\delta}\bar{u}_I^{\alpha}\bar{u}_J^{\beta}\bar{u}_K^{\gamma}\bar{u}_L^{\delta}G_\mu^I G_\nu^I G_\nu^J G^{\mu K}G^{\nu L}$$

and

$$\vartheta_0 = \big[\bar{u}_{0I}\bar{u}_{0J}\big(\partial_i G_j^I\partial^i G^{jJ} - \partial_i G_j^I G^{iJ}\big)\big] + b_\alpha \bar{u}_{\alpha I}\bar{u}_{0J}\partial^i G^{jI}\big(\partial_i G_j^J - \partial_j G_i^J\big)$$

$$+ C_{(\alpha\beta)}\bar{u}^{\alpha}\bar{u}_J^{\beta}\partial_i G_j^I\big(\partial^i G^{jJ} - \partial^j G^{iJ}\big) + D_{(\alpha\beta)}\bar{u}_I^{\alpha}\bar{u}_J^{\beta}\partial_i G_j^I\partial^i G^{iJ}$$

$$+ E_{(\alpha\beta)}u_I^{-\alpha}u_J^{-\beta}\partial_i G_j^I\partial^j G^{iJ} + F_{(\alpha\beta)}u_I^{-\alpha}u_J^{-\beta}\partial_i G^{iI}\partial_j G^{jJ}$$

$$+ (D+E)_{(\alpha\beta)}u_I^{\alpha}u_J^{\beta}\partial_i G_i^I\partial^i G^{iJ} + 1/2u_I^{\alpha}u_J^{\beta}M_{\alpha\beta}^2 G_\mu^I G^{\mu J} \tag{25}$$

However. it is necessary to understand whether or not eq.(25) provides a **unique** Hamiltonian. Therefore, we have to analyse its singular nature. Since eq.(13) appears as an extension of the usual Lagrangian with one potential field associated to the same group, one would expect that it should contain, as boundary conditions. the initial paoperties of the common gauge model. Therefore. we make use of the intermediate constructor model where the correspondent time component of Π_D^μ must still vanish weakly giving the prirnary constraint

$$\lambda_1 = \Pi_d^0(x) \simeq 0 \tag{26}$$

So we have the following correspondent formulation for the physical set II:

$$\lambda_1 = u_{0I}\Pi_{G_I}^0 \simeq 0 \tag{27}$$

Notice once more that. since eq.(27) characterizes the constraint due to the precence of more potential fields in the same group. it preserves the properties obtained for theories which include just one field. This means that, if the primary constraint is obtained for QED, then it should appear again in this U(1)-case with more fields. However, the difference in eq.(27) is that. although the constraint is preserved. it is not any longer associated to a specific field. For this reason, eq.(27) should be called a symmetry constraint, just because it acts over the field system as a whole. Consequently. quantizing the system with this constraint will not in principle violate the covariance property of the commutation relations in terms of physical fields.

Requiring consistency of the primary constraint eq.(26) provides. in terms of the physical fields, the secondary constraint

$$\lambda_2 = u_{J0}\partial_i\Pi^i_{G_J} \simeq 0 \tag{28}$$

Constraints eqs.(27) and (28) are then also first class ones and the consistency conditions show that there are no other constraints.

Considering that first class constraints are associated to local gauge invariance, one should expect

$$\textit{Generator} = \epsilon_1\phi_1 + \epsilon_2\phi_2 \tag{29}$$

where ϕ_1 is given by eq.(26) and $\phi_2 = \partial_i\Pi^i_D$, generates the gauge transformation

$$D_\mu(x) \longrightarrow D_\mu(x) + \partial_\mu\alpha(x) \tag{30}$$

Thus, from the information obtained above through the sector I, and using eq.(8), we get

$$G_{\mu I}(x) \longrightarrow G_{\mu I}(x) + u_{I0}\partial_\mu\alpha(x) \tag{31}$$

Relations eq.(30) or eq.(31) express here that the first class symmetry constraints are taking at most two degrees of freedom. However, they do not select what the particular physical field is which has its dynamical variables reduced by the gauge symmetry. Another information from these relations is about the gauge-fixing terrn. The existence of a single group provides only one gauge parameter. Thus there is only one general expression for expressing such a gauge function parameter as a functional of the N-potential fields that are involved.

Eqs.(27) and (28) indicate the presence of two auxiliary conditions to quantize the system. Considering the Coulomb gauge, we have them given in terms of physical fields

$$\lambda_3 = \bar{u}_{0I} G_I^0 \simeq 0 \tag{32}$$

$$\lambda_4 = \bar{u}_{0I} G_I^i \simeq 0 \tag{33}$$

Eqs. (30) and (31) are made compatible through the equation

$$\partial_0 \lambda_4 = -\Delta \bar{u}_{I0} G_I^0 \simeq 0 \tag{34}$$

Thus. the Dirac bracket for the generalized radiation gauge is

$$\sum_{IJ} \bar{u}_{I0} \bar{u}_{J0} \{ G_{\mu I}(x), \Pi_{\nu G_J}(y) \}^* = (g^{\mu\nu} - g^{\mu} g^{\nu}) \delta^3(x-y) + \partial_x^{\mu} \partial_y^{\nu} \frac{1}{4|\vec{x} - \vec{y}|} \tag{35}$$

and all other combinations are equally zero.

Finally we would observe that eq.(35) represents a Dirac parenthesis derived when the $U(1)$ symmetry is considered as a source acting over all fields. Thus, we leave for sec. 3 a clear description of the Dirac bracket that is associated to each particular field.

3. DEGREES OF FREEDOM

A covariant formulation treats the components of a quadrivector without any distinction. Therefore, the mechanism that must select the non-physical degrees of freedom is fixed by the theory. Normally. arbitrary functions are identified through the canonical momenta and eliminated by a convenient choice of gauge. However such a localization is not immediate when more than one potential field is involved. The difference liere is that, although there exists only one parameter $\alpha(x)$ originated from the gauge group, the transformation is acting over N-fields. This means that at least one degree of freedom is taken by the gauge symmetry, but the theory does not distinguish which is the potential field that suffers such a reduction in its degrees of freedom.

The presence of more potential fields in the same group yields more arbitrariness in the quantization programme. They are the symmetry and the circumstance constraints. The former is represented by the expressions (27) and (28). It generates a gauge arbitrariness due the fact that they are first class constraints. and so, arbitrary variables can be fixed by fixing the gauge. Then. the contribution of such a first mode is just to eliminate at least one degree of freedom from eq.(13). However it does not indicate which field has its phase space reduced. Thus it is still necessary to control the isolation of degrees of freefom (d.f.). Up to now. we have understood that a gauge group transforming N-potential fields generates a minimum of (4N − 1) degrees of freedom. A procedure to systematize the d.f. localization is by splitting the canonical momenta eq.(18) in symmetric and antisymmetric pieces

$$\Pi^{\mu}_{GI} = \Pi^{\mu,A}_{GI} + \Pi^{\mu,S}_{GI} \tag{36}$$

Observe that eq.(36) depends on free coefficients. This means that such coefficients may take any value without violating gauge invariance. From this property, the concept of constraint of circumstances is developed. For instance. a field $G_{\mu I}$ will carry only transverse d.f. if. through a convenient choice of the free coefficients. $\Pi^{\mu,S}_{GI}$ is taken to be zero.

Lagrangian eq.(13) is made of some momenta that are not independent variables. This fact can be identified through the Hessian for the D_μ-sector. As we know, classicaly it means that the accelerations will not be uniquely determined by positions and velocities. However. such an extended model also contains another aspect about its singular nature to be studied. This means considering the Jacobian of the transformation (q, $\dot{q}$) $\longrightarrow$ (q, p) for each field separately. The specific constraint associated to each potential field is determined through

$$W^{\mu\nu}_{G_I} = \frac{\partial^2 \mathcal{L}'}{\partial(\partial_0 G_{\mu I})\partial(\partial_o G_{\nu I})} \tag{37}$$

Thus eq.(37) contains the meaning of identifying the existence of the so-called constraint of circumstances that involves each potential field. Through an engineering offered by the presence of such free coefficients, we can manipulate

$$\Gamma_1 = \Pi^0_{G_I} \simeq 0 \tag{38}$$

Eq.(38) is referred to as constraint of circumstance. For instance. in the case of the non-interacting $U(1)$ case. the Hessian matrix eq.(37) for each $G_{\mu I}$-field is given by

$$W^{\mu\nu}_{G_I} = g^{\mu\nu} - g^{\mu 0} g^{\nu 0} \tag{39}$$

Then note that it verifies eq.(38). The respective secondary constraint to eq.(38) is determined by the consistency condition which gives

$$\Lambda_2 = 2\Big(\partial_i \Pi^i_{G_I} - C\delta_{IJ}\Box G^0_J\Big) - M^2 G_{0J} \simeq 0 \tag{40}$$

eqs.(38) and (40) form a second class constraint. Observe that eq.(40) already fixes G^0_I. Furthermore eq.(40) can also be restricted to the QED-case (many photons)

$$\Gamma_3 = \partial_i \Pi^i_{G_I} \simeq 0 \tag{41}$$

For this it is necessary to demonstrate the existence of transformations which are explicitly controlling such arbitrariness that some components of the quadrivector $G_{\mu I}$ are carrying. For this test we will choose the radiation gauge. From eq.(31) one gets the existence of two parameters

$$\Lambda_I = -\int_0^{x^0} dt G^0_i \tag{42}$$

and

$$\bar{\Lambda}_I = \int dy^3 \frac{1}{4\Pi|\vec{x} - \vec{y}|} \vec{\nabla}_y \vec{G}_I(x^0; x) \tag{43}$$

that together with the equation

$$\nabla \cdot G_I = 0 \tag{44}$$

(satisfied for restricted situations) conduct the following transformation

$$(G^0_I, \vec{G}^0_I) \xrightarrow{\Gamma_I} (G^0_I = 0, \vec{G}'_I) \xrightarrow{\Gamma_I} (G''^0_I = 0, \vec{\nabla} \cdot \vec{G}''_I = 0) \tag{45}$$

Thus, eq.(43) completes the sufficient conditions for assuming the specific set of secondary constraints that each field can carry. The last two are

$$\Gamma_3 = G_I^0 \simeq 0 \tag{46}$$

and

$$\Gamma_4 = \vec{\nabla} \cdot \vec{G}_I \simeq 0 \tag{47}$$

Consequently. the Dirac brackets are computed and the result is as already expected

$$\left\{\Pi_G^\mu(x), G_I^\nu(y)\right\}^* = (g^{\mu\nu} - g^{\mu 0} g^{\nu 0})\delta^3(\vec{x}-\vec{y}) + \partial_\mu \partial_\nu \frac{1}{4\Pi|\vec{x}-\vec{y}|} \tag{48}$$

and

$$\left\{\Pi_{G_I}(x), \Pi_{G_I}(y)\right\}^* = \left\{G_I(x), G_I(y)\right\}^* = 0 \tag{49}$$

Thus. eq.(41) contains non-physical aspects to be considered. Although these transformations provide just one gauge-fixing term that in principle should be involving all $G_{\mu I}$ fields. the theory contains more than one procedure for taking d.f. Considering that a necessary condition to eliminate one d.f.is to construct its correspondent constraint. this case where the gauge group is manifested by including more fields realizes that such task now can be organized by two qualities of constraints. In the first one, equally to QED, the symmetry constraint naturally eliminates the same nurnber of degrees of freedom. However it does not localize on each field such d.f. are subtracted. Nevertheless, the theory contains more information about its spectrum to be analysed. This means that it contains circumstances, relatively to gauge invariance, that generate a second type of constraint to be used. Eqs. (38) and (40), (47) and (48) are conditions for isolating the d.f. of each physical field. Note that they are not imrnediately connected. For instance. subtransformations as eq.(43) suffer restrictions as eq.(44).

To conclude. we should stress that. depending on particular situations for the canonical variables. this model describes systerns of photons interacting with massive vector fields and scalar photons. photons along with massive vectors or simply N-interacting photons.

4. HAMILTONIAN COVARIANCE

In order to prove that the proposed model is relativistically invariant, when expressed in the Hamiltonian form eq(25), we are going to follow (ref.4). For simplicity. the proof will be made in terms of the constructor fields. where the set of constraints is simpler and easier to manipulate. Thus the Hamiltonian is given by

$$\mathcal{H}_C = \mathcal{H}_{Free} + \mathcal{H}_I \tag{50}$$

where

$$\begin{aligned}\mathcal{H}_{Free} = {} & a\Pi_x^{it}\beta^{-1}\Pi_x^i - \Pi_x^{it}\beta^{-1}b\Pi_D^i + (1/4a)\Pi_D^i\big(-1 + b^t\beta^{-1}b\big)\Pi_D^i \\ & + (1/4)\Pi_x^{0t}\big(D+E+F\big)^{-1}\Pi_x^0 - \Pi_x^{0t}\big(D+E+F\big)^{-1}F\partial_i X^i \\ & - \Pi_x^{it}\beta^{-1}\big(bb^t - 4a(C-E)\big)\partial_i X^0 - \Pi_D^i\partial_i D^0 \\ & + (\partial_i X^i)^t\big[F\big(D+E+F\big)^{-1}F\big]\partial_i X^i \\ & + 2\Pi_D^i b^t\beta^{-1}\big(E+D\big)\partial_i X^0 - \vartheta_1 - \partial_i X^{0t}\eta\partial_i X^0\end{aligned} \tag{51}$$

with

$$\begin{aligned}\eta = {} & -4a(e-D)\beta^{-1}bb^t\beta^{-1}(E+D) + 2(bb^t - 4a(C-E))\beta^{-1}bb^t\beta^{-1}(E+D) + \\ & - 2bb^t\beta^{-1}(E+D) - (bb^t - 4a(C-E))\beta^{-1}(C+D)\beta^{-1}(bb^t - 4a(C-E)) + \\ & + 2(bb^t - 4a(C-E))\beta^{-1}(C-E) - (C-D)\end{aligned} \tag{52}$$

and

$$\begin{aligned}\vartheta_1 = {} & \big[\partial_i D_j\partial^i D^j - \partial_i D_j\partial^j D^i\big] + b_\alpha\big[\partial^i X^j\big(\partial_i D_j - \partial_j D_i\big)\big] \\ & + \partial_i X_j^t C(\partial^i X^j - \partial^j X^i) + \partial_i X_j^t D\partial^i X^j \\ & + \partial_i X_j^t E\partial^j X^i + \partial_i X^{it} F\partial_j X^j + \partial_i X_i^t(D+E)\partial^i X^i\end{aligned} \tag{53}$$

Eq.(51) follows the second class set of constraints

$$\begin{aligned}\lambda_1 &= \Pi_D^0 \simeq 0\\ \lambda_2 &= \partial_i \Pi_D^i \simeq 0\\ \lambda_3 &= D^0 \simeq 0\\ \lambda_4 &= \partial_i D^i \simeq 0\end{aligned} \tag{54}$$

The proof of covariance will be made in two stages. In the first one we will consider the equivalence between the canonical quantization method and the functional path integral for the Hamiltonian without the interaction term. In the second step, this equivalence is shown directly for the interaction term due the presence of a space-time derivative in $\mathcal{L}_I$ eq.(3).

So. considering that the functional phase space has redundant constrained paths. the functional integral of departure is

$$< out|S|in >= \int \prod_x \prod_{\mu=0}^{4} \mathcal{D}D_\mu \mathcal{D}X_\mu \mathcal{D}\Pi_x^\mu \mathcal{D}\Pi_D^i \;\; det\,|\partial_i\partial_i\delta^3(\vec{x}-\vec{y})|\delta(\lambda_4)e^{iS} \tag{55}$$

where

$$S = \int dx^4 \left[\Pi_D^i \dot{D}^i + \Pi_x^{it}\dot{X}^i + \Pi_x^{0t}\dot{X}^0 - \mathcal{H}_{Free}\right] \tag{56}$$

It is well-known[4] that the presence of the genuine second class λ_4 in the rneasure through the δ-function does not prevent us from constructing a manifestly relativistic functional integral. Since we have a Gaussian form for the Π_x^0 we can immediately perform the integration. Similarly for the integration over Π_x^i. Then. one is left with

$$< out|S|in >= \int \prod_x \prod_{\mu=0}^{4} \mathcal{D}D_\mu \mathcal{D}X_\mu e^{iS'}\delta(\lambda_4) \;\; det\,|\partial_i\partial_i\delta^3(\vec{x}-\vec{y})|\mathcal{D}\Pi_D^i \tag{57}$$

where

$$S' = -i\int dx^4[1/2\Pi_D^i(-1/2a)\Pi_D^i + B_2\Pi_D^i + C_2] \tag{58}$$

with

$$-b_2 = (1/2a)b^t\dot{X}^i + (1/2a)b^t\beta^{-1}(bb^t - 4a(C-E))\partial_i X^0 + (\dot{D}^i + \partial_i D^0) - 2b^t\beta^{-1}(E+D)\partial_i X^0$$

$$and - C_2 = \dot{X}^{it}(\beta^{-1}/4a)X^i + X^{it}[(bb^t - 4a(C-E))/a]\partial_i X^0 + (1/4a)\partial_i X^{0t}(bb^t - 4a(C-E))\beta^{-1}(bb^t - 4a(C-E))\partial_i X^0 + \partial_i X^{0t}\eta\partial_i X^0 + \vartheta_1 \quad (59)$$

Finally, integrating over Π_D^i we get the expected result

$$< out|S|in >= \int \prod_x \prod_{\mu=0}^{4} \mathcal{D}D_\mu \mathcal{D}X_\mu \delta(\lambda_4) e^{i\int dx^4 \mathcal{L}_{Kin.}} \quad (60)$$

However. **eq.(60)** only demonstrates that the relativistic invariance **is** not lost for the free **part. In** the case where the interaction term **is** switched on, with

$$\mathcal{L}_I(X_{Free}, X_{Free}) = \dot{X}^{0\alpha}(a_{\alpha\beta\gamma} + b_{\alpha\beta\gamma})X^{0\beta}X^{0\gamma} + b_{\alpha\beta\gamma}X_i^\beta X^{i\gamma}) + a_{\alpha\beta\gamma}[-X^{\alpha i}X^{0\beta}X^{i\gamma} + (\partial_i X^{0\alpha})X^{i\beta}X^{0\gamma} + (\partial_i X_j)^\alpha X^{i\beta}X^{j\gamma}] + b_{\alpha\beta\gamma}\partial_i X^{\alpha i}X_\nu^\beta X^{\gamma\nu} + C_{\alpha\beta\gamma\delta}X^\alpha_{\ \mu}X^\beta_{\ \nu}X^{\gamma\mu}X^{\delta\nu} \quad (61)$$

the functional covariance is proved through a general statement[5]. **It** requires that the field variables and momenta of H_I must be written in terms of free variables. This prescription is analogous to obtaining the transition amplitude in terms of $\mathcal{L}_I(\partial_\mu XFree, X_{Free})$. This **equivalence** is well known in terms of the functional generator

$$Z[J] =< 0|T\,exp -i[\mathcal{H}_I(X^\alpha_{Free}, \Pi^\alpha_{Free}) - j_\alpha X^\alpha_{Free}]dx^4|0 >= \, =< 0|\hat{T}\,exp\,i\int [\mathcal{L}_I(X_{Free}, X^\alpha_{Free}) + j_\alpha X^\alpha_{Free}]dx^4|0 > \quad (62)$$

where $\hat{T}$ is the covariant temporal ordering operator which accounts for the fact that the derivatives of the fields are not temporal time-ordered.

Thus considering also the interaction Hamiltonian, H_I, one gets

$$< out|S|in >= \mathcal{N}' \int \prod_x \prod_{\mu=0}^{4} \mathcal{D} D_\mu \mathcal{D} X_\mu \delta(\lambda_4) e^{i \int \mathcal{L}_{Free} + \mathcal{L}(\dot{X}^\alpha_{Free}, (X^\alpha_{Free})) dx^4} \tag{63}$$

Eq.(63) shows that eq.(3), when expressed in Hamiltonian form, is relativistically invariant. A similar result can be obtained for a constraint $\Pi^0_{x\alpha} = 0$. Considering the Jacobian of transformation for physical field is absorbed by the normalization constant. we finally obtain

$$< out|S|in >= \int \prod_x \prod_{\mu=0}^{4} \prod_{I=0}^{N} \mathcal{D} G_{\mu I} \delta(\bar{u}_{I0} \partial_i G^i_I) e^{i \int dx \mathcal{L}'} \tag{64}$$

where L!' is given by eq.(13)

5. AN EXTENDED GAUSS LAW

The local Noether Theorem also provides information about the presence of constraints in a quantum field theory. It implies the three following independent equations to supervise the theory

$$\partial_\mu J^\mu = 0 \tag{65}$$

$$u_{I0} \frac{\partial \mathcal{L}'}{\partial \partial_\mu G_{\nu I}} = -J''' \tag{66}$$

and

$$u_{I0} \frac{\partial \mathcal{L}'}{\partial \partial_\mu G_{\nu I}} = 0 \tag{67}$$

where J_μ is the current associated with the one parameter global $U(1)$ phase invariance. Eq. (65) means a conservation law for the symmetry current. while in eq.(66) it appears as the source for the potential fields. However, through eq.(67), a $U(1)$-symmetry constraint emerges. Its strongest condition would be to consider that the fields $G_{\mu I}$ are not dynamical. Nevertheless. it is possible to weaken this condition by reading eq.(67) as an equation connecting different fields. This condition would represent a constraint between the N-fields that are involved.

On the other hand. the most interesting physical consequence to which the local phase transformation leads is in the case eq.(67) is weakened as

$$u_{I0}\frac{\partial \mathcal{L}'}{\partial\partial_\mu G_{\nu I}}\partial_\mu\partial_\nu\alpha(x) = 0 \tag{68}$$

Substituting eq.(67) in eq.(65) one gets

$$\partial^\mu Z_{[\mu\nu]} = J_\nu \tag{69}$$

where $Z_{[\mu\nu]}$ is an antisymmetric tensor. Observe that eq.(69) represents a Gauss law in a covariant form. Its recognition as a constraint is due the fact that it is not originated from the equations of mstion but from gauge symmetry. Rewriting in terms of canonical momenta such an extended Gauss law is

$$u_{I0}\partial_i\Pi^i_{G_I} = 0 \tag{70}$$

This. through an appropriate utilization of the gauge group, means assuming the possibiiity of introducing more than one potential field. the picture for the $U(1)$-constraint is enlarged. This means that we can surpass the common impression where the experimental Coulomb-Cavendish law is an obligation from gauge symmetry. Eq.(70) shows the existence of a type of Gauss law but it does not necessarily imply the relation

$$\vec{\nabla}\cdot\vec{E} = \rho \tag{71}$$

where $\vec{E}$ is the Maxwell electric field. Eq. (71) is only an equation of motion.

As it is well-known. the gauss law has topologiçal implications, as it is sensitive to field configurations with non-trivial topology. For this reason, in its operational form it has the strong consequence of not allowing the spontaneous breaking of a gauge symmetry. However. whenever weakened by the condition of being valid only on the sector of physical states. the spontaneous breakdown of the local symmetry can take place. In our case. the inclusion of more potentials may circumstancially weaken the Gauss law in its original operatorial form and this is a fact of direct consequence in the formal study of spontaneous breaking of an internal local symmetry.

6. CONCLUSIONS

The possibilities and consequences obtained from the transformations eq.(1) are under development. The strategy of this work was to study physical situations such as constraints, degrees of freedom. Gauss law and Hamiltonian covariance through the classical field viewpoint.

Reflecting that such an introduction of more potential fields is just an extension to the usual case, it becomes intuitive to expect that such new properties that are being built up must appear with boundary conditions to the usual case. For instance. the velocity for the D_μ-sector does not change. Similarly, the discussion about theory constraints emerges. Then two types of constraints are developed. A first one. called symmetry constraint. represents the contact with the usual case. However. with the condition that it is not more acting over just one field. Thus. in order to isolate the dynamical variables of a given field, the theory was also able to provide a second type of constraint. It was called constraint of circumstances. It is basically an engineering mechanism that the theory propitiates through the presence of free coefficients. This mechanism is a regulator for the dynamical variables associated to each involved field. It breaks equal time commutation relations and requires the development of a Dirac bracket for each field separately.

Another aspect to control in this extended model is about the degrees of freedom. Eq.(39) shows that theory contains fields rotating with different weights u_{I0} but under the same parameter $\alpha(x)$. This means that there is just one gauge fixing. Thus symmetry takes at least one d.f. from eq.(13). This fact can also be observed through the Ward Identities. There. just one non-determined longitudinal d.f. is frozen. Therefore we notice again that the introduction of more fields respects the old situation, however globally. Thus the canonical momenta $\Pi^0_{G_I}$ are not expected to be naturally zero. For this, it is necessary to take into account the free coefficients. This means that the theory contains only implicit information on how to control ingredients as first class constraints and the auxiliary conditions for specifying the d.f. Generally speaking, the spectrum contains a perspective of choice.

A third test was to verify consistency between the Hamiltonian and Lagrangian formalisms. The Hamiltonian is not a Lorentz scalar, and this brings about the question of whether playing with the Lorentz group means only a balance between covariant and cantravariant indices. Thus in order to get credibility with group theory arguments like pointing out the involved spin structure, such consistency was necessary. In this work instead of verifying the closure generators algebra we have followed the path integral procedure. Thus a minimum floor exists for possible physical interpretation from this extended model to be assumed.

Finally this work should be concluded by offering some context for debate. An axiomatic approach to defining Gauge Theories is to consider them as theories where the equation

$$\partial_\mu F^{\mu\nu} = j^\nu \tag{72}$$

is obtained as a symmetry constraint. Thus the respective current conservation appears as an identity ($F_{\mu\nu}$ is the Maxwell-field strength). The strongest consequence from this context is that Coulomb's law contains has a non-dynamical origin. This means that gauge theories are generating such law as a definition. Our theme for the debate is that this strong compromise is not required. Considering the viewpoint where the symmetry works as a source for transforming more than one field as eq.(39), the conclusion above may be surpassed. Eq.(70) reexamines this reflex between symmetry and Coulomb's law.

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APPENDIX

This appendix is added in order to make clearer the meaning of the free coefficients. For this purpose we give the velocities for a model involving four potential fields. For the sake of simplicity, symmetric field-strengths and interaction terms are avoided.

The velocity corresponding to the final $G^i_{(1)}$ is

$$\dot{G}^i_{(1)} = u_{01}\dot{D}^i + u_{11}\dot{X}^i_1 + u_{21}\dot{X}^i_2 + u_{31}\dot{X}^i_3 \qquad (A1)$$

where

$$\dot{X}^i_1 = (-R_{22})/(2aR_{11})$$

with

$$R_{22} = 2R_{11}a\partial_i X^0_1 - R_{14}\Pi^i_D - 2aR_1R_{15} + R_{16}C_{(12)} - R_{17}C_{23} + 2R_{18}C_{22}$$
$$+ R_{19}C_{32} - 2R_{20}C_{33}b_2 - R_{21}C_{(13)}b_2 - 2a[C^2_{23} + C^2_{32}]\Pi^i_{x^1}$$
$$R_{21} = \Pi^i_{x^2}b_3 - \Pi^i_{x^3}b_2$$
$$R_{20} = \Pi^i_{x^1}b_2 - \Pi^i_{x^2}b_1$$
$$R_{19} = 2\Pi^i_{x^1}b_2b_3 - \Pi^i_{x^2}b_1b_3 - \Pi^i_{x^3}b_1b_2$$
$$R_{18} = 4aC_{33}\Pi^i_{x^1} - \Pi^i_{x^1}(b_3)^2 + \Pi^i_{x^3}b_1b_3$$
$$R_{17} = 4aC_{32}\Pi^i_{x^1} - 2\Pi^i_{x^1}b_2b_3 + \Pi^i_{x^2}b_1b_3 + \Pi^i_{x^3}b_1b_2$$
$$R_{16} = 4aC_{23}\Pi^i_{x^3} - 4aC_{33}\Pi^i_{x^2}(b_3)^2 - \Pi^i_{x^3}b_2b_3$$
$$R_{15} = 2C_{22}\Pi^i_{x^3} - C_{(23)}\Pi^i_{x^2}$$
$$Rir = R_{12}R_{13} - R_1R_{10} + 4aC_{22}C_{33}b_1 - C^2_{(23)}b_1$$
$$R_{13} = C_{(23)}b_3 - 2C_{33}b_2$$
$$R_{12} = C_{(12)}$$
$$R_{11} = 2R_4C_{12} + 2R_5C_{21} - 2R_7C_{13} - 4R_9C_{11} - R_6[C^2_{13} + C^2_{31}] + R_{10}C_{31}b_1 -$$
$$- R_3[C^2_{12} + C^2_{21}] - 4C_{22}C_{33}b^2_1 + (C_{(23)}b_1)^2$$
$$R_{10} = 2C_{22}b_3 - C_{(23)}b_2$$
$$R_9 = R_8C_{23} - R_3C_{22} + a[C^2_{23} + C^2_{32}] - C_{32}b_2b_3 + C_{33}b^2$$
$$R_8 = 2aC_{32} - b_2b_3$$
$$R_7 = R_6C_{31} - 2C_{22}b_1b_3 + C_{(23)}b_1b_2$$
$$R_6 = 4aC_{22} - b^2_2$$

$$R_5 = R_1R_2 - C_{(23)}b_1b_3 + 2C_{33}b_1b_2$$
$$R_4 = R_1R_2 - R_3C_{21} - C_{(23)}b_1b_3 + 2C_{33}b_1b_2$$
$$R_3 = 4aC_{33} - b_3^2$$
$$R_2 = 2aC_{(23)} - b_2b_3$$
$$R_1 = C_{(13)}$$

$$\dot{X}_2^i = (-S_{26})/(2aS_{11}$$

with

$$S_{26} = 2aS_{11}\partial_i X_2^0 - S_{17}\Pi_D^i + 2aS_{18}C_{31} + S_{19}C_{(12)} - 2aS_{20}C_{13}$$
$$- 2S_{21}C_{11} - 4aS_{22}C_{11} - S_{23}C_{(13)} + 2S_{24}C_{33}b_1 - S_{25}C_{(23)}b_1 +$$
$$- 2aC_{13}^2\Pi_{x^3}^i$$
$$S_{25} = \Pi_{x^1}^i b_3 - \Pi_{x^3}^i b_1$$
$$S_{24} = \Pi_{x^1}^i b_2 - \Pi_{x^2}^i b_1$$
$$S_{23} = \Pi_{x^1}^i b_2b_3 - 2\Pi_{x^2}^i b_1b_3 + \Pi_{x^3}^i b_1b_2$$
$$S_{22} = C_{32}\Pi_{x^3}^i - 2C_{33}\Pi_{x^2}^i$$
$$S_{21} = 2aC_{23}\Pi_{x^3}^i + \Pi_{x^2}^i b_3^2 - \Pi_{x^3}^i b_2b_3$$
$$S_{20} = 2C_{31}\Pi_{x^2}^i - C_{(23)}\Pi_{x^1}^i$$
$$S_{19} = 2aC_{(13)}\Pi_{x^3}^i - 4aC_{33}\Pi_{x^1}^i + \Pi_{x^1}^i(b_3)^2 - \Pi_{x^3}^i b_1b_3$$
$$S_{18} = S_{16}\Pi_{x^1}^i - C_{31}\Pi_{x^2}^i$$
$$S_{17} = S_{12}S_{13} - S_{14}S_{13} - 2S_{15}C_{11} + S_{16}C_{31}b_1 - [C_{(13)}]^2 b_2$$
$$S_{16} = C_{(23)}$$
$$S_{15} = C_{(23)}b_3 - 2C_{33}b_2$$
$$S_{14} = 2C_{31}b_2 - 2C_{(23)}b_1$$
$$S_{13} = C_{(13)}b_3 - C_{33}b_1$$
$$S_{12} = C_{(12)}$$
$$S_{11} = 2S_4C_{12} + 2S_5C_{21} - 2S_7C_{13} - 4S_9C_{11} - S_6[C_{13}^2 + C_{31}^2]$$

$$+ 2S_{10}C_{31}b_1 - S_3[C_{12}^2 + C_{21}^2] - 4C_{22}C_{33}(b_1)^2 + [C_{(23)}b_1]^2$$

$$S_{10} = 2C_{22}b_3 - C_{(23)}b_2$$

$$S_9 = S_8C_{23} - S_3C_{22} + a[C_{23}^2 + C_{32}^2] - C_{32}b_2b_3 + C_{33}b_2^2$$

$$S_8 = 2aC_{32} - b_2b_3$$

$$S_7 = S_6C_{31} - 2C_{22}b_1b_3 + C_{(23)}b_1b_2$$

$$S_6 = 4aC_{22} - b_2$$

$$S_5 = S_1S_2 - C_{(23)}b_1b_3 + 2C_{33}b_1b_2$$

$$S_4 = S_1S_2 - S_3C_{21} - C_{(23)}b_1b_3 + 2C_{33}b_1b_2$$

$$S_3 = 4aC_{33} - b_3^2$$

$$S_2 = 2aC_{(23)} - b_2b_3$$

$$S_1 = C_{(13)}$$

$$\dot{X}_3^i = (-T_{23})/(2aT_{11})$$

with

$$T_{23} = 2aT_{11}\partial_i X_3^0 + T_{14}\Pi_D + 2aT_{16}C_{21} - 2aT_{17}C_{12} - T_{18}C_{13}$$

$$+ 2T_{19}C_{11} - T_{20}C_{(21)} - T_{21}C_{(23)}b_1 + 2T_{22}C_{22}b_1 - 2aC_{12}^2\Pi^i_{x^3}$$

$$T_{22} = \Pi^i_{x^1}b_3 - \Pi^i_{x^3}b_1$$

$$T_{21} = \Pi^i_{x^1}b_2 - \Pi^i_{x^2}b_1$$

$$T_{20} = \Pi^i_{x^1}b_2b_3 + \Pi^i_{x^2}b_1b_3 - 2\Pi^i_{x^3}b_1b_2$$

$$T_{19} = 4aC_{22}\Pi^i_{x^3} - 2aC_{(23)}\Pi^i_{x^2} + \Pi^i_{x^2}b_2^ib_3 - \Pi^i_{x^3}b_2^2$$

$$T_{18} = 4aC_{22}\Pi^i_{x^1}b_3 - \Pi^i_{x^1}(b_2)^2 + \Pi^i_{x^2}b_1b_2$$

$$T_{17} = 2C_{21}\Pi^i_{x^3} - C_{(13)}\Pi^i_{x^2} - C_{(23)}\Pi^i_{x^1}$$

$$T_{16} = T_{15} - C_{21}\Pi^i_{x^3}$$

$$T_{15} = C_{(13)}\Pi^i_{x^2} + C_{(23)}\Pi^i_{x^1}$$

$$T_{14} = T_{12}C_{12} - T_{13}C_{21} - 2T_{10}C_{11} + [C_{12}^2 + C_{21}^2]b_3 + 2C_{(13)}C_{22}b_1$$

$$T_{13} = C_{(13)}b_2 + C_{(23)}b_1$$

$$T_{12} = 2C_{21}b_3 - C_{(13)}b_2 - C_{(23)}b_1$$
$$T_{11} = 2T_4C_{12} + 2T_5C_{21} - 2T_7C_{13} - 4T_9C_{11} - T_6[C_{13}^2 + C_{31}^2] + 2T_{10}C_{31}b_1 - T_3[C_{12}^2 + C_{21}^2] - 4C_{22}C_{33}b_1^2 + [C_{23}^2 + C_{32}^2]b_1^2$$
$$T_{10} = 2C_{22}b_3 - C_{(23)}b_2$$
$$T_9 = T_8C_{23} - T_3C_{22} + a[C_{23}^2 + C_{32}^2] - C_{32}b_2b_3 + C_{33}b^2$$
$$T_8 = 2aC_{32} - b_2b_3$$
$$T_7 = T_6C_{31} - 2C_{22}b_1b_3 + C_{(23)}b_1b_2$$
$$T_6 = 4aC_{22} - b_2$$
$$T_5 = T_1T_2 - C_{(23)}b_1b_2 + 2C_{33}b_1b_2$$
$$T_4 = T_1T_2 - T_3C_{21} - C_{(23)}b_1b_3 - 2C_{33}b_1b_2$$
$$T_3 = 4aC_{33} - b_3^2$$
$$T_2 = 2aC_{(23)} - b_2b_3$$
$$T_1 = C_{(13)}$$

and

$$\dot{D}^i = -[2a\partial_i D^0 + \Pi_D^i + b_1(\dot{X}_1 + \partial_i X_1^0) + b_2(\dot{X}_2 + \partial_i X_2^0) + b_3(\dot{X}_3 + \partial_i X_3^0)]$$

For the fields $G_{(2)}^i, G_{(3)}^i$ and $G_{(4)}^i$, the same linear combination as eq.(A1) are obtained by just changing the respective coefficients.

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Resumo

Dois tipos de vlnculos regulados pela invariância de gauge são identificados para sistemas envolvendo mais de um potencial vetor que se transformam sob um único grupo. São eles os vlnculos da simetria e da circunstância. Uma lei de Gauss estendida e a covariância do Hamiltoniano são também estudados.