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# Extension of Glauber Theory to Large Angle Scattering of "Dirac" Protons

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**Abstract** The recently developed theory of the symmetrical T-matrix is extended here to the relativistic description of proton-nucleus scattering. The 4x4 symmetric T-matrix is reduced to an effective symmetric 2x2 matrix using projection techniques. The resulting non-relativistic--looking $T$-matrix contains relativistic (virtual pair) effects to all orders. Glauber theory is then applied to develop a reasonable approximation which could be valid even at large angles.

During the last several years a great amount of effort has been devoted towards the construction of a relativistic theory of nuclear structure and reactions[1]. In such a theory nucleons and mesons appear explicitly, with the Dirac equation used to describe the former's motion. Though several problems still remain to be solved, many major advantages over the conventional non-relativistic theory with effective two-body interactions have been unambiguously established. A major test of the relativistic theory has been its predictive power of spin observables in elastic and inelastic proton nucleus scattering at intermediate cnergies, where the basic vector and scalar interactions which arises from w- and $\sigma$-meson exchange, respectively, can be constructed using the impulse approximation. For a more detailed and stringest comparison of the theory with experiment data at relatively large angles are required.

Quite recently, data of 200 MeV proton scattering from $^{208}Pb$ were measured at the Indiana University Cyclotron up to $\theta = 90^{\circ}$ where the cross-section drops to about $10^{-15}$ mb/sr [2] 500 MeV proton data are also available[3]. To perform an exact relativistic calculation of the

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cross-section up to these angles, a quite costly and lengthy numerical effort must be allocated. On the other hand, the much simpler Glauber (eikonal) approximation is obviously inadequate at the large angles involved. In this contribution we develop a new modified Dirac eikonal amplitude which may work well at large angles. This is accomplished through the use of a symmetrized version of the T-matrix with respect to the exact ingoing and outgoing scattering wave functions.

The starting point of our discussion is a relativistic generalization of the recently proposed symmetrical $T$-matrix, developed in great detail by Hussein and Marques[4].

$$<\vec{k}'|T|\vec{k}> = \int_0^1 d\lambda \quad <\psi^{(-)}_{\vec{k}'}(\lambda U^{\dagger})|U|\psi^{(+)}_{\vec{k}}(\lambda U)> \tag{1}$$

where $|\psi^{(+)}_{\vec{k}}(\lambda U)>$ is a four-component scattering wave function which satisfies Dirac's equation

$$|\psi^{(+)}_{\vec{k}}(\lambda U)> = |\vec{k}+> + (\not{p}-m+i\eta)^{-1}\lambda U|\psi^{(+)}_{\vec{k}}(\lambda U)> \quad , \tag{2}$$

where $|\vec{k}+>$ is a positive-energy plane wave solution of the free Dirac equation and the scaled interaction $\lambda U$ is left here as general as possible. consistent with relativistic covariance. The ingoing wave function $<\psi^{(-)}_{\vec{k}}(\lambda U^{\dagger})|$ is just the time-reversed version of eq.(2). Of course the 4×4 T-matrix itself, satisfies the Lippmann-Schwinger Diracequation with the unscaled potential

$$T = U + U(\not{p}-m+i\eta)^{-1}\, T \tag{3}$$

Eq.(1) is an alternative *exact* representation of the $T$-matrix.

Since what interests us here is just a 2x2 sub-matrix of the $T$-matrix, eq. (3), which describes the upper component only, we have to first project out from eq.(3) the lower component. Introducing the notation $T^{++}$ (upper), $T^{--}$ (lower), $T^{+-}$ (mixed upper-lower) etc., and the following spectral representation of the free Dirac Green function

$$(\not p - m + i\eta)^{-1} = \int d\vec{p}\,' \left\{ \frac{|\vec{p}\,'+\rangle\langle\overline{\vec{p}\,'+}|}{E_p - E_{p'} + i\eta} + \frac{|\vec{p}\,'-\rangle\langle\overline{\vec{p}\,'-}|}{E_p + E_{p'} - i\eta} \right\} \tag{4}$$

$$\equiv G_0^{(+)+} + G_0^{(+)-}$$

where

$$\langle\overline{\vec{p}\pm}| \quad \langle\vec{p}\pm|\,\gamma^0$$

we can write down two coupled matrix integral equations for $T^{++}$ and $T^{+-}$. Eliminating $T^{+-}$ in favor of $T^{++}$ we finally find

$$T^{++} = V^{++} + V^{++}\, G_0^{(+)+}\, T^{++} \tag{5}$$

with the 2x2 matrix interaction $V^{++}$ given by

$$V^{++} = U^{++} + U^{+-}\,(G_0^{(+)} - U^{--})^{-1}\, U^{-+} \; . \tag{6}$$

The second term in eq. (6) takes into account the virtual nucleon-antinucleon pair creation. This is the term which introduces genuinely relativistic correction to the now apparently non-relativistic $T^{++}$--matrix which satisfies the L-S equation[5].

We use eq. (5), instead of eq. (3), to derive the symmetrical form of the 2x2 rnatrix $T^{++}$, using the operator manipulations of ref.4:

$$\langle\vec{k}\,'|T^{++}|\vec{k}\rangle = \int_0^1 d\lambda \langle\psi^{(-)}_{\vec{k}\,'}(\lambda V^{++})^{+}|V^{++}|\psi^{(+)}_{\vec{k}}(\lambda V^{++})\rangle \quad . \tag{7}$$

The wave functions $|\psi^{(+)}_{\vec{k}}(\lambda V^{++})\rangle$ and $|\psi^{(-)}_{\vec{k}}(\lambda V^{++})\rangle$ satisfy the more familiar Lippmann-Schwinger equations

$$\begin{aligned} |\psi^{(+)}_{\vec{k}}(\lambda V^{++})\rangle &= |k+\rangle + G_0^{(+)+}(E_k)\,\lambda V^{++}|\psi^{(+)}_{\vec{k}}(\lambda V^{++}\rangle \\ |\psi^{(-)}_{\vec{k}}(\lambda V^{++})\rangle &= |k+\rangle + G_0^{(-)+}(E_k)\,\lambda V^{++}|\psi^{(-)}_{\vec{k}}(\lambda V^{++})\rangle \; . \end{aligned} \tag{8}$$

In what follows we shall develop modified eikonal expressions for the T-matrix both in the original Dirac form, eq. (1) and its Schrödinger equivalent form, eq. (7). To proceed, we write the following form for the wave functions[5]

$$\psi^{(\pm)}(\lambda V^{++})$$

$$\psi_{\vec{k}}^{(+)}(\lambda V^{++},\vec{r}) = e^{ik'.\vec{r}}\, e^{i\delta_{\lambda,\vec{k}}(\vec{r},V^{++})}\, \chi_s \tag{9}$$

$$\psi_{\vec{k}}^{(-)*}(\lambda V^{++},\vec{r}) = e^{-ik'.\vec{r}}\, e^{i\delta_{\lambda,\vec{k}'}(\vec{r},V^{++})}\, \chi_{s'},$$

where $\chi_s$ and $\chi$ , are the Pauli spinors for the ingoingandoutgoing solutions (of course both depend on the directions of the momenta, and thus the prime on s in the second spinor).

We are now in a position to derive an expression for the integrand

$$<\psi_{\vec{k}'}^{(-)}(\lambda u)|u|\psi_{\vec{k}}^{(+)}(\lambda u)>$$

in eq. (1)

$$<\psi_{\vec{k}'}^{(-)}(\lambda u)|u|\psi_{\vec{k}}^{(+)}(\lambda u)> = \int d\vec{r}\left(\frac{E+m}{2m}\right) e^{i\delta_{\lambda,\vec{k}'}(\vec{r})}\, e^{-i\vec{k}'.\vec{r}} \cdot \Big\{(V_s+V_v)$$

$$+\ \vec{\sigma}\cdot\vec{p}\,\frac{1}{E+m-\lambda(V_v-V_s)}\,(V_v-V_s)\,\frac{1}{E+m-\lambda(V_v-V_s)}\,\vec{\sigma}\cdot\vec{p}\Big\}\ .$$

$$\cdot\ e^{i\vec{k}\cdot\vec{r}}\, e^{i\delta_{\lambda,\vec{k}'}(\vec{r})} \tag{10}$$

where the lower components of the Dirac wave function have been eliminated.

It is now a simple matter to derive the equations that would determine the ingoing and outgoing eikonals, with the aid of eq.(3),(or, more precisely, the corresponding Dirac equation)

$$\frac{1}{m}\vec{k}.\vec{\nabla}\delta_{\lambda,\vec{k}'}(\vec{r}) + V_c(\lambda) + V_{so}(\lambda)\left[\vec{\sigma}.\ \vec{r}\wedge\vec{k} - i\vec{r}.\vec{k}\right] + V_{so}(\lambda)\left[\vec{\sigma}.(\vec{r}\wedge\vec{\nabla})\right.$$

$$\left.-\ i\vec{r}'.\vec{\nabla}\right]\delta_{\lambda,k'}(r') = 0 \tag{11}$$

and similarly for $\delta_{\lambda,\vec{k}'}(\vec{r})$. Eq. (11) was obtained, as usual, after dropping second order derivative terms and assuming the usual scalar vector form for $u$. In eq. (11), the scaled central and spin-orbit potentials are given by

$$V_C(\lambda) = A\ V_s + \lambda \frac{E}{m} V_v + \frac{\lambda^2}{2m} (V_-^2 - V_{,,}^2)$$

$$V_{so}(\lambda) = \frac{1}{2m} \frac{1}{E+m-\lambda(V_v-V_s)} \frac{1}{r} \frac{d}{dr} \left[\lambda(V_v - V_s)\right] \qquad (12)$$

We note here that the last term in $V_c$ (quadratic in the V's) is scaled by $h^2$. Secondly, the scaled spin-orbit interaction depends on $\lambda$ only through the potential term $\lambda(V_v - V_s)$.

Notice that eq. (12) and eq. (16) to follow are exact and have not appeared before in the literature.

The *eikonal* form of

$$<\psi_{\vec{k}'}^{(-)}(\lambda u)|u|\psi_{\vec{k}}^{(+)}(\lambda u)>$$

is now easily obtained by using eq. (11) in (10). We find

$$\int d\vec{r}\ \frac{E+m}{2m}\ e^{i\delta_{\lambda,\vec{k}'}(\vec{r})}\ e^{-i\vec{k}'.\vec{r}} \Big\{ V_s + V_v -$$

$$- \vec{\sigma}.\vec{p}\ \frac{\vec{\sigma}.\vec{r}}{r}\ \frac{1}{E+m-\lambda(V_v-V_s)}\ (V_v-V_s)\ \frac{1}{\frac{d}{dr}\ |\lambda(V_v-V_s)|}\ .$$

$$.\ (2i\vec{k}.\vec{\nabla}\delta_{\lambda,\vec{k}'} + 2miV_c) \Big\} e^{i\vec{k}.\vec{r}}\ e^{i\delta_{\lambda,\vec{k}}(\vec{r})} \qquad (13)$$

$$= <\psi_{\vec{k}}^{(-)}(\lambda u)|u|\psi_{\vec{k}}^{(+)}(\lambda u)>$$

We observe that a mixture of $\lambda$-scaled and unscaled quantities appears in the integrand. It would be interesting to compare eq. (13) with the corresponding (and formally identical) one for

$$<\psi^{(-)}(\lambda v^{++})|v^{++}|\psi^{(+)}(\lambda v^{++})>$$

of eq.(7). We find

$$<\psi^{(-)}(\lambda v^{++})|v^{++}|\psi^{(+)}(\lambda v^{++})> =$$

$$= \int d\vec{r} \left(\frac{E+m}{2m}\right) e^{i\delta_{\lambda,\vec{k}'}(\vec{r})} e^{-i\vec{k}'.\vec{r}} \;.$$

$$. \left\{V_c - iV_{so}\left[\vec{\sigma}.\vec{r}\wedge\vec{\nabla} - i\vec{r}.\vec{\nabla}\right]\right\} e^{i\delta_{\lambda,\vec{k}}(\vec{r})} e^{i\vec{k}.\vec{r}} \qquad (14)$$

$$= \int d\vec{r}\left(\frac{E+m}{2m}\right) e^{i\vec{q}.\vec{e}} e^{i\delta_{\lambda,\vec{k}'}(\vec{r})} \;.$$

$$. \left\{- \frac{1}{\lambda m}\vec{\nabla}(\vec{k}.\vec{r}) \;.\; \vec{\nabla}\delta_{\lambda,\vec{k}}(\vec{r})\right\} e^{i\delta_{\lambda,\vec{k}}(\vec{r})} \quad ; \qquad (15)$$

$$\vec{q} = \vec{k} - \vec{k}'$$

The equation that determines $S_\lambda$ in this case is

$$\frac{1}{m}\vec{\nabla}(\vec{k}.\vec{r}).\vec{\nabla}S_{\lambda,\vec{k}} + \lambda V_c + \lambda V_{so}\vec{\sigma}.(\vec{r}\times\vec{\nabla})(S_{\lambda,\vec{k}})$$
$$(16)$$
$$-i\lambda V_{so}\vec{r}.\vec{\nabla}(S_{\lambda,\vec{k}}) + \lambda V_{so}\vec{\sigma}.(\vec{r}\times\vec{k}) - i\lambda V_{so}\vec{r}.\vec{k} = 0$$

where $V_c$ and $V_{so}$ are given by eq. (12) with A equal to 1. The difference between the symmetrical Dirac representation and the symmetrical Schrödinger representation can be more easily seen by comparing the way $\lambda$ modifies the central spin-orbit interaction. We exhibit in fig.1 a plot of $V_c(\lambda)/V_c(\lambda=1)$ vs. $\lambda$, with $V_v$ and $V_s$ taken from + $^{40}$Ca elastic scattering Dirac fit at $E$ = 500 MeV. ($V_v$ - + 270 MeV and $V_s \simeq$ $\simeq$ - 400 MeV) and using for the radial density shape a constant value of 1 (nuclear matter)[6]. We see clearly that the Dirac scaled central interaction is attractive in the interval $0 < \lambda < 0.5$ and repulsive in the interval $0.5 < \lambda \leqslant 1$. This is to be compared with the Schrödinger equívalent scaled potential which is purely repulsive in the whole $\lambda$--interval.

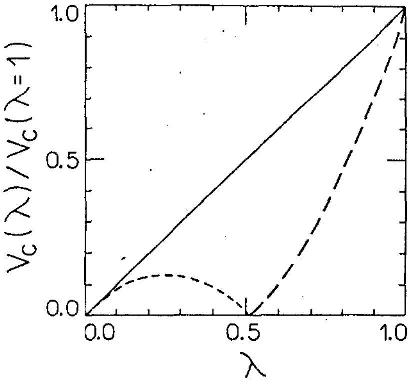


Fig.1 - A plot of $V_c(\lambda)/V_c(\lambda=1)$ vs. λ. Short dashed line represents negative values. Full line, the scaled Schrödinger $V_c$. See text for details.

The imaginary part of $V_c(\lambda)$, however is always negative in the whole λ-interval, guaranteeing thus the absorptive nature of the scattering process. In fact, we have for the same system,

$$\text{Im}\, V_c^{\text{Dirac}}(\lambda) = -14\,\lambda + 3.75\,\lambda^2 \text{ (MeV)}$$

$$\text{Im}\, V_c^{\text{Schrödinger}} = -10.251 \text{ (MeV)} \qquad (17)$$

As a final remark, the modified Glauber expression for $\langle \vec{k}'|T|\vec{k}\rangle$ developed above should be a more adequate large angle high-energy approximation. Which one of the representations, eq. (13) or eq. (15) is more convenient can only be settled through a detailed numerical comparison, which will be carried out soon. Further, we believe that our modified symmetrical Glauber model supplies a better representation of the $T$-matrix at large angles as compared to other approaches. The reason is that in our derivation using the symmetrical form of $T$, nowhere have we used the small-angle approximation. In fact, we consider our approach to be based on a high-energy approximation to the ingoing and outgoing wave functions, valid at all angles.

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### Resumo

Estende-se a Teoria de Matriz T Simétrica, recentemente desenvolvida, para a descrição relativística do espalhamento próton- núcleo. Utilizando-se técnicas de projeção reduz-se a matrix 4x4 simétrica a uma matriz efetiva 2x2 também simétrica. A matriz T resultante, apesar de sua aparência não relativística, contém efeitos relativísticos (pares virtuais) em todas as ordens. Aplica-se a Teoria de Glauber ,para se desenvolver uma aproximação razoável e que poderia ser válida para grandes ângulos.