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Non-Abelian Spinning Particle in an Einstein-Yang-Mills-Higgs Field

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Abstract Applying the method of moments of Papapetrou, the equations of a non-Abelian charged spinning test particle in an Einstein-Yang-Mills-Higgs field are derived from covariant energy-momentum and charge conservation.

INTROOUCTION

Some time ago we derived' the equations of motion of a non-Abelian spinning test particle in a Yang-Mills field by the method of moments of Papapetrou[2]. In this paper we extend those results to the case of the coupled Einstein-Yang-Mills-Higgs field. The case of the Yang-Mills-Higgs field was treated by Drechsler, Havas and Rosemblum[3] following the procedure first introduced by Mathisson[4]. We generalize that paper's results by superimposing a gravitational field. We shall follow Papapetrou's method of moments of the energy-momentum tensor and of the current, which has been used[5] in the analysis of a pole - dipole charged particle in an Einstein-Maxwell field.

The problem of the gravitational motion of a Yang-Mills particle has been treated by Wospakrik[6]. However, subsidiary conditions for the spin and for the non-Abelian dipole moment tensor were used in the derivation of the equations of motion and one is not sure if these equations are independent of the side conditions. We generalize Wospakrik's paper by superimposing a Higgs field and derive the equations with no subsidiary conditions. We make use of the subsidiary conditions only to simplify the resulting equations.

The method will also yive the volume integral of the energy-momentum tensor and of the current, and of their first moments.

1. THE EQUATIONS OF MOTION

Our starting point is the space-time covariant generalization of the divergence relations given in ref.3 in flat space-time, for the energy-momentum tensor $\tilde{T}^{\alpha\beta}$ and the non-Abelian Yang-Mills current $\vec{\tilde{J}}^{\mu}$ of the system, $\vec{\tilde{\rho}}$ being its Higgs charge density. In terms of the densities $T^{\alpha\beta} = \sqrt{-g}\,\tilde{T}^{\alpha\beta}$, $\vec{J}^{\nu} = \sqrt{-g}\,\vec{\tilde{J}}^{\nu}$ and $\vec{\rho} = \sqrt{-g}\,\vec{\tilde{\rho}}$, the divergence conditions become

$$\partial_{\beta}T^{\alpha\beta} + \Gamma^{\alpha}_{\mu\nu}\,T^{\mu\nu} = \vec{F}^{\alpha}{}_{\beta}\cdot\vec{J}^{\beta} - \vec{\rho}\cdot D^{\alpha}\,\vec{\Phi} \tag{1.1}$$

and

$$\partial_{\alpha}\vec{J}^{\alpha} - b\,\vec{A}_{\alpha}\times\vec{J}^{\alpha} = b\,\vec{\rho}\times\vec{\Phi}\quad , \tag{1.2}$$

where

$$\vec{F}_{\alpha\beta} = \nabla_{\alpha}\vec{A}_{\beta} - \nabla_{\beta}\vec{A}_{\alpha} - b\,\vec{A}_{\alpha}\times\vec{A}_{\beta} \tag{1.3}$$

is the Yang-Mills field tensor expressed in terms of its potentials, b is a dimensionless constant, ∇_{α} is the space-time covariant derivative and

$$D^{\alpha}\,\vec{\Phi} = (\nabla^{\alpha} - b\,\vec{A}^{\alpha})\times\vec{\Phi} \tag{1.4}$$

is the covariant derivative in SU(2) space. Notice that as $\vec{\Phi}$ is a scalar in space-time, $\nabla^{\alpha}\,\vec{\Phi} = \partial^{\alpha}\vec{\Phi}$.

We consider an extended system with reference point $X^{\alpha}(s)$ and with velocity

$$u^{\alpha} = dX^{\alpha}/ds \tag{1.5}$$

where $ds^2 = g_{\alpha\beta}\,dX^{\alpha}\,dX^{\beta}$, and we shall consider moments of $T^{\alpha\beta}$ and of $\vec{J}^{\alpha}$ about X^{α} up to first order. This is a good approximation for systems which are small in comparison with the length scale variation of the external field. By demanding that the dimensions of the system tend to zero around X^{α} at the very end of the calculation, this point will give the world line of our point-like charged spinning particle[7].

Now we proceed as usual[2].

Integrating eq. (1.1) over the three-dimensional space volume of our system, for t = const, we have

$$\frac{d}{dt}\int T^{\alpha o}dV + \int \Gamma^{\alpha}_{\mu\nu}T^{\mu\nu}\,dV = \int (\vec{F}^{\alpha}{}_{\beta}\cdot\vec{j}^{\beta} - \vec{\rho}\cdot D^{\alpha}\vec{\Phi})dV \ . \tag{1.6}$$

We now consider the space integral over our system of the divergencies $\partial_\mu(x^\beta T^{\alpha\mu})$ and $\partial_\mu(x^\beta x^\lambda T^{\alpha\mu})$, which can be calculated by means of eq. (2). We obtain

$$\frac{d}{dt}\int x^\beta T^{\alpha o}dV = \int T^{\alpha\beta}dV + \int x^\beta(-\Gamma^{\alpha}_{\mu\nu}T^{\mu\nu} + \vec{F}^{\alpha}{}_{\mu}\cdot\vec{j}^{\mu} - \vec{\rho}\cdot D^{\alpha}\vec{\Phi})dV \ , \tag{1.7}$$

and

$$\frac{d}{dt}\int x^\beta x^\lambda T^{\alpha o}dV = \int(x^\lambda T^{\alpha\beta} + x^\beta T^{\alpha\lambda})dV + \int x^\beta x^\lambda(-\Gamma^{\alpha}_{\mu\nu}T^{\mu\nu} + \vec{F}^{\alpha}{}_{\mu}\cdot\vec{j}^{\mu} - \vec{\rho}\cdot D^{\alpha}\vec{\Phi})dV \tag{1.8}$$

Next we write

$$x^a = X^\alpha + \delta x^\alpha \ , \tag{1.9}$$

with $X^0 = t$, that is $\delta x^0 = 0$, since all integrals refer to the hyperplane t = constant.

We now substitute eq. (1.9) into eq. (1.7) for x^β and make use of eq. (1.6), then into eq. (1.8) first for x^β and make use of eq. (1.7); afterwards, we do the same for x^λ and make use of the previous one derived from eq. (1.7) involving $\int\delta x^\beta T^{\alpha o}dV$. In this way we obtain the equation

$$\frac{dX^\beta}{dt}\int T^{\alpha o}dV + \frac{d}{dt}\int \delta x^\beta T^{\alpha o}dV = \int T^{\alpha\beta}dV + \int \delta x^\beta(-\Gamma^{\alpha}_{\mu\nu}T^{\mu\nu} + \vec{F}^{\alpha}{}_{\mu}\cdot\vec{j}^{\mu} - \vec{\rho}\cdot D^{\alpha}\vec{\Phi})dV \tag{1.10}$$

and, to first order in δx^α,

$$\frac{dX^\beta}{dt}\int \delta x^\lambda T^{\alpha o}dV + \frac{dX^\lambda}{dt}\int \delta x^\beta T^{\alpha o}dV = \int \delta x^\lambda T^{\alpha\beta}dV + \int \delta x^\beta T^{\alpha\lambda}dV \ . \tag{1.11}$$

We now expand $\Gamma^{\alpha}_{\mu\nu}(x)$, $\vec{F}^{\alpha\beta}(x)$ and $D^{\mu}\vec{\Phi}(x)$ around X^{α},

$$\Gamma^{\alpha}_{\mu\nu}(x) = \Gamma^{\alpha}_{\mu\nu}(X) + \delta x^{\sigma}\partial_{\sigma}\Gamma^{\alpha}_{\mu\nu}(X) + \dots \qquad (1.12)$$

where $\partial_{\sigma} = \partial/\partial X^{\sigma}$, with similar expressions for $\vec{F}^{\alpha\beta}$ and $D^{\alpha}\vec{\Phi}$.

Taking these expansions into eqs. (1.6) and (1.10) we obtain, to first order in δx^{a} and taking the X dependence as implicitly understood,

$$\frac{d}{dt}\int T^{\alpha 0}dV + \Gamma^{\alpha}_{\mu\nu}\int T^{\mu\nu}dV + \partial_{\sigma}\Gamma^{\alpha}_{\mu\nu}\int \delta x^{\sigma}T^{\mu\nu}dV = \vec{F}^{\alpha}{}_{\beta}\cdot\int \vec{j}^{\beta}dV + \partial_{\sigma}\vec{F}^{\alpha}{}_{\beta}\cdot\int \delta x^{\sigma}\vec{j}^{\beta}\,dV - D^{\alpha}\vec{\Phi}\cdot\int\vec{\rho}\,dV - \partial_{\sigma}D^{\alpha}\vec{\Phi}\cdot\int\delta x^{\sigma}\vec{\rho}\,dV \qquad (1.13)$$

and

$$\frac{dX^{\beta}}{dt}\int T^{\alpha 0}dV + \frac{d}{dt}\int \delta x^{\beta}T^{\alpha 0}dV = \int T^{\alpha\beta}dV - \Gamma^{\alpha}_{\mu\nu}\int\delta x^{\beta}T^{\mu\nu}dV + \vec{F}^{\alpha}{}_{\mu}\cdot\int\delta x^{\beta}\vec{j}^{\mu}dV - D^{\alpha}\vec{\Phi}\cdot\int\delta x^{\beta}\vec{\rho}dV\,. \qquad (1.14)$$

We now introduce the notation $(u^{0} = dt/ds)$

$$M^{\alpha\beta} = u^{0}\int T^{\alpha\beta}\,dV \quad , \quad M^{\lambda\alpha\beta} = u^{0}\int\delta x^{\lambda}T^{\alpha\beta}dV \qquad (1.15)$$

$$\vec{N}^{\alpha} = u^{0}\int\vec{j}^{\alpha}\,dV \quad , \quad \vec{N}^{\lambda\alpha} = u^{0}\int\delta x^{\lambda}\vec{j}^{\alpha}dV \qquad (1.16)$$

and

$$\vec{f} = u^{0}\int\vec{\rho}\,dV \quad , \quad \vec{R}^{\alpha} = u^{0}\int\delta x^{\alpha}\vec{\rho}\,dV \qquad (1.17)$$

where $\vec{f}$ and $\vec{R}^{\alpha}$ are the Higgs charge and dipole moment of the system in its rest frame, respectively. Also

$$\vec{q} = \frac{\vec{N}^{0}}{u^{0}} = \int\vec{j}^{0}\,dV \quad , \quad \vec{d}^{\lambda} = \frac{\vec{N}^{\lambda 0}}{u^{0}} = \int\delta x^{\lambda}\vec{j}^{0}\,dV \qquad (1.18)$$

are the Yang-Mills charge and electric-dipole moment of the system respectively. Note that the two quantities in eq. (1.15) are symmetric in a, β and that, as $\delta x^{0} = 0$, we have

$$M^{0\alpha\beta} = 0 \quad , \quad \vec{N}^{0\alpha} = 0 \quad , \quad \vec{F}^{0} = 0 \ . \tag{1.19}$$

Using eqs. (1-15) - (1.18) we can write eqs. (1.13), (1.14) and (1.11) as

$$\frac{d}{ds}\frac{M^{\alpha 0}}{u^0} + \Gamma^{\alpha}_{\mu\nu} M^{\mu\nu} + \partial_\sigma \Gamma^{\alpha}_{\mu\nu} M^{\sigma\mu\nu} = \vec{F}^{\alpha}{}_{\beta}\cdot\vec{N}^{\beta} + \partial_\sigma \vec{F}^{\alpha}{}_{\beta}\cdot\vec{N}^{\sigma\beta} - \vec{f}\cdot\vec{D}^{\alpha}\vec{\Phi} - \vec{R}^{\sigma}\cdot\partial_\sigma D^{\alpha}\vec{\Phi} \tag{1.20}$$

$$\frac{u^\beta}{u^0} M^{\alpha 0} + \frac{d}{ds}\frac{M^{\beta\alpha 0}}{u^0} = M^{\alpha\beta} - \Gamma^{\alpha}_{\mu\nu} M^{\beta\mu\nu} + \vec{F}^{\alpha}{}_{\mu}\cdot\vec{N}^{\beta\mu} - D^{\alpha}\vec{\Phi}\cdot\vec{R}^{\beta} \tag{1.21}$$

and

$$u^\beta M^{\lambda\alpha 0} + u^\lambda M^{\beta\alpha 0} = (M^{\lambda\alpha\beta} + M^{\beta\alpha\lambda}) u^0 \tag{1.22}$$

Before we go on with these equations, we derive those which follow from eq. (1.2). From this equation we can calculate $\partial_\mu (x^\alpha \vec{j}^\mu)$ and $\partial_\mu (x^\alpha x^\beta \vec{j}^\mu)$. Integrating the resulting exprersions, and also eq. (1.2), over our system we obtain

$$\frac{d}{dt}\int \vec{j}^0 \, dV = b\int (\vec{A}_\alpha \times \vec{j}^\alpha + \vec{\rho}\times\vec{\Phi}) \, dV \ , \tag{1.23}$$

$$\frac{d}{dt}\int x^\alpha \vec{j}^0 dV = \int \vec{j}^\alpha \, dV + b\int x^\alpha (\vec{A}_\beta \times \vec{j}^\beta + \vec{\rho}\times\vec{\Phi}) \, dV \quad , \tag{1.24}$$

and

$$\frac{d}{dt}\int x^\alpha x^\beta \vec{j}^0 dV = \int (x^\alpha \vec{j}^\beta + x^\beta \vec{j}^\alpha) dV + b\int x^\alpha x^\beta (\vec{A}_\mu \times \vec{j}^\mu - \vec{\rho}\times\vec{\Phi}) dV \tag{1.25}$$

We now proceed as before. We introduce eqs. (1.9) into (1.24) and make use of eq. (1.23), then into eq. (1.25), first for x^α and afterwards for x^β. Next we expand $\vec{A}_\mu$ around X^a. Keeping only terms to first order in δx^a we obtain, in the notation of eqs. (1.16)-(1.18), the following set of equations analogous to eqs. (1.20)-(1.22):

$$\frac{d\vec{q}}{ds} = b\, \vec{A}_\alpha \times \vec{N}^{\alpha} + b\partial_\alpha \vec{A}_\beta \times \vec{N}^{\alpha\beta} + b\vec{f}\times\vec{\Phi} + b\vec{F}^{\alpha}\times\partial_\alpha\vec{\Phi} . \tag{1.26}$$

$$\vec{q}u^{\alpha} + \frac{d}{ds}\frac{\vec{N}^{\alpha 0}}{u^0} = \vec{N}^{\alpha} + b\vec{A}_{\beta}\times\vec{N}^{\alpha\beta} + b\vec{F}^{\alpha}\times\vec{\Phi} \quad , \tag{1.27}$$

and

$$u^{\alpha}\,\vec{N}^{\beta 0} + u^{\beta}\,\vec{N}^{\alpha 0} = (\vec{N}^{\beta\alpha} + \vec{N}^{\alpha\beta})\,u^0 \quad . \tag{1.28}$$

2. THE CHARGE EQUATION

The second term on the ríght-hand side of eq. (1.26) can be written as

$$\partial_{\alpha}\vec{A}_{\beta} \times \vec{N}^{\alpha\beta} = \partial_{\alpha}\vec{A}_{\beta} \times \vec{J}^{\alpha\beta} + \frac{d\vec{A}_{\beta}}{ds} \times \frac{\vec{N}^{\beta 0}}{u^0} \tag{2.1}$$

where

$$\vec{J}^{\alpha\beta} = \vec{N}^{\alpha\beta} - \frac{u^{\alpha}}{u^0}\vec{N}^{\beta 0} \tag{2.2}$$

By symmetrizing and antisymmetrizing $\vec{N}^{\alpha\beta}$ and by making use of eq. (1.28) we can see that $\vec{J}^{\alpha\beta}$ is an antisymmetric quantity

$$\vec{J}^{\alpha\beta} = \frac{1}{2}\,(\vec{N}^{\alpha\beta} - \vec{N}^{\beta\alpha}) + \frac{1}{2u^0}\,(u^{\beta}\vec{N}^{\alpha 0} - u^{\alpha}\vec{N}^{\beta 0}) \quad . \tag{2.3}$$

From the second relation in eqs. (1.16) and (1.18) we see that

$$\vec{J}^{\alpha\beta} = \frac{u^0}{2}\int(\delta x^{\alpha}\vec{j}^{\beta} - \delta x^{\beta}\vec{j}^{\alpha})dV + \frac{1}{2}\,(u^{\beta}\vec{d}^{\alpha} - u^{\alpha}\vec{d}^{\beta}) \tag{2.4}$$

From eq. (2.2) and the relation in (1.18) for $\vec{d}^{\lambda}$ and in eq. (1.19) for $\vec{N}^{0\alpha}$, it follows that

$$\vec{J}^{\alpha 0} = \vec{N}^{\alpha 0} = u^0\,\vec{d}^{\alpha} \quad . \tag{2.5}$$

Therefore $\vec{J}^{i0}$ is the Yang-Mills electric dipole moment and $\vec{J}^{ij}$ is the usual nonrelativistic magnetic dipole moment of the system in its rest frame. Taking eq. (2.5) into eq. (2.2) we obtain

$$\vec{N}^{\alpha\beta} = \vec{J}^{\alpha\beta} + \frac{u^{\alpha}}{u^0}\vec{J}^{\beta 0} \tag{2.6}$$

As $\vec{J}^{\alpha\beta}$ is antisymmetric, and recalling eq. (1.3) we can write eq. (2.1) as

$$\partial_\alpha \vec{A}_\beta \times \vec{N}^{\alpha\beta} = \frac{d\vec{A}_\beta}{ds} \times \frac{\vec{N}^{\beta 0}}{u^0} + \frac{1}{2} \vec{F}_{\alpha\beta} \times \vec{J}^{\alpha\beta} + \frac{b}{2} (\vec{A}_\alpha \times \vec{A}_\beta) \times \vec{J}^{\alpha\beta} . \tag{2.7}$$

From eqs. (1.27), (2.5) and (2.6) we obtain

$$\vec{N}^\alpha = \vec{Q} u^\alpha + \frac{d}{ds} \frac{\vec{J}^{\alpha 0}}{u^0} - b\vec{A}_\beta \times \vec{J}^{\alpha\beta} - b\vec{R}_\alpha \times \vec{\Phi} \tag{2.8}$$

where

$$\vec{Q} = \vec{q} - \frac{b}{u^0} \vec{A}_\beta \times \vec{J}^{\beta 0} \tag{2.9}$$

is the generalized Yang-Mills charge.

Now we substitute eqs. (2.7) and (2.8) into eq. (1.26). Making use of the Jacobi identity for $\vec{A}_\alpha$, $\vec{A}_\beta$ and $\vec{J}^{\alpha\beta}$, and for $\vec{A}_\alpha$, $\vec{\Phi}$ and $\vec{R}^\beta$, and of the antisymmetric character of $\vec{J}^{\alpha\beta}$ we obtain the following equation for the charge $\vec{Q}$:

$$\frac{D\vec{Q}}{Ds} = b(\frac{1}{2} \vec{F}_{\alpha\beta} \times \vec{J}^{\alpha\beta} + \vec{G} \times \vec{\Phi} + \vec{R}^\alpha \times D_\alpha \vec{\Phi}) \tag{2.10}$$

where

$$\vec{G} = \vec{f} - b\vec{A}_\alpha \times \vec{R}^\alpha \tag{2.11}$$

is the generalized Higgs charge and

$$\frac{D\vec{Q}}{Ds} = u^\alpha D_\alpha \vec{Q} = u^\alpha (\partial_\alpha - b\vec{A}_\alpha \times)\vec{Q} \tag{2.12}$$

is the covariant derivative of the charge $\vec{Q}$.

Eqs. (2.8) and (2.6) give, respectively, the space integral of the current $\vec{J}^\alpha$ and of its first moment in terms of the dipole moment tensors $\vec{J}^{\alpha\beta}$ and $\vec{R}^\alpha$.

3. THE SPIN EOUATION

The definition of rnomenturn arises in a natural way if we look first for the spin equation. For that purpose we interchange a and β in eq. (1.21) and subtract the resulting equation from eq.(1.21). Recalling that $M^{\beta\alpha} = M^{\alpha\beta}$ and using eq.(2.6), we obtain

$$\frac{u^\beta}{u^0}(M^{\alpha 0} - \vec{F}^{\alpha}{}_{\mu}\cdot\vec{J}^{\mu 0}) - \frac{u^\alpha}{u^0}(M^{\beta 0} - \vec{F}^{\beta}{}_{\mu}\cdot\vec{J}^{\mu 0}) + \frac{dS^{\beta\alpha}}{ds} =$$

$$= -\Gamma^{\alpha}_{\mu\nu} M^{\beta\mu\nu} + \vec{F}^{\alpha}{}_{\mu}\cdot\vec{J}^{\beta\mu} - D^{\alpha}\vec{\Phi}.\vec{F}^{\beta} - (\alpha \leftrightarrow \beta) , \qquad (3.1)$$

where

$$S^{\alpha\beta} = \frac{1}{u^0}(M^{\alpha\beta 0} - M^{\beta\alpha 0}) = \int (\delta x^{\alpha} T^{\beta 0} - \delta x^{\beta} T^{\alpha 0}) dV \qquad (3.2)$$

is the spin tensor of our system. Notice that

$$S^{\alpha 0} = \frac{1}{u^0} M^{\alpha 0 0} = \int \delta x^{\alpha} T^{00}\, dV \qquad (3.3)$$

Now use express $M^{\beta\mu\nu}$ in terms of the velocity and spin by the following procedure[2]. Add to eq. *(1.22)* the equation obtained from it by exchanging α and β and substract the one obtained by exchanging *a* and *A*. One obtains, using eq. (3.2),

$$2M^{\lambda\alpha\beta} = u^{\alpha}S^{\lambda\beta} + u^{\beta}S^{\lambda\alpha} + \frac{u^{\lambda}}{u^0}(M^{\alpha\beta 0} + M^{\beta\alpha 0}) \quad . \qquad (3.4)$$

To obtain the latter sum in terms of the spin we put $\alpha = 0$ in eq. (1.22) and use eq. (3.3). We obtain

$$M^{\lambda\beta 0} + M^{\beta\lambda 0} = u^{\beta}S^{\lambda 0} + u^{\lambda}S^{\beta 0} \qquad (3.5)$$

With this result eq. (3.4) becomes

$$2M^{\lambda\alpha\beta} = u^{\alpha}S^{\lambda\beta} + u^{\beta}S^{\lambda\alpha} + \frac{u^{A}}{u^0}(u^{\alpha}S^{\beta 0} + u^{\beta}S^{\alpha 0}) \quad . \qquad (3.6)$$

Substituting this relation into eq. (3.1) we obtain for the spin tensor the equation

$$\frac{DS^{\alpha\beta}}{Ds} = p^{\alpha}u^{\beta} - p^{\beta}u^{\alpha} + \vec{F}^{\alpha}{}_{\mu}\cdot\vec{J}^{\mu\beta} - \vec{F}^{\beta}{}_{\mu}\cdot\vec{J}^{\mu\alpha} + D^{\alpha}\vec{\Phi}\cdot\vec{R}^{\beta} - D^{\beta}\vec{\Phi}\cdot\vec{R}^{\alpha} \qquad (3.7)$$

where

$$p^{\alpha} = \frac{1}{u^0}(M^{\alpha 0} + \Gamma^{\alpha}_{\mu\nu} u^{\mu}S^{\nu 0} - \vec{F}^{\alpha}{}_{\mu}\cdot\vec{J}^{\mu 0}) \qquad (3.8)$$

is the momentum of the system and $DS^{\alpha\beta}/Ds$ is the covariant derivative of $S^{\alpha\beta}$,

$$\frac{DS^{\alpha\beta}}{Ds} = \frac{dS^{\alpha\beta}}{ds} + u^{\mu}\,\Gamma^{\alpha}_{\mu\nu}\,S^{\nu\beta} + u^{\mu}\,\Gamma^{\beta}_{\mu\nu}\,S^{\alpha\nu} \quad . \tag{3.9}$$

4. THE MOMENTUM EQUATION

Let us consider first eq. (1.21). For the second term on its left-hand side we need eq. *(3.6)* with $\beta = 0$, that is,

$$2M^{\lambda\alpha 0} = u^{\alpha}S^{\lambda 0} + u^{0}S^{\lambda\alpha} + u^{\lambda}S^{\alpha 0} \quad . \tag{4.1}$$

Substituting eqs. (2.6), (3.6) and *(4.1)* into eq. (1.21) and using eq. *(3.8)* we obtain

$$M^{\alpha\beta} = p^{\alpha}u^{\beta} + \frac{d}{2ds}\left\{S^{\alpha\beta} + \frac{u^{\alpha}S^{\beta 0} + u^{\beta}S^{\alpha 0}}{u^{0}}\right\} + \Gamma^{\alpha}_{\mu\nu}u^{\mu}S^{\beta\nu}$$

$$+ \vec{F}^{\alpha}{}_{\mu}\cdot\vec{J}^{\mu\beta} + D^{\alpha}\vec{\Phi}\cdot\vec{R}^{\beta} \quad . \tag{4.2}$$

We now introduce eqs. (2.6), (2.8), (3.6) and (4.2) into eq. *(1.20)*. We obtain, for the momentum, the equation

$$\frac{Dp^{\alpha}}{Ds} + (\partial_{\sigma}\,\Gamma^{\alpha}_{\mu\nu} + \Gamma^{\alpha}_{\sigma\lambda}\,\Gamma^{\lambda}_{\mu\nu})u^{\mu}S^{\sigma\nu} = \vec{Q}\vec{F}^{\alpha\beta}u_{\beta} + (\partial_{\sigma}\vec{F}^{\alpha}{}_{\beta} + \Gamma^{\alpha}_{\mu\sigma}\,\vec{F}^{\mu}{}_{\beta}$$

$$- b\vec{A}_{\sigma}\times\vec{F}^{\alpha}{}_{\beta}).\vec{J}^{\sigma\beta} - \vec{R}^{\sigma}.(\partial_{\sigma}D^{\alpha}\vec{\Phi} + \Gamma^{\alpha}_{\sigma\mu}D^{\mu}\vec{\Phi}) - \vec{f}.D^{\alpha}\vec{\Phi} + \vec{R}^{\sigma}.(\vec{F}^{\alpha}{}_{\sigma}\times\vec{\Phi}) . \tag{4.3}$$

As $\vec{J}^{\beta\sigma}$ is antisymmetric we can subtract from the second term of the right-hand side the term $\Gamma^{\mu}_{\beta\sigma}\vec{F}^{\alpha}{}_{\mu}.\vec{J}^{\beta\sigma}$, which is equal to zero. In this way the covariant derivative of $\vec{F}^{\alpha}{}_{\beta}$ appears in this second term. Then we add and subtract the quantity $b\vec{R}^{\sigma}.(\vec{A}_{\sigma}\times D^{\alpha}\vec{\Phi})$ to the third term. The covariant derivative of $D^{\alpha}\vec{\Phi}$ will appear in this term and the fourth one become $-\vec{G}.D^{\alpha}\vec{\Phi}$, where

$$\vec{G} = \vec{f} + b\vec{A}_\sigma \times \vec{R}^\sigma \qquad (4.4)$$

is the generalized Higgs charge. For the last term of eq. (4.3) we use the identity

$$(D^\alpha D^\sigma - D^\sigma D^\alpha)\vec{\Phi} = -b\,\vec{F}^{\alpha\sigma} \times \vec{\Phi} \qquad (4.5)$$

Then eq. (4.3) becomes

$$\frac{Dp^\alpha}{Ds} = \frac{1}{2} R^\alpha{}_{\mu\nu\sigma} u^\mu S^{\nu\sigma} + \vec{Q}\vec{F}^{\alpha\beta} u_\beta - D_\sigma \vec{F}^\alpha{}_\beta . \vec{J}^{\beta\sigma} - \vec{G}.D^\alpha\vec{\Phi} - \vec{R}^\sigma . D^\alpha D_\sigma \vec{\Phi} \qquad (4.6)$$

where

$$R^\alpha{}_{\mu\nu\sigma} = \partial_\sigma \Gamma^\alpha_{\mu\nu} + \Gamma^\alpha_{\sigma\lambda}\Gamma^\lambda_{\mu\nu} - \partial_\nu \Gamma^\alpha_{\mu\sigma} - \Gamma^\alpha_{\nu\lambda}\Gamma^\lambda_{\mu\sigma} \qquad (4.7)$$

is the Riemann curvature tensor.

Eqs. (4.2) and (3.6) give, respectively, the space integral of the energy-momentum tensor and of its first moment in terms of dynamical quantities of our problem.

Notice now that the Yang-Mills field tensor satisfies the equation

$$D_\alpha \vec{F}_{\beta\sigma} + D_\sigma \vec{F}_{\alpha\beta} + D_\beta \vec{F}_{\sigma\alpha} = 0 \qquad (4.8)$$

As $\vec{J}^{\beta\sigma}$ is antisymmetric the momentum equations (4.6) can also be written

$$\frac{Dp^\alpha}{Ds} = \frac{1}{2} R^\alpha{}_{\mu\nu\sigma} u^\mu S^{\nu\sigma} + \vec{Q}.\vec{F}^{\alpha\beta} u_\beta + \frac{1}{2} D^\alpha \vec{F}_{\beta\sigma} . \vec{J}^{\beta\sigma} - \vec{G}.D^\alpha\vec{\Phi} - \vec{R}^\sigma . D^\alpha D_\sigma \vec{\Phi}$$
$$(4.9)$$

Eqs. (2.10), (3.7) and (4.6) are the generalizations ta curved space-time of the corresponding ones derived in ref.3, and generalize also those derived in ref.6 in the absence of the Higgs field ãnd using subsidiary conditions for the spin and the dipole tensors (see eqs. (4.10) and (4.11) beiow). Eqs. (2.10), (3.7) and (4.6) reduceto those derived in ref.1 in flat space-time without the Higgs field, and eqs.(3.7) and (4.6) reduce to those of ref.5 for the usual Einstein-Maxwell field which were derived by the same method.

Considering that $\vec{G}$, $\vec{R}^{\alpha}$ and $\vec{J}^{\alpha\beta}$ are given we have seventeen quantities to be determined: $\vec{Q}$, p^{α}, u^{α} and $S^{\alpha\beta}$. Together with $u^{\alpha}u_{\alpha}=1$, eqs. (2.10), (3.7) and (4.6) add to fourteen equations and therefore we need three more equations to solve for the unknown quantities. These extra subsidiary conditions are related to the particular choice of the reference point X^{α} of the extended system. One possibility is to choose

$$u_{\alpha}S^{\alpha\beta} = 0 \tag{4.10}$$

This is the condition chosen by Mathisson[4] in the case of the motion of a neutral spinning particle[8]. As contraction with u_{β} gives an identity, eq. (4.10) gives only three additional equations which in fact we need to determine our problem. This relation was adopted in ref. 3 and 6 and we shall do the same. We shall also adopt a relation between $\vec{J}^{\alpha\beta}$ and $S^{\alpha\beta}$ which is analogous to the electromagnetic case, that is

$$\vec{J}^{\alpha\beta} = \vec{K}\, S^{\alpha\beta} \tag{4.11}$$

With eq. (4.10) we have

$$u_{\alpha}\,\vec{J}^{\alpha\beta} = 0\ . \tag{4.12}$$

5. DISCUSSION AND SIMPLIFICATION OF THE EQUATIONS OF MOTION

We now analyse the motion of a non-Abelian charged particle which is characterized by a single vector in isospace, the isospin $\vec{\tau}$ of constant magnitude. That is, we adopt the following relations

$$\vec{Q} = Q\,\vec{\tau} \tag{5.1}$$

$$\vec{G} = G\,\vec{\tau} \tag{5.2}$$

$$\vec{R}^{\alpha} = R^{\alpha}\,\vec{\tau} \tag{5.3}$$

$$\vec{J}^{\alpha\beta} = \vec{K}\,S^{\alpha\beta} = K\,S^{\alpha\beta}\,\vec{\tau} \tag{5.4}$$

where $\vec{\tau}$ is a vector of constant magnitude equal to one,

$$\vec{\tau}\cdot\vec{\tau} = 1\ . \tag{5.5}$$

In this situation eq. (2.10) takes the form

$$\dot{\vec{Q}} = b(\frac{1}{2} K S^{\alpha\beta} \vec{F}_{\alpha\beta} - G\vec{\Phi} - R^{\alpha} D_{\alpha}\vec{\Phi}) \times \vec{\tau}, \tag{5.6}$$

where, as will be used from now on, a dot designates covariant derivative.

Contracting eq. (5.6) with eq. (5.1), we see that $\vec{Q}.\dot{\vec{Q}}=0$ or $\vec{Q}^2=Q^2\tau^2=$ = constant. As we have chosen τ^2 a constant we conclude that the Yang-Mills charge has a constant magnitude,

$$Q = \text{constant} . \tag{5.7}$$

Conversely, if one choses eq. (5.7) then τ^2 is a constant which can be normalized to one, that is, eq. (5.5) follows.

With eq. (5.7), eq. (5.6) becomes an equation for $\vec{\tau}$,

$$\frac{D\vec{\tau}}{Ds} = \frac{b}{Q} (\frac{1}{2} K S^{\alpha\beta} \vec{F}_{\alpha\beta} - G\vec{\Phi} - R^{\alpha} D_{\alpha}\vec{\Phi}) \times \vec{\tau} . \tag{5.8}$$

With eqs. (5.1) - (5.4) the spin and momentum equations (3.7) and (4.9) become,

$$\frac{DS^{\alpha\beta}}{Ds} = p^{\alpha}u^{\beta} - p^{\beta}u^{\alpha} + \vec{\tau}.(K\vec{F}^{\alpha}{}_{\lambda} S^{\lambda\beta} - K\vec{F}^{\beta}{}_{\lambda}S^{\lambda\alpha} + R^{\beta}D^{\alpha}\Phi - R^{\alpha} D^{\beta}\Phi) \tag{5.9}$$

and

$$\frac{Dp^{\alpha}}{Ds} = \frac{1}{2} R^{\alpha}{}_{\sigma\mu\nu} u^{\mu} S^{\nu\sigma} + \vec{\tau}.(Q\vec{F}^{\alpha}{}_{\beta}u^{\beta} + \frac{1}{2} K S^{\beta\sigma}D^{\alpha}\vec{F}_{\beta\sigma} - GD^{\alpha}\vec{\Phi} - R_{\beta}D^{\alpha}D^{\beta}\vec{\Phi}) \tag{5.10}$$

We shall also assume that the magnitude of the spin, Higgscharge and dipole moment and of the Yang-Mills electric-magnetic dipole moment are constants

$$S^{\alpha\beta} S_{\alpha\beta} = \text{constant} , \tag{5.11}$$

$$G = \text{constant} , \tag{5.12}$$

$$R^{\alpha} R_{\alpha} = \text{constant} , \tag{5.13}$$

and

$$\vec{J}^{\alpha\beta}\,\vec{J}_{\alpha\beta} = \text{constant}$$

or, with eqs. (5.4) and (5.5),

$$K = \text{constant} \qquad (5.14)$$

As we show below, the assumptions that we have made will lead to a system with a constant mass, mechanical plus Yang-Mills and Higgs energies of interaction.

First of all we note that as $\vec{R}^0 = 0$ we have

$$u_\alpha \vec{R}^\alpha = 0 \;, \qquad (5.15)$$

since this is valid in the particle rest frame. From here we have

$$\dot{u}_\alpha \vec{R}^\alpha + u_\alpha \dot{\vec{R}}^\alpha = 0 \;.$$

As

$$d\vec{R}^0/ds = 0$$

we have

$$u_\alpha d\vec{R}^\alpha/ds = 0$$

and, with eq. (5.15),

$$u_\alpha \vec{R}^\alpha = 0 \;.$$

Therefore

$$u_\alpha \vec{R}^\alpha = 0 \quad \text{and} \quad \dot{u}_\alpha \vec{R}^\alpha = 0 \;. \qquad (5.16)$$

Recalling eq. (5.3) and noting that

$$\vec{R}^\alpha = \dot{R}^\alpha\,\vec{\tau} + R^\alpha\,\dot{\vec{\tau}}$$

eqs. (5.15) and (5.16) give

$$u_\alpha R^\alpha = 0 \;, \qquad (5.17)$$

and

$$u_\alpha \dot{R}^\alpha = 0 \quad \text{and} \quad \dot{u}_\alpha R^\alpha = 0 \qquad (5.18)$$

From eq. (4.10) we have

$$u_\alpha \dot{S}^{\alpha\beta} + \dot{u}_\alpha S^{\alpha\beta} = 0 \qquad (5.19)$$

Contracting eq. (5.9) with u_α and using eqs. (4.10), (5.17) and (5.19) we have for the momentum

$$p^{\beta} = mu^{\beta} + \vec{\tau}.(Ku_{\alpha}\vec{F}^{\alpha}{}_{\lambda}S^{\lambda\beta} + R^{\beta}\dot{\vec{\Phi}}) + \dot{u}_{\alpha}S^{\alpha\beta} \tag{5.20}$$

where

$$m = u_{\alpha}\, p^{\beta} \tag{5.21}$$

We now look for an equation for m.

Contracting eq. (5.20) with $\dot{u}_{B}$ and using $\dot{u}_{B}u^{B} = 0$ we have, using eq. (5.18) and eq. (5.20) for $S^{\lambda\beta}\,\dot{u}_{\beta}$ afterwards, we obtain

$$\dot{u}_{\beta}\,p^{\beta} = u_{\alpha}\vec{K}.\vec{F}^{\alpha}{}_{\lambda}S^{\lambda\beta}\dot{u}_{\beta} = u_{\alpha}\vec{K}.\vec{F}^{\alpha}{}_{\lambda}(-p^{\lambda} + \dot{\vec{\Phi}}.\vec{R}^{\lambda})\ , \tag{5.22}$$

where $\vec{K} = K\,\vec{\tau}$ and in the last step we have made use of the fact that $\vec{K}.\vec{F}^{\alpha}{}_{\lambda}S^{\lambda\rho}\vec{K}.\vec{F}^{\beta}{}_{\rho}$ is antisymmetric in α and β.

From eq. (5.9) we obtain, using eq. (4.11),

$$\vec{K}.\vec{F}_{\alpha\beta}\dot{S}^{\alpha\beta} = 2\vec{K}.\vec{F}_{\alpha\beta}(p^{\alpha}u^{\beta} + D^{\alpha}\vec{\Phi}.\vec{R}^{\beta}) \tag{5.23}$$

Taking this result into eq. (5.22) we get

$$\dot{u}_{\beta}p^{\beta} = \frac{1}{2}\,\vec{K}.\vec{F}_{\alpha\beta}\dot{S}^{\alpha\beta} - \vec{K}.\vec{F}_{\alpha\beta}D^{\alpha}\vec{\Phi}.\vec{R}^{\beta} + u_{\alpha}\vec{K}.\vec{F}^{\alpha}{}_{\lambda}\dot{\vec{\Phi}}.\vec{R}^{\lambda} \tag{5.24}$$

We now contract eq. (5.9) with $S_{\alpha\beta}$. Using eq. (4.10) and eq. (5.11) we obtain

$$S^{\alpha\beta}R_{B} = 0\ . \tag{5.25}$$

From here we have

$$\dot{S}^{\alpha\beta}\,R_{\beta} + S^{\alpha\beta}\,\dot{R}_{\beta} = 0 \tag{5.26}$$

$$\dot{R}_{\alpha}\,R_{\beta}\,\dot{S}^{\alpha\beta} = 0 \tag{5.27}$$

Using eq. (5.9) we obtain, recalling eqs. (5.13), (5.17), (5.18) and (5.25),

$$\dot{R}_{\alpha}\,R_{\beta}(-\vec{K}.\vec{F}^{\beta}{}_{\lambda}S^{\lambda\alpha} + D^{\alpha}\vec{\Phi}.\vec{R}^{\beta}) = 0 \tag{5.28}$$

Using here eq. (5.26) for $\dot{R}_\alpha S^{\lambda\alpha}$ and eq. (5.9) afterwardr we obtain, recalling eq. (5.17),

$$\dot{R}_\alpha R_\beta D^\alpha \vec{\Phi}.\vec{F}^\beta = R_\beta R_\alpha \vec{K}.\vec{F}^\beta{}_\lambda (p^\alpha u^\lambda - R^\alpha \vec{\tau}.\, D^\lambda \vec{\Phi}) \tag{5.29}$$

From eqs. (5.20), (5.17) and (5.25) we get

$$R\beta\, p^\beta = \vec{\tau}.\vec{\Phi}\; R_\beta R^\beta\,. \tag{5.30}$$

Taking this result into the first term of the right-hand sideof eq. (5.29) we obtain

$$R_\beta \vec{K}.\vec{F}^\beta{}_\lambda \vec{\tau}.\dot{\vec{\Phi}} u^\lambda = \dot{R}_\alpha \vec{\tau}.D^\alpha \vec{\Phi} + R_\beta \vec{K}.\vec{F}^\beta{}_\lambda \vec{\tau}.D^\lambda \vec{\Phi}\,. \tag{5.31}$$

Using this result in the last term of eq. (5.22) we obtain

$$\dot{u}_\beta\, p^\beta = (\tfrac{1}{2} K \vec{F}_{\alpha\beta}\, \dot{S}^{\alpha\beta} - \dot{R}^\alpha D_\alpha \vec{\Phi}).\vec{\tau}\,. \tag{5.32}$$

Now from eq. (5.10) we have

$$u_\beta\, \dot{p}^\beta = (\tfrac{1}{2} \dot{\vec{F}}_{\beta\sigma}\, K S^{\beta\sigma} - G\dot{\vec{\Phi}} - R^\sigma \frac{D}{Ds} D_\sigma \vec{\Phi}).\vec{\tau}\,. \tag{5.33}$$

Adding these two relations and recalling eqs. (5.12) and (5.14) weobtainfor m in eq. (5.21),

$$\dot{m} = \vec{\tau}\,.\,\frac{D}{Ds}\,(\tfrac{1}{2} K \vec{F}_{\alpha\beta}\, S^{\alpha\beta} - G\vec{\Phi} - R^\alpha D_\alpha \vec{\Phi})\,. \tag{5.34}$$

From the $\vec{\tau}$ eq. (5.8) we have

$$(\tfrac{1}{2} K \vec{F}_{\alpha\beta}\, S^{\alpha\beta} - G\vec{\Phi} - R^\alpha D_\alpha \vec{\Phi})\vec{\tau} = 0\,. \tag{5.35}$$

From eqs. (5.34) and (5.35) we get the relation

$$\frac{D}{Ds}\left[m + (-\tfrac{1}{2} K \vec{F}_{\alpha\beta}\, S^{\alpha\beta} + G\vec{\Phi} + R^\alpha D_\alpha \vec{\Phi}).\vec{\tau}\right] = 0. \tag{5.36}$$

From here we conclude that the quantity

$$M = m + \vec{\tau}.(-\tfrac{1}{2} K \vec{F}_{\alpha\beta}\, S^{\alpha\beta} + G\vec{\Phi} + R^\alpha D_\alpha \vec{\Phi}) \tag{5.37}$$

is a constant of motion, which we identify with the mass of our system. It contains the mechanical mass M, the Yang-Mills electric-magnetic dipole interaction energy and the Higgs charge and dipole interaction energy.

With eqs. (5.37), (5.20) becomes, with the help of eq. (5.19)

$$p^\beta = [M + \vec{\tau}.(\frac{1}{2} K \vec{F}_{\alpha\lambda} S^{\alpha\lambda} - G\vec{\Phi} - R^\alpha D_\alpha \vec{\Phi}] u^\beta + \vec{\tau}.(K u_\alpha \vec{F}^\alpha{}_\lambda S^{\lambda\beta} + R^\beta \dot{\vec{\Phi}}) + \dot{S}^{\beta\alpha} u_\alpha \ . \tag{5.38}$$

With this relation the translation eq. (5.10) becomes

$$\begin{aligned} &\frac{D}{Ds} \{ [M + \vec{\tau} (\frac{1}{2} K\vec{F}_{\alpha\lambda} S^{\alpha\lambda} - G\Phi - R^\alpha D_\alpha \vec{\Phi}] u^\alpha \\ &+ \vec{\tau}.(K u_\beta \vec{F}^\beta{}_\lambda S^{\lambda\alpha} + R^\alpha \dot{\vec{\Phi}}) - u_\lambda \dot{S}^{\lambda\alpha} \} = \\ &= \frac{1}{2} R^\alpha{}_{\sigma\mu\nu} u^\mu S^{\nu\sigma} + \vec{\tau}.(Q \vec{F}^\alpha{}_\beta u^\beta + \frac{1}{2} K S^{\beta\sigma} D^\alpha \vec{F}_{\beta\sigma} \\ &- G D^\alpha \vec{\Phi} - R_\beta D^\alpha D^\beta \vec{\Phi}) \ . \end{aligned} \tag{5.39}$$

Taking eq. (5.38) into eq. (5.9), the spin equation becomes

$$\begin{aligned} &\dot{S}^{\alpha\beta} - \dot{S}^{\alpha\rho} u_\rho u^\beta + \dot{S}^{\beta\rho} u_\rho u^\alpha = \\ &= \vec{\tau}.\{ R^\alpha (\dot{\vec{\Phi}} u^\beta - D^\beta \vec{\Phi}) - R^\beta (\dot{\vec{\Phi}} u^\alpha - D^\alpha \vec{\Phi}) \\ &+ K S^{\alpha\lambda} (\vec{F}^\rho{}_\lambda u_\rho u^\beta - \vec{F}_\lambda{}^\beta) - K S^{\beta\lambda} (\vec{F}^\rho{}_\lambda u_\rho u^\alpha - \vec{F}_\lambda{}^\alpha) \} \end{aligned} \tag{5.40}$$

Eqs. (5.8), (5.39) and (5.40) are the generalizations to curved space-time of the corresponding equations obtained in ref.3.

We can go a step forward in the rotational equation by noting that because of eqs. (4.10) and (5.25) we can write the following relation between the spin and the Higgs dipole moment,

$$S^{\alpha\beta} = L\, \varepsilon^{\alpha\beta\lambda\nu} u_\lambda R_\nu \ . \tag{5.41}$$

Contracting this equation with itself and using eq. (5.17) we obtain

$$S^{\alpha\beta} S_{\alpha\beta} = 2L^2 \; R^{\nu} R_{\nu} \tag{5.42}$$

This shows, with eqs. (5.11) and (5.13), that L is a constant. Then

$$\dot{S}^{\alpha\beta} = L\, \varepsilon^{\alpha\beta\lambda\nu}(\dot{u}_{\lambda} \; R_{\nu} + u_{\lambda} \; \dot{R}_{\nu}) \tag{5.43}$$

Taking this result into eq. (5.40) and making use of the identity

$$u^{\alpha}\varepsilon^{\beta\lambda\nu\rho}u_{\lambda}\dot{u}_{\nu}R_{\rho} - u^{\beta}\varepsilon^{\alpha\lambda\nu\rho}u_{\lambda}\dot{u}_{\nu}R_{\rho} = \varepsilon^{\alpha\beta\lambda\nu}R_{\lambda}\dot{u}_{\nu}$$

the left-hand side of eq. (5.40) reduces to $L\varepsilon^{\alpha\beta\lambda\nu}u_{\lambda}\dot{R}_{\nu}$ and we obfain

$$L\varepsilon^{\alpha\beta\lambda\nu}u_{\lambda}\dot{R}_{\nu} = \vec{\tau}.\{R^{\alpha}(\dot{\vec{\Phi}}\, u^{\beta} - D^{\beta}\vec{\Phi}) - R^{\beta}(\dot{\vec{\Phi}}\, u^{\alpha} - D^{\alpha}\vec{\Phi})$$
$$+ KL\varepsilon^{\alpha\lambda\gamma\sigma}u_{\gamma}R_{\sigma}(\vec{F}_{\lambda}{}^{\rho}u_{\rho}u^{\beta} - \vec{F}_{\lambda}{}^{\beta}) - KL\varepsilon^{\beta\lambda\gamma\sigma}u_{\gamma}R_{\sigma}(\vec{F}_{\lambda}{}^{\rho}\, u_{\rho}u^{\alpha} - \vec{F}_{\lambda}{}^{\alpha}). \tag{5.44}$$

This equation is the generalization to curved space-time, including the Yang-Mills field, of the corresponding equation obtained in the case of a usual nucleon interacting with a scalar meson field, which was obtained before[9] using the momentum method of Papapetrou, consisting in an alternative derivation of the one derived previously by Harish-Chandra[10].

With eqs. (5.41), (5.8) and (5.39) together with eq. (5.44) are three equations for u^{α}, R^{α} and $\vec{\tau}$ totalizing eleven unknowns, in termsof the constants M, Q, G, K and L. As contraction of eq. (5.44) with *u,* gives three identities, eq. (5.44) gives not six but only three independent equations. These three together with $u^{\alpha}u_{\alpha} = 1$ and the seven relations coming from eqs. (5.8) and (5.391, give the eleven equations one needs to determined the eleven unknown quantities.

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Resumo

Usando-se o método dos momentos de Papapetrou, as equações do movimento para uma partícula teste não Abeliana carregada na presença de um campo de Einstein-Yang-Mills-Higgs são obtidas a partir de conservação de carga e de energia-momento.