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Cosmological Solutions with Quadratic Viscosity

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Abstract Starting with a non-linear viscous fluid we present an isotropic and homogeneous solution of Einstein's equations which exhibits smooth behaviour and is free from singularities,

1. INTRODUCTION

The influence of viscosity on the evolution of cosmological models has been considered with an increasing interest by several authors in the last years[1-17]. Among the characteristics possessed by viscous models, one can quote the quite appealing possibility of removing metric singularities in perfect fluid models by adding terms corresponding to viscous processes to its energy-momentum tensor. This has been done for the first time by Murphy, who considered a Friedmann universe with plane spatial section and a fluid with bulk viscosity terms[2]. The linearity of viscosity terms present in Murphy's model is not essential to the non-existence of singularities. In fact, one can have non-linear viscous fluids showing smooth and non-singular behaviour. The question of, how to provide a physical realization of such a fluid will not be treated here. In this paper, we investigate a Friedmannian metric with zero-spatial curvature generated by a viscous fluid in a quadratic regime with constant coefficients. This kind of fluid has been examinated by Novello-Araújo using Dynamical System Theory, following the method of Belinskii-Khalatnikov[3,6]. We solve Einstein's equations and discuss special solutions.

2. VISCOUS FLUIDS IN QUADRATIC REGIME

We start by taking a Friedmann-like metric with Euclidean spatial section written in the standard form

$$ds^2 = dt^2 - A(t)^2(dr^2 + r^2d\theta^2 + \mathrm{sen}^2\theta d\phi^2)$$

and an isotropic pressure $\tilde{p} = p + \alpha\theta + \beta\theta^2$, where the coefficients of the expansion factor θ are constants and p is the thermodynamic pressure. This phenomenological form of $\tilde{p}$ has been associated to the stationary case of constant creation of particles in the universe, inducing viscous phenomena in a steady-state regime[6].

Thus, Einstein's equations are simply

$$3\left(\frac{\dot{A}}{A}\right)^2 = \rho - \Lambda \tag{1.a}$$

$$2\frac{\ddot{A}}{A} + \left(\frac{\dot{A}}{A}\right)^2 = -\tilde{p} - \Lambda \tag{1.b}$$

We take the cosmological constant $\Lambda = 0$ and adopt the equation of state $p = \lambda\rho$, where ρ is the density of mass-energy. Furthermore, in a comoving coordinate system ($V^\mu = \delta^\mu_0$), we have $\theta = V^\mu{}_{;\mu} = 3\,\dot{A}/A$. Thus, the above system reduces to

$$\theta^2/3 = \rho \tag{2.a}$$

$$\dot{\theta} + \frac{\theta^2}{2} + \frac{3}{2}\left(\lambda\rho + \alpha\theta + \beta\theta^2\right) = 0 \tag{2.b}$$

Before carrying out the general solution, we see immediately that both $\theta = 0$ and $\theta = -a/b$, where $a = -\frac{3}{2}\alpha$ and $b = -\frac{1}{2}(1 + \lambda + 3\beta)$, satisfy eq. (2). It is clear that these correspond to Minkowski and de Sitter geometries respectively. In the latter, the effective pressure $\tilde{p} = -\frac{1}{3}\frac{a^2}{b^2}$ acts as a negative pressure. The general solution is promptly obtained

$$A(t) = A_0\left(K - be^{at}\right)^{-1/3b} \tag{3.a}$$

where $a \neq 0$, A and K are integration constants. If $a = 0$, we get the solution

$$A(t) = A_0^*\,(N + t)^{-1/3b} \tag{3.b}$$

where A_0^* and N are integration constants.

From eqs. (3.a) and (3.b) we can evaluate the scalar of curvature R and the kinematical expansion parameter θ in a straightforward way. With a suitable choice of the constants K, a and β, we can obtain a class of non-singular solutions. Thus, we have the following general scheme:

i) For K = 0, we get a class of de Sitter-type solutions

$$A(t) = A(t_0)\, e^{-\frac{a}{3b}(t-t_0)} \quad ; \qquad (4)$$

ii) For K > 0, a < 0 and $\beta \geq 0$, this solution constitutes an expanding non-singular model, in which the universe evolves from a finite radius at the infinite past and continues to expand indefinitely in the future (see fig.1).

iii) For $K < 0$ and $\beta \geq 0$, we just mention the less interesting cases with the occurrence of singularities (see fig.2).

3. CONCLUSION

If we look into the modern theories of Cosmology, those so-called Grand Unification Theories[17], we find out the existence of a generalized trend to appeal to Elementary Particle Physics in order to explain the presumable exponential growth of the Cosmos at its beginning — the well-known *inflationary era*. As a matter of fact, even with the help of these powerful symbiotic theories, we still do not know what is really going on at that stage. Nonetheless, it seem at least amazing that, without a knowledge of the physical process involved, and with rather simplified assumptions (as the more introduction of dissipative effects in the dynamics of the cosmological fluid), a simple model built on purely classical grounds is able to account for a rough description of the inflationary era.

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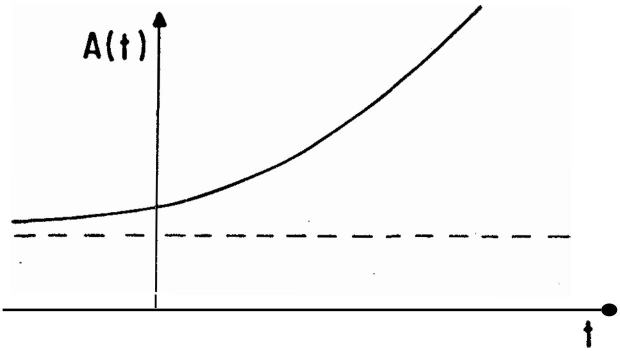

Fig.1 - Expanding non-singular model with quadratic viscosity.

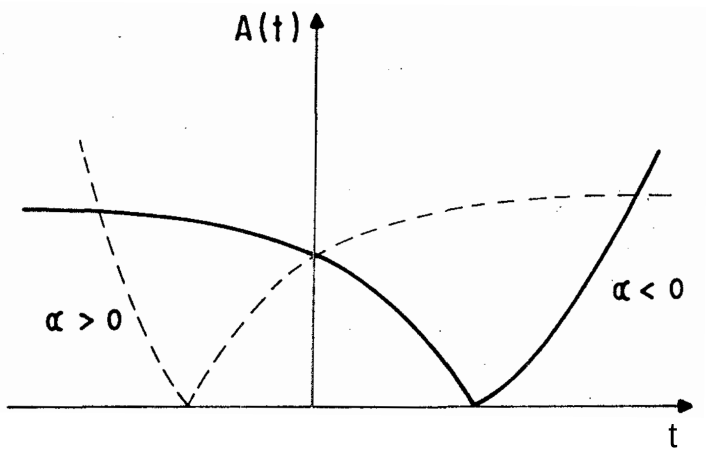

Fig.2 - Singular solutions with quadratic viscosity.

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Resumo

A partir de um fluido com viscosidade não-linear, apresentamos uma solução das equações de Einstein, exibindo homogeneidade, isotropia e não-singularidade.