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On the Internal Size of Compact Dimensions in Polyakov's Bosonic String Moving in SO(2) Group Space

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Abstract We argue that the parameter related to the size of the compactified dimensions in a Polyakov Bosonic string moving in thegroup manifold SO(2) satisfies an asymptotic freedom law, thus implying the existente of a mass gap in the above mentioned strong theory. As a result we show that the internal size of the group manifold is fixed by quantum dynamical effects.

Recently, the analysis of Kaluza-Klein string theories[1,2,3,4] has become an important issue in the construction of grand unification string theories[5,6].

Fundamental points in those grand unification string theories are the questions of existence and of size of internal compactified dimensions.

In this comment we aim to remark that the ratio between the Regge slope parameter and the radius of the group manifold O(2) in the simplest Kaluza-Klein string theory, a Polyakov bosonic stringmoving in $R^D \times SO(2)$[4,6], satisfies an asymptotic freedom law. On the basis of this result we show that the internal size of SO(2) is not fixed but it is determined by a intrinsic parameter.

Let us start our study by considering the covariant string action for the compactified dimension:

$$S\left[X^c(\xi),\ g_{ab}(\xi),\ \lambda(\xi)\right] =$$

$$\frac{1}{2}\int_D\left[\sqrt{g}\ (X^c_\mu(-\Delta_g)X^c_\mu\ +\ \lambda\ \frac{\alpha'}{R^2}\ (X^c_\mu . X^c_\mu\ -\ 1))\right](\xi)\,d^2\xi \qquad (1)$$

where $\{X^c_\mu(\xi)\}$ denotes the striop vector position, $\{g_{ab}(\xi)\}$ the intrinsic metric field; $\lambda(\xi)$ the auxiliary Lagrange multiplier field irnplementing the constraint that the string moves in an O(2) group manifold and α'/R^2

is the ratio between the Regge slope parameter and the radius of the group manifold 0(2).

Our strategy now will be to consider the ratio $\beta = \alpha'/R^2$ as a theory coupling constant and then determine its renormalization group law.

As a first step we integrate out the *spherical* vector position $\{X^c_\mu(\xi)\}$ and obtain an unrenormalized effective action in terms of the auxiliary field $\lambda(\xi)$.

$$S_{EFF}[\lambda(\xi),g_{ab}(\xi)] = +\frac{1}{2}\log \mathrm{DET}_F(-\Delta_g + \frac{\alpha'}{R^2}\lambda) \tag{2}$$

In order to analyse the counterterms of the above written effective action, we define the unrenormalized functional determinant in eq. (2) by a proper-time representation[7]. Explicitly:

$$S_{EFF}[\lambda(\xi),g_{ab}(\xi)] = -\frac{1}{2}\int_0^\infty \frac{dT}{T}\,\mathrm{Tr}_F(\exp(-T(-\Delta_g + \frac{\alpha'}{R^2}\lambda))) \tag{3}$$

Now it is well known that the counterterms of $S^{EFF}[\lambda(\xi),g_{ab}(\xi)]$ are determined by the asymptotic expansion of the diagonal part of the operator in eq. (3), which is tabulated[8].

$$\lim_{T\to 0^+} \langle\xi|\exp(-T(-\Delta_g + \frac{\alpha'}{R^2}\lambda))|\xi\rangle$$

$$= \lim_{T\to 0^+} \frac{g}{2\pi}(\frac{1}{T}) - \frac{1}{2\pi}\sqrt{g}\,R + \frac{1}{2\pi}\frac{\alpha'}{R^2}\sqrt{g}\,\lambda \tag{4}$$

Substituting eq. (4) into eq. (3), we get straightforwadly the counterterm associated to the coupling constant, namely

$$\frac{1}{2}\cdot\frac{1}{2\pi}\beta\cdot\log(\frac{1}{\varepsilon})\int_D(\sqrt{g}\,\lambda)(\xi)d^2\xi \tag{5}$$

where ε is an ultraviolet cutoff,

So, on the basis of eq. (5) we have the following renormalization law for the coupling constant β [9]

$$\frac{1}{\beta_R} = \frac{1}{\beta_0} - \frac{1}{4\pi}\,(\log \frac{1}{\varepsilon}) \tag{6}$$

and standard renormalization group arguments give its momentum dependence

$$\beta_R(p^2) = \frac{\beta_0(p^2=\varepsilon^2)}{1 - \frac{1}{2}\,\beta_0(p^2=\varepsilon^2)\,(\log(\varepsilon^2/p)} \tag{7}$$

Equation (7) yieds the fact that the internal size $\beta = \alpha'/R^2$ satisfies an asymptotic freedom law in the intrinsic string's energy phase space which, in turn strongly suggests the existence of a mass gap $\bar\lambda(p^2)$ in the theory[9,10] satisfying the constraint

$$\bar\lambda(p^2) = \lambda_0(\varepsilon^2)\,\exp\left[-\frac{1}{4\pi}\,(\frac{\alpha'}{R})^2\right] \tag{8}$$

This is our main result: the integral size for ***a bosonic being compactified to the M-Tori***

$$\prod_{i=1}^{M} (O(2))_i$$

is not arbitrary but it is fixed by the new purely intrinsic (dimensionless) parameter. It is instructive to point out that the existence of the parameter *signals* the well known dynamical breking by quantum corrections of the string's two *dimensional conformal group*[10].

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Resumo

Mostramos que a razão entre o parâmetro de inclinação de Regge e o raio da variedade SO(2) para um string bosônico de Polyakov movendo-se em $R^D \times SO(2)$ satisfaz uma lei de renormalização levando a um comportamento de liberdade assintótica. Este resultado, por sua vez, sugere que a razão acima mencionada é fixa pela dinâmica quântica bidimensional do string.