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Heavy Ion Fusion Reactions: Comparison Among Different Models

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Abstract A comparison among different heavy ion fusion models is presented. In particular, the multistep aspects of the recently proposed Dinucleus Doorway Model are made explicit and the model is confronted with other compound nucleus limitation models. It is suggested that the latter models provide effective one-step descriptions of heavy ion fusion.

1. INTRODUCTION

The popular picture of heavy ion fusion reactions statcs that at low energies the fusion cross section σ_F follows the trend of the total reaction cross section, σ_R, exhausting almost all of it. This comes about as a consequence of the fact that in this energy region (usually referred to as region I) few direct reaction channels compete with fusion in the distribution of the incoming flux. After a certain critical energy is reached, however, σ_R continues exhibiting its geometrical behaviour,

$$\sigma_R = \frac{\pi}{k^2}(\ell_g + 1)^2 = \pi R_b^2 \left(1 - \frac{E_b}{E}\right)$$

where ℓ_g is the grazing angular momentum and R_b and E_b are the radius and height of the Coulomb barrier; while the fusion cross

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section bends down and eventually takes the approximate form

$$\sigma_F = \frac{\pi}{k^2} (\ell_c + 1)^2 = \pi R_c^2 \left(1 - \frac{E_c}{E}\right) \quad . \tag{2}$$

In the above equation ℓ_c refers to the critical angular momentum for fusion and R_c and E_c are convenient parameters. The energy region where σ_F/σ_R gradually drops below unity is called region II. The authors of most of the recent publications concerned with heavy ion fusion address themselves to the question of whether the fact that in region II, $\ell_c < \ell_g$ (and accordingly $\sigma_F < \sigma_R$) is just a trivial consequence of unitarity, namely the increasing contribution to σ_R of direct process dominated by deep inelastic and incomplete fusion reactions, or whether it hides more subtle dynamics. Further, one may ask whether $\sigma_F/\sigma_R < 1$ in region II indicates a statistical density of states restriction on σ_F, or whether it reflects coherence effects involved in entrance channel coupling to the compound nucleus states.

The standard representation for the fusion cross section is

$$\sigma_F = \frac{\pi}{k^2} \sum_{\ell=0}^{\infty} (2\ell + 1) T_\ell P_\ell \quad , \tag{3}$$

where T_ℓ is the elastic channel transmission coefficient associatedwith the R-th partial wave and P_ℓ is the probability that the system fuses after having overcome the potential barrier associated with T_R. Assuming the sharp cut-off approximation to the product $T_R P_\ell$,

$$T_\ell P_\ell = \theta(\ell_c - \ell) \quad , \tag{4}$$

eq. (3) reduces to eq. (2).

Two distinct classes of models for the origin of R have been discussed extensively in the literature. The first class, loosely called entrance channel models, purports to explain the limitation of σ_F as arising from the general characteristics of the interacting ions in the approaching state. Through the explicit or implicit use of both conservative and dissipative components of the ion-ion interaction, it is possible to account reasonably well for most of the gross features of the fusion excitation function'. In this class of models, we group the critical distance model of Glas and Mosel[2] (infinite short-range dissipation), finite friction models' and the extra-push model of Swiatecki and Bjørnholm[3]. No reference is made to the properties of the fused system (the compound nucleus).

The second class of models ignores the details of en'trance channel dynamics, except for the effects of the potential barrier and kinematic constraints. They describe R_c as an angular momentum related to the Yrast line[4-7] of the fused system. Microscopically such identification is connected with the degree of overlap of the compound nucleus resonances, as will be discussed fully later.

Recently a new model for fusion has been proposed'and successfully applied to a variety of heavy ion systems[8-11]. A very important difference betwqen this model and the ones previously mentioned is the introduction of'the dinucleus acting as a doorway to fusion. It is easily seen that this enables the simultaneous consideration of both the entrance channel characteristics and those of the compound nucleus. Further, it has now become possible to discuss the observable consequences of the limitation to fusion on the competing processes[12], namely the deeply inelastic channels (DIC). In order to better appreciate the potential implications of the multistep compound model on other observables besides σ_F, and, accordingly, to further understand the role of the dinucleus, it is of paramount importance to make a detailed comparison between this model and those discussed earlier. The purpose of this paper is to supply such a comparison and pin down the features which are connected with the compound nucleus (CN) and those with the entrance channel.

The paper is organized as follows: in section 2 we describe briefly the entrance channel and the compound nucleus limitation models. In section 3 we summarize the main features of the dinucleus doorway model and explicitly show its multistep nature. In section 4 we make a detailed comparison of these models, and finally we present our concluding remarks in section 5.

2. ENTRANCE CHANNEL AND CN LIMITATION MODELS

2.1 - Entrance channel models

In the infinite short-range dissipation model of Glas-Mosel[2], it is assumed that the heavy ion system must reach a certain critical distance R_C for fusion to occur. This defines a critical ℓ_c, which specifies P_ℓ. On the other hand, in a finite friction classical calculation of the type discussed in ref.1, one invariably associates ℓ_c with the angular momentum at which orbiting occurs. However, the physics involved in both models is the same. The sharp cut-off behaviour of P_ℓ assumed in the above models is a consequence of the classical (deterministic) physics involved. The joint action of the conservative and dissipative pieces of the nuclear force together with the centrifugal and Coulomb repulsions defines the dynamics of the nuclear system which culminates in the formation of the mononucleus, geometrically representing the fused system (or, at least, the initial stage of its formation). No reference to the detailed characteristics of the mononucleus is, however, explicitly made, in the sense of its density of states and its average lifetime. One would, therefore, expect that such models could, at most, account for the gross behaviour of σ_F.

2.2 - CN limitation models

The CN limitation models express the fusion probability in terms of the product $\rho(E^*,\ell)\Gamma(E^*,\ell)$, where E^* is the total excitation of the CN, ρ is its density of states and Γ its corresponding correlation width. This function determines the availability of states in the Compound Nucleus.

The Extreme Yrast Line limitation model[4] (EYL) identifies ℓ_c with the angular momentum value on the CN Yrast line. For a given collision energy E_{cm}, R_c is determined from the relation

$$E_{cm} + Q = E^* = \frac{\hbar^2 \ell_c(\ell_c + 1)}{2J_c} \tag{5}$$

where J_c is the CN moment of inertia. This criterion implies

$$\rho(E^*,\ell_c)\Gamma(E^*,\ell_c) \ll 1 \ . \tag{6}$$

The statistical Yrast Line (SYL) limitation model[5] requires a larger density of states for CN formation. It imposes that the CN has an intrinsic excitation energy ΔQ,

$$\Delta Q = E^* - \frac{\hbar^2 \ell_c(\ell_c + 1)}{c} \approx 10 \text{ MeV} \tag{7}$$

and extracts ℓ_c from the above relation. It is, of course, difficult to know a priori the amount of intrinsic excitation energy (thermal energy) which must be supplied to the compound nucieus. Lee et *al.*[5] treated ΔQ, the shift from the Yrast line Q-value, as an adjustable parameter and fit it to several sets of fusion data. For the systems considered, they found values in the range $8 \text{ MeV} \leq \Delta Q \leq 12 \text{ MeV}$.

A different formulation of the SYL model was introduced by Vandenbosch[6,7]. This author determines ℓ_c from the condition

$$\rho(E^*,\ell_c)\Gamma(E^*,\ell_c) \approx 1 \ . \tag{8}$$

This corresponds to the boundary between overlapping resonances ($\rho\Gamma \approx \Gamma/D \gg 1$) and isolated resonances ($\Gamma/D \ll 1$) of the CN. The condition of eq. (8) corresponds to requiring that the CN has an intrinsic excitation energy E^*, above the Q-value Yrast line. This defines an *effective* Yrast-line, from which R_c can be extracted. As shown in sec.4 (see fig.5), this line is very similar to the statistical Yrast line and, therefore, Vandenbosch's model (V) is essentially equivalent to the SYL.

3. THE DINUCLEUS DOORWAY MODEL OF FUSION

The Dinucleus Doorway Model (DDM) of heavy-ion fusion developed recently in São Paulo[9-11] asserts that the compound nucleus is reached from the entrance channel only via a superdeformed dinucleus configuration. Such a doorway picture allows for a consistent coupling between the entrance channel and the compound nucleus. Since the dinucleus may break up intodeeply inelastic channels, one has a simple and direct way to constrain a_F.

According to the statistical multistep treatment of ref.9, the fusion cross-section is written in the general form of eq. (3) with the fusion probability P_ℓ given by

$$P_\ell = \left[\frac{t}{2\pi(\rho_d\Gamma_d^\uparrow + \rho_c\Gamma_c^\uparrow)t + (2\pi\rho_d\Gamma_d^\uparrow)(2\pi\rho_c\Gamma_c^\uparrow)}\right] 2\pi\rho_c\Gamma_c^\uparrow \quad , \tag{9}$$

where $\Gamma_c^\uparrow$ is the total escape width (to particle channels) of the compound nucleus, $\Gamma_c^\uparrow$ the corresponding one for the dinucleus (to break up and particle channels), the ρ's are level densities and t is an average dinucleus-compound nucleus coupling coefficient which goes as $\sqrt{\rho_c\rho_d}$.

The physical significance of the seemingly complicated denominator appearing in eq.(9) can be understood through an expansion in terms of the *round-trip* probability propagator y,

$$y = \left(\frac{t}{t + 2\pi\rho_d\Gamma_d^\uparrow}\right)\left(\frac{t}{t + 2\pi\rho_c\Gamma_c^\uparrow}\right) \quad , \tag{10}$$

and the branching ratios

$$P_\ell^{(0)} = \frac{t}{t + 2\pi\rho_d\Gamma_d^\uparrow} \tag{11}$$

and

$$\Delta_\ell = \frac{2\pi\rho_c\Gamma_c^\uparrow}{t + 2\pi\rho_c\Gamma_c^\uparrow} \quad . \tag{12}$$

It is then easy to write for P_ℓ the following series

$$P_\ell = P_\ell^{(0)} \left[1 + y + y^2 + \ldots\right] \Delta_\ell \quad . \tag{13}$$

It is important to emphasize that $P_\ell^{(0)}$ reprerents the probability for the dinucleus to form the compound nucleus while Δ_ℓ measures the summed particle decay probability of the CN. In the limit $2\pi\rho_c\Gamma_c^\uparrow \gg t$, the round-trip probability propagator, y, becomes very small. In this case

$\Delta_\ell \gg 1$ and the fusion probability P_R approaches the probability of CN formation $P_R^{(0)}$. A pictorial representation of eq. (13) is shown in fig. 1. The first term represents the contribution of the direct DN-CN transition to fusion. The higher order terms express contributions to fusion arising from multistep processes containing round-trip DN-CN transitions.

In fig.2 we compare the probabilities of CN formation, $P_\ell^{(0)}$, and the fusion probability, P_ℓ, for the system $^{27}Al + {}^{12}C$, at E_{cm} = 50 MeV. We note that the two have approximately the same shape but the CN formation probability extends to higher R-values. In the region where the two differ, the CN is formed but subsequently decays to the dinucleus, which we do not interpret as fusion. In fig.3 we show the corresponding fusion and compound nucleus formation cross-sections vs. $1/E_{cm}$. As expected, σ_{CN} is consistently larger than the fusion cross-section. Therefore, in this region, multistep effects reduce σ_F to about 80% of its CN counterpart.

4. COMPARISON OF STATISTICAL FUSION MODELS

In this section we confront the Dinucleus Doorway Model with the other CN limitation models discussed in sec.2. In this comparison, we disregard the Yrast Line Model since it is a rather extreme case, never actually realized.

To be specific, we consider the systems $^{12}C + {}^{27}Al$, $^{24}Mg + {}^{32}S$ and $^{40}Ca + {}^{40}Ca$. These systems represent the three typical behaviors of σ_F in region II. In fig. 4 we show σ_F calculated within the DDM, the SYL and the V models, for the above mentioned systems. Also shown are the CN formation cross-section of the DDM and the data points of ref.1. The criterion $\Gamma_c \rho_c = 1$ was used to generate $\sigma_F^{(V)}$. The parameters used in the DDM calculation were taken from ref. 11. Although one would expect $\sigma_F^{(SYL)}$ ($\simeq \sigma_F^{(V)}$) to be close to the CN limit of the DDM calculation, the results shown in these figures, however, indicate clearly that $\sigma_F^{(SYL)} \sim \sigma_F^{(V)} \approx \sigma_F^{(DDM)}$. This shows that the SYL and the V models contain implicitly, through their adjustable parameters, the multistep effects explicitly contained in $\sigma_F^{(DDM)}$.

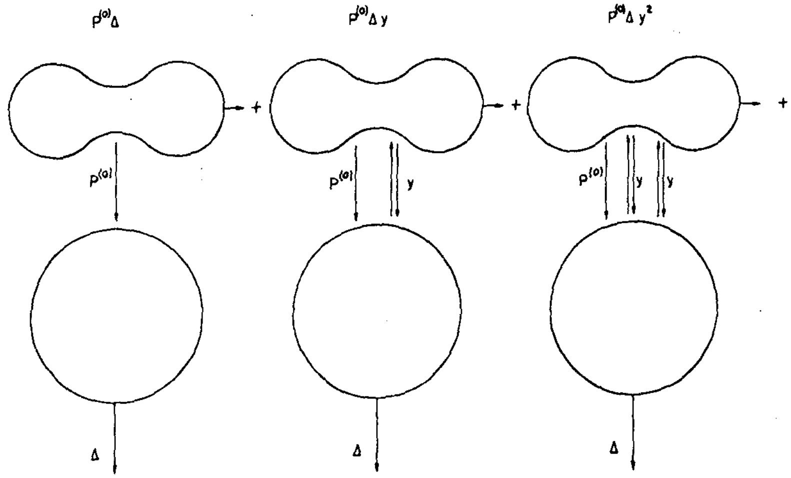


Fig.1 - A pictorial representation of the contributions to the fusion cross-section (eq.(13)).

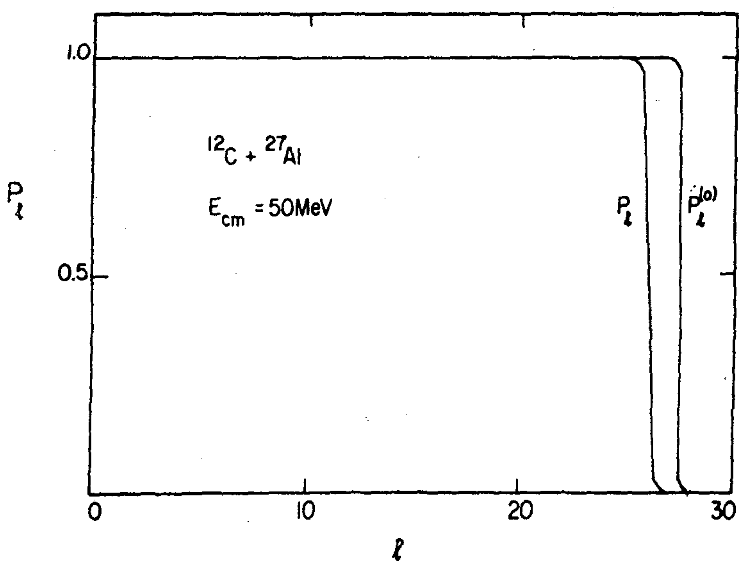


Fig.2 - The ℓ-dependence of the fusion (P_ℓ) and CN formation ($P_\ell^{(0)}$) probabilities according to the Dinucleus Doorway Model, for the system $^{12}C + ^{27}Al$ at E_{cm} = 50 MeV.

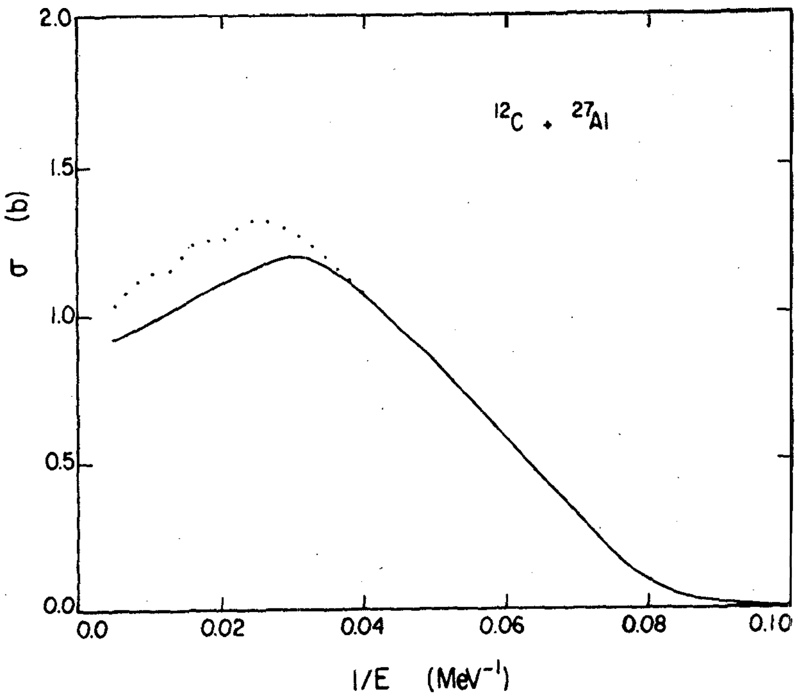
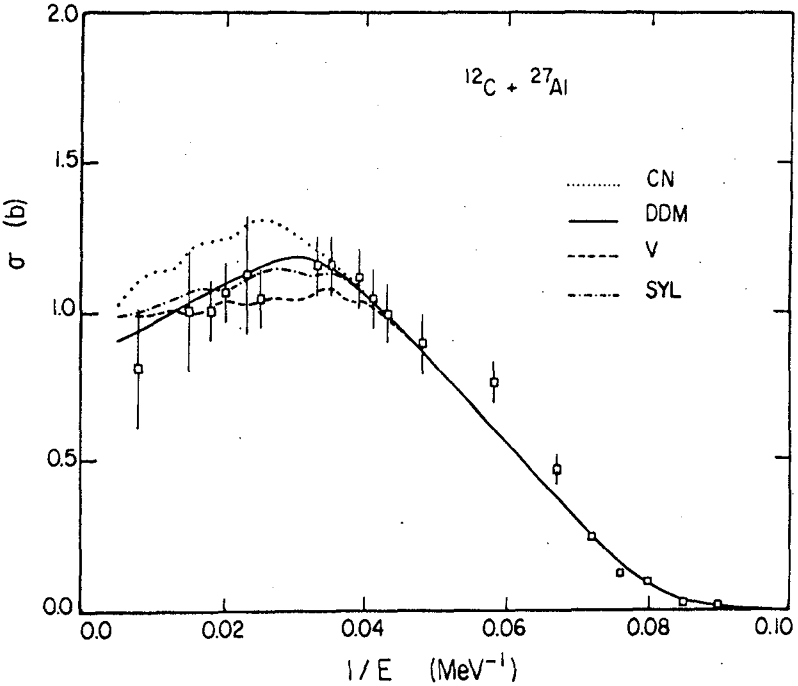


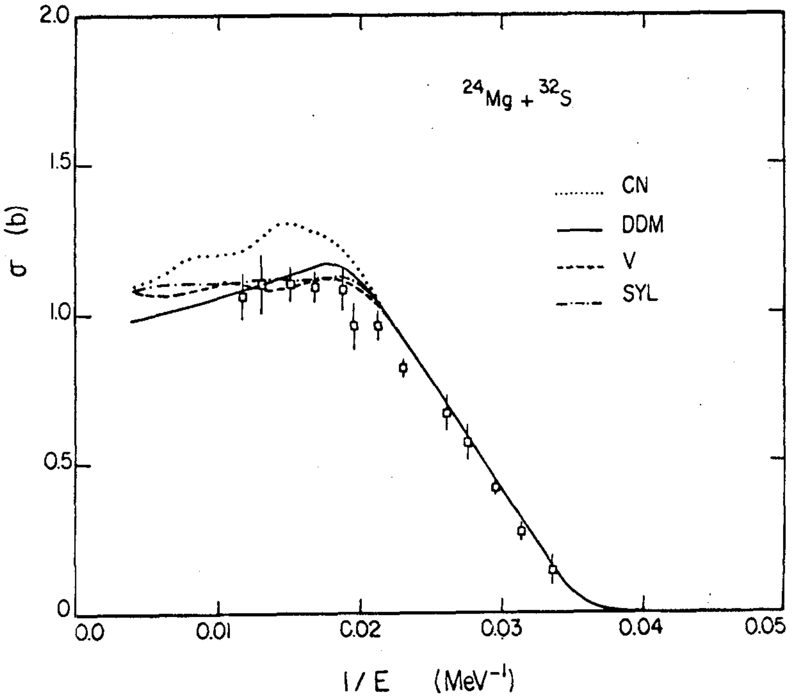
Fig.3 - The fusion (solid line) and the CN formation (dotted line) cross-sections for the system $^{12}C + ^{27}Al$, according to the Dinucleus Doorway Model.

We further compare the statistical fusion models by showing in fig.5 the *effective* Yrast line which corresponds to the critical angular momentum for fusion in ^{12}C + ^{27}Al. In fact we verified that ℓ_c is quite well defined, as the diffuseness of P_ℓ is small (see fig.2). It is clearly seen that all three models (SYL, V and DDM) yield very similar curves in the E* vs. $J(J+1)$ plot of fig.5. The CN limit of the DDM starts appreciably deviating from the three curves at J ≈ 25 (or $J(J+1)$ = 600). Also shown in the figure are the CN Yrast line and the barrier line (which determines the total reaction cross-section).

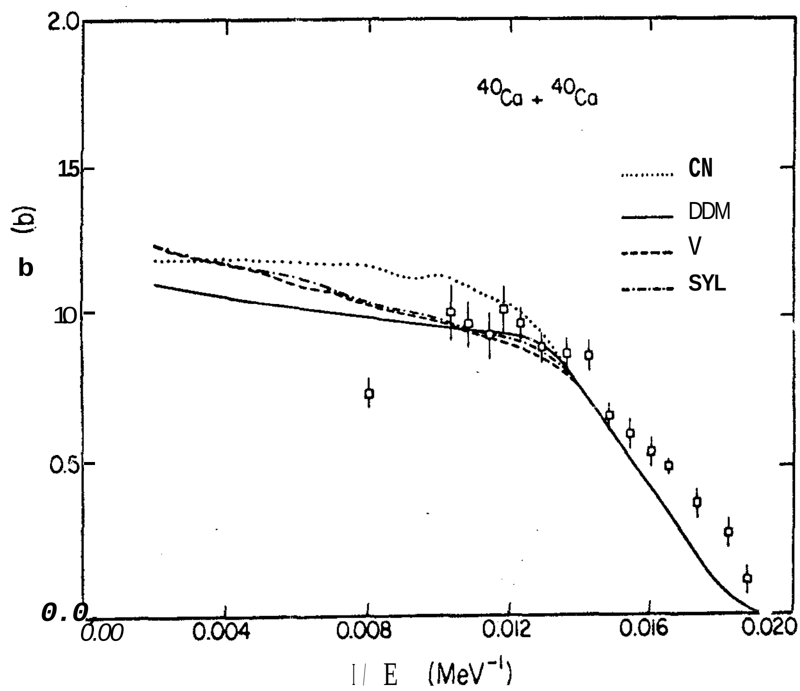
From the above results we conclude that the most effective test of the multistep nature of the heavy ion fusion process is to look at the consequence of the doorway aspect of the dinucleus on other competing processes which accompany fusion in region II, namely the DIC channels. We come to this question in the next section.



(4a)



(4b)



(4c).

Fig.4 - The fusion cross-sections for the systems: (a) $^{12}C+^{27}Al$, (b) $^{24}Mg+^{32}S$, and (c) $^{40}Ca+^{40}Ca$. Also shown is the CN cross--section (see text for details).

5. DISCUSSION

In this paper we have made a detailed cornparison among statistical models of heavy ion fusion. We have seen that the statistical Yrast Line Model as well as the Vandenbosch Model are basically effective one--step models which contain implicitly, through their parameters, the rnultistep effects explicitly contained in the Dinucleus Doorway Model.

Fusion, being a highly inclusive process, is not very sensitive to the details of the dynamics involved. One therefore needs observables describing other reaction channels, as well as a_F, in order to test the physics. The dominant channels cornpeting with fusion in region II are the deeply inelastic processes. If the dinucleus were to play as important a role in these processes as it does in fusion, an analysis of DIC excitation functions should reveal Ericson-type fluctuations. The charac-

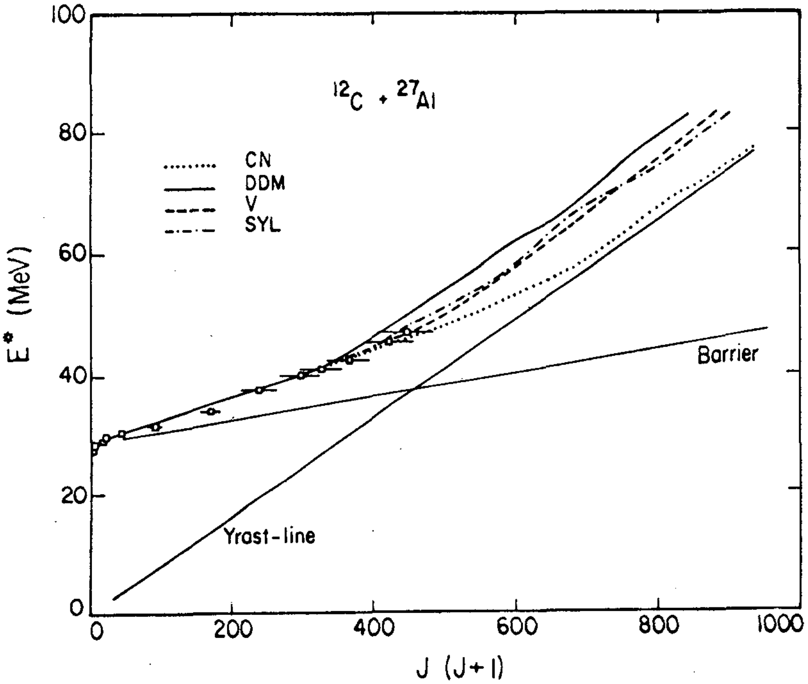


Fig.5 - The effective Yrast lines for the $^{12}C + ^{27}Al$ (see text for details).

teristic correlation width of these fluctuations would measure the dinucleus lifetime directly. Such analyses have been performed[12] for the DIC cross-sections of $^{12}C + ^{24}Mg$ at $30 < E_{cm} < 42$ MeV and $^{28}Si + ^{64}Ni$ at $120 < E_{Lab} < 126.75$ MeV. The values obtained for the dinucleus lifetime are $\tau \approx 1.0.10^{-21}$s and $2.0.10^{-21}$s respectively. These values of τ correspond to correlation widths of 240 keV and 200-800 keV, respectively, which are quite close to the values obtained from adjusting the fusion cross-section of the DDM to the experimental data.

Other fluctuations studies of DIC excitation functions are certainly called for to further establish this phenomenon.

REFERENCES

1. J.R.Birkelund and J.R.Huizenga, Ann.Rev.Nucl.Part.Sci. 33,265 (1983); P.Fröbrich, Phys.Reports *116*, 337 (1984); D.H.E. Gross and H. Kalinowski, Phys. Reports *45C*, 175 (1978).

2. D.Glas and U.Mösel, Nucl.Phys. A237, 429 (1975).
3. W.J.Swiatecki, Physica Scripta 24, 113 (1981); S.Bjørnholm and W.J. Swiatecki, Nucl.Phys. A391, 471 (1982).
4. J.Lefort, J.Phys. (Paris), Colloq. bf 33 C5-73 (1972).
5. S.M.Lee, T.Matsuse and A.Arima, Phys. Rev. Lett. 45, 165 (1980).
6. R.Vandenbosch, Phys. Lett. 87B, 183 (1979).
7. R.Vandenbosch and A.J.Lazzarini, Phys. Rev. 23C, 1074 (1981).
8. O.Civitarese, B.V.Carlson, M.S.Hussein and A.Szanto de Toledo,Phys. Lett. 125B, 22 (1983).
9. M.S.Hussein, B.V.Carlson, O.Civitarese and A.Szanto de Toledo, Phys. Rev.Lett. 54, 2659 (1985).
10. M.S.Hussein, Proceedings of the International Conference on Nuclear Reaction Mechanisms, Varenna, Italy, June 1985, to appear.
11. B.V.Carlson, O.Civitarese, M.S.Hussein and A.Szanto de Toledo, Ann. Phys. (NY) 169, 167 (1986).
12. R.Bonetti and M.S.Hussein, Phys. Rev. Lett. 57, 194 (1986).

Resumo

É apresentada uma comparação entre vários modelos de fusão de íons pesados. Especificamente, são explicitados os aspectos de múltiplas etapas do recém proposto Modelo de *Doorway* Dinuclear, o qual é confrontado com modelos que limitam a formação do núcleo composto. É sugerido que os últimos fornecem descrições efetivas a uma etapa da fusão de íons pesados.