Revista Brasileira de Física, Vol. 18, nº 3, 1988

Laser Cavity with Absorptive Medium and Output Coupling

J.C. PENAFORTE and B. BASEIA
Departamento de Física, Universidade Federal da Paraíba, 58000, João Pessoa, PB, Brasil

Recebido em 9 de dezembro de 1987

Abstract The modes, lineshape and linewidth of an optical cavity having an absorptive medium plus output coupling, are investigated. While standard procedures include the losses after the normal modes of the free-field have been defined, in the present approach the normal modes already contain the field losses. It is shown that the normal of the lossv cavity are mainly determined by the high reflecting dielectric window which couples the optical cavity to the outside world and the contributions to the linewidth, due to both simultaneous loss rnechanisms, are not additive. The development of a laser theory using this lossy cavity is discussed.

1. INTRODUCTION

A basic question which appears in laser theory is concerned with the model of optical cavity. Laser theories for several types of optical cavities have been developed. Two rnain models are the gas laser cavity (where He-Ne laser is a typical example) and the cavity as a long dielectric slab (Rubi laser is typical example of this).

Whataver be the case, conventional semiclassical and quanturn treatments[1] simulate the radiation loss by including artificial mechanisms, since they assume optical cavities to have ideally reflecting mirrors. So, strictly speaking, these approaches give no information on the radiation field outside the laser vacity, since the coupling of the optical cavity to the outside region requires the presence of a transmitting window.

On the other hand, more realistic models of laser cavities have been investigated. Recently[2], the modes of a laser cavity composed by a 'long' dielectric slab with an absorptive medium have been investigated. Also recently[3], an alternative rnodel has been introduced, which deals with laser transrnission loss, through a semitransparent mirror simulated by a thin dielectric slab[4] placed at one end of the optical cavity. However, in all these (phenomenological or realistic) models the radiation loss is assumed to be small so that truncation in perturbative series and some algebraic manipulations leading to analytic solutions are allowable.

The conventional laser theory - while artificially simulating the radiation loss due to the laser beam that emerges from the laser cavity - is able to attain some results, such as the laser threshold and the energy balance in a proper way. The correctness of this approach is supported by the fact that different (artificial) loss mechanisms lead to equivalent results[5]. Thus, the usual procedure is justified by claiming an isensitive nature to details of the loss mechanism. Although it simplified the treatment and leads to some correct results, the conventional procedure is not efficient up to ultimate consequences since some questions are left unanswered, as for example: why is the laser line so narrow[6]? Does the laser threshold correspond to a phase transition[7]? So, this inequivalence of models (the realistic and the conventional) leads one to investigate the influence on the normal modes of the laser cavity when the two (simultaneous) loss mechanisms are present in the lossy cavity. Here the absorbing medium is neither a simulation of the transmitted light beam, as occurs in the serniclassical treatment[6], nor is it simulated by ficticious loss-reservoirs, as is the case in the quantum theory[1,5] but instead it is assumed as being a real loss-mechanism, in addition to the loss due to the transmission throughout the window.

Contrary to usual treatments which deal with 'bare modes', i.e, the modes of the free-field, in this paper we work with a kind of 'dressed-modes', namely, the cavity modes in presence of an absorptive medium. We also investigate the composite lineshape and linewidth that result from the two processes (transmission loss and absorption loss). In order to attain mathematical simplifications, we consider the loss mechanisms as having small magnitude[6].

In section 2, we summarize the results for the normal mode spectrum of a cavity having a thin dielectric slab acting as a semitransparent mirror in the absence of an absorbing medium. The reader will find further details in ref,3. In section 3, the absorbing medium is included inside the cavity, in order to investigate the influence of both effects on the normal modes of the system. Section 4 contains the concluding remarks.

2. EMPTY CAVITY WITH OUTPUT COUPLING

A. Cavity Model

The model consists of two parallel plates one of which is totally reflecting, located at $z = \$$ and the other is a semitransparent plate (a thin dielectric slab) located at $z = 0$, having very small thickness with large dielectric constant η. The model is analytically described by a dielectric permittivity as

$$\varepsilon(z) = \varepsilon_0 [1 + \eta\delta(z)] \ , \tag{1}$$

where η determines the transparency of the window, E is the vacuum permitivitty and $\delta(z)$ is the Dirac δ-function.

B. Normal Mode Spectrum

The normal field modes $U_k(z)$ are found by solving Maxwell's equations with proper boundary conditions. The details of the calculation are given in ref.3. For monochromatic waves of circular frequency ω_k,

$$E(z,t) = U_k(z)e^{-i\omega_k t} \ , \tag{2}$$

Maxwell's equation give a wave-equation governing the above electromagnetic field in the entire cavity, z E $(-\infty$, R., as

$$d^2U_k(z)/dz^2 + \mu_0\varepsilon_0 [1 + \eta\delta(z)]\omega_k^2 U_k(z) = 0; \quad \omega_k = ck \tag{3}$$

subject to the boundary conditions $U_k(z) = 0$, at $z = R$, and

$$U_k(0^+) = U_k(0^-) \tag{4}$$

$$U_k'(0^+) - U_k'(0^-) = -\eta k^2 U_k(0) \tag{5}$$

where the discontinuity in eq. (5) arises from the Dirac δ-function in eq. (3).

The solutions of eq. (3) have the form

$$U_k(z) = \begin{cases} L_k \sin[k(z-\ell)] , & 0 \leq z \leq \ell \\ (2/\pi)^{1/2} \sin(kz-\delta_k) , & -\infty < z \leq 0 . \end{cases} \tag{6}$$

Applying the boundary conditions to these solutions we get

$$\sin \delta_k = (\pi/2)^{1/2} L_k \sin k\ell \tag{7}$$

$$(\pi/2)L_k^2 = (1 + \Lambda^2 \sin^2 k\ell - \Lambda \sin 2k\ell)^{-1} \tag{8}$$

where

$$\Lambda = \Lambda(k) = \eta k \quad . \tag{9}$$

Setting $t = \tan(k\ell)$ in eqs. (7), (8) we find, after some algebraic manipulations

$$(\pi/2)L_k^2 = (1 + t^2)/[t^2 + \Lambda^2(t - 1/\Lambda)^2] \; . \tag{10}$$

The function L_k^2 has peaks at the points

$$t_{0n} = \tan(k_{0n}\ell) = (\eta k_{0n})^{-1} = \Lambda_{0n}^{-1} \ll 1 \tag{11}$$

where $\Lambda_{0n} \gg 1$ means that one assumes a semitransparent window having low transrnission. The peak values of L_k^2 are given by

$$(\pi/2)[L_k(t_{0n})]^2 = 1 + \Lambda_{0n}^2 \simeq \Lambda_{0n}^2 \quad , \tag{12}$$

with the peak half-widths

$$\Delta t_n = 1/\Lambda_{0n}^2 \simeq \ell \Delta k_n \quad . \tag{13}$$

By making suitable approximatíons in order to solve eq. (11) we find (see ref. 3)), in the neighbourhood of

$$\omega_k = \omega_{0n} = (n\pi + \Lambda_{0n}^{-1})c/\ell$$

$$L_k^2 \simeq M_k^2 = (1/\pi)\Gamma_n^2 \Lambda_{0n}^2/\left[(\omega_k - \omega_{0n})^2 + \Gamma_n^2\right] = \frac{c}{\pi}\frac{\Gamma_n}{(\Delta\omega_k)^2 + \Gamma_n^2} \qquad (14)$$

where ω_{0n} is the resonant frequency associated with the n-th Fox-Li quasimode ($n \sim 10^6 >> 1$ in the optical domain) and

$$\Gamma_n = c/\ell\Lambda_{0n}^2 \qquad (15)$$

plays the role of the bandwidth of the laser cavity, with Λ_{0n} depending on η

$$\Lambda_{0n} \simeq n\pi\eta/\ell >> 1 . \qquad (16)$$

The expression for M_k^2 shows a Lorentzian lineshape with linewidth Γ_n which depends on the window transparency according to eq.(15).

Semiclassical[3] and quantum[8] laser theories using this lossy cavity have been recently developed and the reader is referred to refs. 3 and 8 for further details.

3. CAVITY WITH ABSORPTIVE MEDIUM PLUS OUTPUT COUPLING

A - Cavity Model

Let us consider the optical cavity model, as previously treated in section 2, with all the intracavity region (the space $z \in [0, \ell]$) filled with a homogeneously distributed absorbing medium. So, instead of eq. (1), the cavity model is analytically described by the dielectric permittivity

$$\varepsilon(z) = \varepsilon_0\left[\theta(-z) + \eta\delta(z) + \varepsilon\theta(z)\theta(\ell-z)\right] , \qquad (17)$$

where ε_0, $\delta(z)$, η have the same significance as in section 2, $\theta(z)$ is the Heaviside step-function and ε is the dielectric constant of the absorbing medium.

B - Normal Mode Spectrum

Next, we consider the normal modes for the combined system constituted by an optical cavity having an absorptive medium plus output coupling to the outside world. The electromagnetic field in the entire optical cavity, defined in the region $z \in (-\infty, \ell]$, is governed by the wave equation

$$\frac{\partial^2 E(z,t)}{\partial z^2} - \mu_0 \varepsilon(z) \frac{\partial^2 E(z,t)}{\partial t^2} = 0 , \tag{18}$$

where $\varepsilon(z)$ is given in eq.(17).

The radiation field can be written as an expansion in terms of normal modes of the 'universe' as

$$E(z,t) = \int_0^\infty U_k(z)\, e^{-i\omega_k t}\, dk \tag{19}$$

and its substitution into eq. (18) leads to

$$\frac{d^2 U_k(z)}{dz^2} + \mu_0 \varepsilon(z) \omega_k^2 U_k(z) = 0 , \tag{20}$$

where ω_k is the circular frequency for monochromatic waves in eq. (19).

The normal mode functions $U_k(z)$ are stationary solutions of eq. (20) subject to the boundary conditions $U_k(z) = 0$ at $z = \ell$ and

$$U_k(0^+) = U_k(0^-) \tag{21}$$

$$U_k'(0^+) - U_k'(0^-) = -\eta k^2 U_k(0) , \tag{22}$$

Eq. (21) means that the electric field is continuous at the window placed at $z = 0$ and eq.(22) shows a discontinuity in the magnetic field related to the 'height' of the dielectric bump, represented by the Dirac δ-function in eq. (17). As can be seen, these houndary conditions are the same as those obtaioed in the previous section (see eqs. (4), (5)).

Thus, considering the two different regions $z \in [0,\ell]$ and $z \in (-\infty,0]$ of the optical cavity, eq. (20) can be written for the internal region as

$$\frac{d^2 U_k^{(i)}(z)}{dz^2} + k_{(i)}^2 U_k^{(i)}(z) = 0 , \tag{23}$$

where[9]

$$k^2_{(i)} = \varepsilon\omega^2_{k(i)}/c^2 \quad ;$$

the label (i) stands for the internal region: $z \in [0,\ell]$. So, eq. (23) describes the field modes inside the cavity and

$$\frac{d^2U_k^{(0)}(z)}{dz^2} + k^2_{(0)}U_k^{(0)}(z) = 0 \quad , \tag{24}$$

with

$$k^2_{(0)} = \omega^2_{k(0)}/c^2 \quad ;$$

the label (0) refers to the outside region, $z \in (-\infty,0]$ and eq. (24) stands for the wave equation governing the field modes outside the cavity.

The normal modes are stationary solutions of Maxwell's wave eqs. (23) and (24) satisfying the boundary conditions (21) and (22), plus the additional condition that $U_k(\ell) = 0$. Under these conditions, we try solutions of eqs. (23) and (24) as

$$\tilde{U}_k(z) = \begin{cases} \tilde{L}_{k(i)} \sin k_{(i)}(z-\ell) & , \ 0 \leq z \leq \ell \\ \tilde{N}_{k(0)} \sin (k_{(0)}z - \delta_k) & , \ -\infty \leq z \leq 0 \end{cases} \tag{25}$$

where the quantities $\tilde{L}_{k(i)}$, $\tilde{N}_{k(\ )}$ and $k_{(i)}$ are complex functions. Applying the boundary conditions (21) and (22) to the solutions (25) we obtain

$$\tilde{M}_k \sin k_{(i)}\ell = \sin \delta_k \tag{26}$$

$$K\tilde{M}_k \cos k_{(i)}\ell - \tilde{M}_k \sin k_{(i)}\ell = \cos \delta_k \tag{27}$$

with $\tilde{M}_k = \tilde{L}_{k(i)}/\tilde{N}_{k(0)}$, $K = k_{(i)}/k_{(0)}$ and $\Lambda = \eta k_{(0)}$, as defined in the previous section (see eq.(9)).

Multiplying eqs. (26) and (27) by its corresponding complex conjugate the expression for the lineshape function $\tilde{M}_k$ follows, after a minor algebra,

$$|\tilde{M}_k|^2 = 1/\Big[(1+\Lambda^2)|\sin k_{(i)}\ell|^2 + K^2|\cos k_{(0)}\ell|^2$$

$$- \Lambda(K\cos k_{(i)}\ell\,\overline{\sin k_{(i)}\ell} + \text{C.C.})\Big] \quad , \tag{28}$$

where the bar and C.C. mean complex conjugate operation.

We set $K = K' + iK''$ and consider the absorptive medium in such a way that $K'' \ll 1$ and the assumption $ck''_{(i)} \simeq \Gamma_n$, which means that the loss due to the absorptive medium is comparable with the loss due to transmission throughout the dielectric window. This latter assumption leads (cf. eq.(15)) to

$$k''_{(i)}\ell \simeq \Gamma_n \ell/c = 1/\Lambda^2_{on} \ll 1 \; . \tag{29}$$

Hence, we may approximate

$$\cosh 2k''_{(i)}\ell \simeq 1 + 2(k''_{(i)}\ell)^2 \tag{30}$$

and, after convenient manipulations, we obtain from eq. *(28)*

$$|\tilde{M}_k|^2 = (1+t^2)/\{\alpha(1+t^2) + [t^2 + \Lambda^2(t-1/\Lambda)^2]\} \; , \tag{31}$$

where the approximation $K' - k'_{(i)}/k_{(0)} \simeq 1$ has been used and

$$t = \tan(k'_{(i)}\ell) \tag{32}$$

with

$$\alpha = (2+\Lambda^2)(k''_{(i)}\ell)^2 \simeq \Lambda^2(k''_{(i)}\ell)^2 \; . \tag{33}$$

It is worthwhile noting that for $k''_{(i)} = 0$ (absence of absorptive medium) eq. (31) recovers the result given in eq. (14) of previous section.

The lineshape function $|\tilde{M}_k|^2$ has peaks at the points

$$t_n = \tan(k'_{(i)}\ell) = 1/\Lambda_n \; ; \quad \Lambda_n \gg 1 \; , \tag{34}$$

which shows that the peaks have their locations unchanged (see eq. (11)), while the peak values of $|\tilde{M}_k|^2$ are now given by

$$|\tilde{M}_k|^2 = (1 + \Lambda_n^2)/|1 + \alpha(1 + \Lambda_n^2)| \ . \tag{35}$$

In the limit a $\to 0$ (i.e., $k''_{(i)} \to 0$) the foregoing result coincides with eq. (12).

Substituting eq. (34) into eq. (31) we find the normalized lineshape function

$$|\tilde{M}_k|^2 = \frac{c}{\pi} \cdot \frac{\tilde{\Gamma}_n}{(\Delta\omega_k)^2 + (\tilde{\Gamma}_n)^2} \ , \tag{36}$$

where the substitution $(t-1/\Lambda)$ in eq. (31) by $\ell(\omega_{kn}-\omega_{0n})/c$ has been done (cf. eqs. A.13, A.14 in ref.3) and the new bandwidth $\tilde{\Gamma}_n$ is given (in terms of the 'old' bandwidth Γ_n) by

$$\tilde{\Gamma}_n = \Gamma_n(1 + \alpha\Lambda_n^2)^{1/2} \tag{37}$$

The foregoing result shows that the mode functions $|\tilde{M}_k|^2$ has a Lorentzian profile with line width $\tilde{\Gamma}_n$ which depends on the window transparency Γ_n and, also, on the loss magnitude of the absorptive medium.

It is interesting to investigate these dependencies in limit cases: firstly, we consider the case $k''_{(i)} = 0$ (absence of the absorptive medium) which gives

$$|\tilde{M}_k|^2 \xrightarrow[\alpha\to 0]{} M_k^2 = \frac{c}{\pi} \frac{\Gamma_n}{(\Delta\omega_k)^2 + \Gamma_n^2} \tag{38}$$

This lineshape function has a Lorentzian profile which means that the line-broadening mechanism is of homogeneous type and the linewidth has a value Γ_n as expected (see eq. (14)). On theother hand, in the limit of an ideally reflecting mirror, i.e., in the limit $\Lambda \to \infty$ the loss due to transmission disappears and eq. (36) gives

$$|M_k|^2 \xrightarrow[\Lambda\to\infty]{} M_k^2 = \frac{c}{\pi} \cdot \frac{ck''_{(i)}}{(\Delta\omega_k)^2 + (ck''_{(i)})^2} \ , \tag{39}$$

which also has a Lorentzian profile and the absorptive medium introduces a homogeneous broadening, with linewidth given by ck''_i.

4. COMMENTS AND CONCLUSION

We have investigated the normal modes of an optical cavity having output coupling plus an absorptive medium. From Maxwell's equations we obtain a fundamental equation which governs the radiation field (eq.(18)). This equation of motion obviously differs from that obtained when we consider the normal modes of an empty optical cavity (see eq. (3)). However, the boundary conditions are not modified by the inclusion of a dissipative medium inside the laser cavity (cf. eqs. (21), (22) and (4), (5)).

The normal modes are given by eq. (25), where the quantities $\tilde{L}_{k(i)}$, $\tilde{N}_{k(0)}$ and $k(i)$ are complex functions. As can be seen, by comparison of eqs. (14) and (36), the imaginary part $k''_{(i)}$ of the wave vector $k_{(i)}$ introduces an 'additional' broadening in the mode linewidths, as expected.

For an absorptive medium having comparable loss in magnitude with that due to transmission throughout the semitransparent mirror, we verify, according to eqs. (11) and (34), that the lineshape function $|\tilde{M}_k|^2$ has peaks at the same positions as those of M_k^2 obtained for the lineshape function of the modes in the empty cavity of section 2. Also, there is a line-broadening when the absorptive medium is included inside the cavity. However, the linewidth $\tilde{\Gamma}_n$ that results from the composition of both loss-mechanisms does not exhibit an additive character (see eq. (37)) and the composite lineshape does not emerge as a true convolution, as one finds for example in the atomic case when two Lorentzian line-broadening mechanisms are present[10].

In the limit $k''_{(i)} \to 0$, which implies the absence of the absorptive medium, we have $z = 0$ and eq. (35) yields $\tilde{\Gamma}_n = \Gamma_n$, as expected. In the limit of a very high value of Λ, which leads to a perfectly reflecting mirror placed at $z = 0$, we obtain $\tilde{\Gamma}_n = ck''_{(n)}$. The intermediate case, where $\Gamma_n = ck''_{(n)}$ (both loss-mechanisms having the same magnitude) yields $\tilde{\Gamma}_n = (2)^{1/2}\,\Gamma_n$, which shows the non-additive character of the two simultaneous Lorentzian loss-mechanisms.

We would like to emphasize that a laser theory including these two simultaneous loss mechanisms (transmission loss and absorption loss) can be developed in a straightforward analogy with that of ref. 3, by defining a new (and classical) collective variable

$$A(t) = \int_0^\infty \tilde{M}_k \, \varepsilon_k(t) dk \quad , \tag{40}$$

where $\tilde{M}_k$ is given by eq. (35) and $\varepsilon_k(t)$ is the slowly varying component of the electric field, or, alternatively, following ref.8 by defining a new (and quantum) collective operator

$$E^-(z,t) = -i \int_0^\infty (\hbar\omega_k/2)^{1/2} \, \tilde{U}_k(z) a_k^+(t) dk \quad , \tag{41}$$

where $\tilde{U}_k(z)$ is given in eq. (25) and $a_k^\dagger(t)$ is the photon creation operator. $E^-(z,t)$ stands for the negative frequency part of the electric field operator.

The above mentioned analogies are supported by the formal identities between the collective variable (operator) of ref.3 (ref.8) and the new collective variable (operator) defined in eq. (40) (eq. (41)).

While the normal modes $U_k(z)$ given in eq. (6) are *bare* modes, i.e., modes of the free-field, the normal modes $\tilde{U}_k(z)$ given by eq. (25) are *dressed-modes*, i.e. , the modes in the presence of an absorptive medium. How to obtain *dressed-modes* of an optical cavity taking into account the presence of a linear active medium, having in mind the development of a laser theory in a cavity with high transmission-loss, is the next step which is under investigation.

The financial support received from by the Conselho Nacional de Desenvolvimento Científico e Tecnológico (CNPq) in acknowledged.

REFERENCES

1. See, e.g., M.Sargent III, M.O.Scully and W.E.Lamb, Jr., *Laser Physics* (Addison-Wesley, Reading Massachusetts, (1974).
2. K.Ujihara, Phys. Rev. *A12*, 148 (1975).
3. B.Baseia and H.M.Nussenzveig, Opt. Acta *31*, 39 (1984).

4. M.B.Spencer and W.E.Lamb, Jr., Phys. Rev. AS, 884 (1972).
5. H.M. Nussenzveig, *Introduction* to *Quantum Optics*, Gordon and Breach, Sc. Publ., N.Work (19731, pg.142.
6. R.Lang, M.O.Scully and W.E.Lamb, Jr., Phys. Rev. A7, 1788 (1973).
7. V.DeGiorgio and M.O.Scully, Phys. Rev. A2, 1170 (1970).
8. J.C.Penaforte and B.Baseia, Phys.Rev. A30, 1401 (1984); Phys.Lett. *107*A, 250 (1985).
9. H.Fröhlich, *Theory of Dielectrics*, Clarendon Press (1958).
10. R.Loudon, The *Quantum Theory of Light*, Oxford University Press (1983), pg.76.

Resumo

Investigamos os modos, de forma de linha e largura, de uma cavidade ótica contendo um meio absorvente e acoplamento externo. Enquanto procedimentos usuais incluem a perda da cavidade *após* estarem definidos os modos de campo livre, neste artigo os modos normais já contêm a perda do campo. Mostramos que esses modos são principalmente determinados pela influência da janela altamente refletora - qug acopla a cavidade, fracamente, com o meio exterior - e as contribuições à largura de linha são influenciadas pelos dois mecanismos de perda, não sendo porém *aditivos*. O desenvolvimento de uma teoria de laser usando essa cavidade ótica é discutido.