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Electromagnetic Moments in Odd Tin Isotopes

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Abstract This work presents a systematic study of E2 and M1 transitions and electric quadrupole and magnetic dipole moments of odd tin isotopes in the context of a quasiparticle-vibration model.

1. INTRODUCTION

The tin isotope region is perhaps the best studied of the mass table, from both experimental[1-3] and theoretical[4-7] points of view. Nevertheless, previous calculations have not analysed systematically the detailed phonon contributions to the electromagnetic moments.

In the present work the transition probabilities between the lowest $1/2^+$, $3/2^+$, $5/2^+$, $7/2^+$ and $11/2^-$ levels in the odd tin isotopes from A = 115 to A = 123 are obtained in a model[8] in which the states of the odd nucleus are represented by a quasiparticle (QP) coupled to components of zero, one and two vibrational 2^+ phonons. Although this model is able to describe other properties of these nuclei as energy levels and spectroscopic factor systematics, the focus of the present work is the test of the wave functions from a electromagnetic point of view. Also presented are the static multipole moments of these levels.

2. ELECTROMAGNETIC MOMENT OPERATORS

The wavefunctions of odd tin nuclei were assumed[8] as:

$$|\psi_\alpha\rangle = A_\alpha C_\alpha^+|\psi_0\rangle + \sum_c A_{ac2}[C_c^+B_2^+]_\alpha|\psi_0\rangle + \sum_{cI} A_{acI}[C_c^+[B_2^+B_2^+]_I]_\alpha |\psi_0\rangle$$

with the usual meaning attached to the quasiparticle and one phonon creation operators.

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The matrix elements of the electric transition operator can be expressed, in a compact way that puts in evidence the contributions of processes involving the emission and absorption of phonons, as

$$<\psi_\beta|M(E\lambda 0)|\psi_\alpha> = \sum_{N_1,N_2} (\alpha\beta E\lambda;N_1,N_2)$$

where N_1 and N_2 represent, respectively, the number of phonons coupled to the quasiparticle in the final and the initial state, so that

$$(\alpha\beta E\lambda;0,0) = A_b A_a Q^E_\lambda(ab) P(\alpha\beta E\lambda)$$

$$(\alpha\beta E\lambda;0,1) = A_b A_{ab2} R(\alpha\beta E\lambda)(-1)^{j_a-j_b}\frac{2j_a+1}{2j_b+1}$$

$$(\alpha\beta E\lambda;1,0) = A_{ba2} A_a R(\alpha\beta E\lambda)$$

$$(\alpha\beta E\lambda;1,1) = \sum_{cc'} A_{bc2} A_{ac'2} \begin{Bmatrix} j_a & \lambda & j_b \\ j_c & 2 & j_{c'} \end{Bmatrix} T_{cc'}(\alpha\beta E\lambda)$$

$$(\alpha\beta E\lambda;1,2) = 2\sum_{cI} A_{bc2} A_{acI} \begin{Bmatrix} j_b & 2 & j_a \\ I & j_c & 2 \end{Bmatrix} S_c(\alpha\beta E\lambda)$$

$$(\alpha\beta E\lambda;0,2) = (\alpha\beta E\lambda;20) = 0 ,$$

since the operator $M(E\lambda 0)$ is not able to create or annihilate two phonons,

$$(\alpha\beta E\lambda;2,1) = 2\sum_{cI} A_{bcI} A_{ac2} \begin{Bmatrix} j_b & 2 & j_a \\ 2 & j_c & I \end{Bmatrix} S_c(\alpha\beta E\lambda)$$

$$(\alpha\beta E\lambda;2,2) = 2\sum_{cc'} A_{bcI} A_{ac'I} \begin{Bmatrix} j_a & \lambda & j_b \\ j_c & I & j_{c'} \end{Bmatrix} T_{cc'}(\alpha\beta E\lambda)$$

where

$$P(\alpha\beta E\lambda) = i^{-\lambda} e_{\text{eff}} \sqrt{\frac{2\lambda+1}{2j_b+1}}\, C^{j_a\ \lambda\ j_b}_{m_a\ 0\ m_b} (u_a u_b - v_a v_b)$$

$$R(\alpha\beta E\lambda) = i^{-\lambda} \left[B(E2,\ 2^+ \to 0)\right]^{\frac{1}{2}} C^{j_a\ 2\ j_b}_{m_a\ 0\ m_b}\ \delta_{\lambda 2}$$

$$S_c(\alpha\beta E\lambda) = \sqrt{(2j_a+1)(2I+1)}\ (-1)^{j_c+j_b}\ R(\alpha\beta E\lambda)$$

$$T_{cc'}(\alpha\beta E\lambda) = Q^E_\lambda(cc') \sqrt{\frac{2j_a+1}{2j_b+1}}\ (-1)^{j_c+j_a-\lambda}\ P(\alpha\beta E\lambda)$$

The one *phonon* reduced transition probability from the first excited state $|\psi_{JM}>$ to the ground state was, for the even tin isotopes, obtained as :

$$B(E\lambda,\ J \to 0) = \delta_{\lambda J}\ \left|\sum_{ab} \frac{e_{\text{eff}}}{2}\ U^{(-)}_{ab}\ Q^E_\lambda(ab)(X_{ab\lambda} + Y_{ab\lambda})\right|^2$$

where

$$Q^E_\lambda(ab) = \frac{<a||i^\lambda\ r^\lambda\ Y_\lambda||b}{\sqrt{2\lambda+1}}$$

$$U^{(-)}_{ab} = u_a u_b - v_a v_b$$

$$u_a = \sqrt{1 - v_a^2}\ ,$$

v_a is the usual occupation probability, for each single particle level and $X_{ab\lambda}$ and $Y_{ab\lambda}$ are, respectively, quasi particle pair creation and pair annihilation amplitudes, with values fixed by the Random Phase Approximation (RPA),

The occupation probabilities v_a and the RPA amplitudes were calculated exactly as in reference 8, using the same model space (corresponding to the major shell 50-82) and the same schematic interactions (delta + quadrupole-quadrupole).

The RPA amplitudes X_{ab2} and Y_{ab2} for a typical nucleus, ^{115}Sn, are presented in table I.

The effective charge was taken as $e_{\text{eff}} = 1$ for the neutron.

Table 1 - RPA amplitudes X_{ab2} and Y_{ab2} for the ^{115}Sn nucleus, related to the quasi-particles a and b in the specified orbitals.

^{115}Sn			
j_a^π	j_b^π	X_{ab2}	Y_{ab2}
$\frac{3}{2}^+$	$\frac{1}{2}^+$	-0.5131925	-0.144968
$\frac{5}{2}^+$	$\frac{1}{2}^+$	-0.4098769	-0.1512218
$\frac{3}{2}^+$	$\frac{3}{2}^+$	0.3695617	0.1183848
$\frac{5}{2}^+$	$\frac{3}{2}^+$	-0.2214978	-0.0882475
$\frac{7}{2}^+$	$\frac{3}{2}^+$	-0.5669232	-0.1874887
$\frac{5}{2}^+$	$\frac{5}{2}^+$	0.1918547	0.0883295
$\frac{7}{2}^+$	$\frac{5}{2}^+$	-0.0894182	-0.0363534
$\frac{7}{2}^+$	$\frac{7}{2}^+$	0.5061929	0.1724978
$\frac{11}{2}^-$	$\frac{11}{2}^-$	0.4611327	0.1845116

The analysis made on the electric matrix elements applies also to the magnetic matrix elements

$$\langle\psi_\beta|M(M\lambda 0)|\psi_\alpha\rangle = \sum_{N_1,N_2} (\alpha\beta M\lambda;N_1,N_2) \ .$$

In this case

$$P(\alpha\beta M\lambda) = i^{-\lambda+1}\,\mu_N \left[\frac{2\lambda+1}{2j_b+1}\right]^{\frac{1}{2}} C^{j_a\ \lambda\ j_b}_{m_a\ 0\ m_b}\,(u_a u_b + v_a v_b)$$

$$R(\alpha\beta M\lambda) = i^{-\lambda+1}\,\delta_{\lambda 2}\left[B(M2,\ 2^+\to 0)\right]^{\frac{1}{2}} C^{j_a\ 2\ j_b}_{m_a\ 0\ m_b}$$

$$S_c(\alpha\beta M\lambda) = \sqrt{(2j_a+1)(2I+1)}\ (-1)^{j_c+j_b}\ R(\alpha\beta M\lambda)$$

$$T_{cc'}(\alpha\beta M\lambda) = Q_\lambda^M(cc')\sqrt{\frac{2j_a+1}{2j_b+1}}\ (-1)^{j_c+j_a-\lambda}\ P(\alpha\beta M\lambda)$$

$$Q_\lambda^M(ab) = \langle a\| i^{\lambda-1}\ \sqrt{\lambda}\ r^{\lambda-1}\left[\left(g_s - \frac{2g_\ell}{\lambda+1}\right)(Y_{\lambda-1}s) + \frac{2g_\ell}{\lambda+1}(Y_{\lambda-1}j)\right]\| b\rangle.$$

The electric quadrupole moment is, in the notation adopted, given as

$$Q = \sqrt{\frac{16\pi}{5}}\sum_{N_1,N_2}(a,\ a,\ E!;\ N_1,\ N_2)$$

while the magnetic dipole moment is calculated by

$$\mu = \sqrt{\frac{4\pi}{3}}\sum_{N_1,N_2}(\alpha,\ \alpha,\ M1;\ N_1,\ N_2)$$

3. RESULTS AND DISCUSSIONS

In table 2 the reduced transition probabilities between the lowest $1/2^+$, $3/2^+$, $5/2^+$, $7/2^+$ and $11/2^-$ levels in odd tin isotopes are presented in comparison with existing experimental data[1-3], Also shown are the detailed contributions of the several processes, as obtained in the present calculation, represented by the number of phonons (N_1, N_2) in the final and initial states. Colums 4 presents the single particle transition probability, calculated with oscillator wave functions ($\hbar\omega_{osc} = 41.A^{-1/3}$ MeV).

The prominent difference between the single particletransition probability $B(E2)_{sp}$ and the (0,0) part of $B(E2)_{theor}$ indicates a large influence of the combined effects of pairing and collective motion, as put forward by a considerable decrease in the single particle amplitudes The transitions $3/2^+ \to 1/2^+$, $5/2^+ \to 1/2^+$, $5/2^+ \to 3/2^+$ and $5/2^+ \to 7/2^+$ show good agreement with the experimental data, for all isotopes for which these are available. Particularl'y, for the $3/2^+ \to 1/2^+$ transitions, Table.2 - Reduced electric quadrupole transitions. Column 1 gives the initial and final state spin-parity of each transition. Columns 2 and 3 list the mass number and the experimental transition probabilities. Column 4 the single particle transition probability calculated with oscillator wave functions. Columns 5 to 9 list the contributions of the several processes represented by the number of phonons (N_1,N_2) in the final and initial state. Last column gives the total theoretical transition probability.

Transition	A	$B(E2)_{exp}$ (e^2fm^4)	$B(E2)_{sp}$ (e^2fm^4)	$B(E2)_{theor}$ (e^2fm^4) (0,0)	(0,1)+(1,0)	(1,1)	(1,2)+(2,1)	(2,2)	Total
$\frac{3}{2}^+ \to \frac{1}{2}^+$	115	70.1^c	53.9	6.07	49.7	0.0240	0.151	$\sim 10^{-8}$	101
	117	3.10^a	54.5	0.194	1.68	$\sim 10^{-5}$	$\sim 10^{-3}$	$\sim 10^{-4}$	2.64
	119	$< 22^a$	55.1	1.96	14.3	0.0665	0.140	$\sim 10^{-4}$	34.0
$\frac{5}{2}^+ \to \frac{1}{2}^+$	115	75.7^c	53.9	6.82	104	0.0899	2.69	$\sim 10^{-4}$	219
	117	207^a	54.5	10.2	233	0.0878	13.0	0.0121	505
	119	200^a	55.1	10.2	288	0.188	22.3	0.0297	650
$\frac{5}{2}^+ \to \frac{3}{2}^+$	115	11.0^c	16.6	0.224	1.92	$\sim 10^{-5}$	0.0422	$\sim 10^{-4}$	2.71
	117	44^a	16.8	0.781	8.16	$\sim 10^{-6}$	0.0254	$\sim 10^{-4}$	15.1
	119	$[56]^a$	17.0	1.31	14.7	$\sim 10^{-3}$	0.0647	$\sim 10^{-4}$	26.8
$\frac{5}{2}^+ \to \frac{7}{2}^+$	115	87.3^b	6.60	2.40	20.6	$\sim 10^{-5}$	0.307	$\sim 10^{-8}$	44.2
$\frac{7}{2}^+ \to \frac{3}{2}^+$	115	4.7^b	44.6	$\sim 10^{-3}$	0.145	$\sim 10^{-6}$	$\sim 10^{-3}$	$\sim 10^{-7}$	0.260
	117	11.0^b	45.1	1.63	17.7	$\sim 10^{-3}$	0.254	$\sim 10^{-5}$	36.4
	119	11.0^b	45.6	4.96	82.2	0.0229	0.663	$\sim 10^{-6}$	150
	121	2.8^b	46.1	6.97	195	0.179	2.35	$\sim 10^{-4}$	345
		$> 5^b$	46.6	6.60	257	0.557	5.98	$\sim 10^{-3}$	478

a = ref. (1) ; b = ref. (2) ; c = ref. (3)

the abrupt variation in $B(E2)_{exp}$ observed between $A = 115$, $A = 117$ and $A = 119$ could be reproduced. This fact can be understood by analysing the variation of the coefficient $(u_{3/2}\,u_{1/2} - v_{3/2}\,v_{1/2})$ in the (0,0) component of $B(E2)_{theor}$. Also for the $5/2^+ \to 1/2^+$ and $5/2^+ \to 3/2^+$ transition the calculations predict, in both cases, the correct trend, although the absolute values of $B(E2)_{exp}$ are not totally reproduced. It can be remarked that an alternative procedure which avoids the inconvenient problem of attributing effective charges and spin gyromagnetic factors is the comparison between ratios of theoretical and experimental results, taking a nucleus (for example ^{115}Sn) as reference [10].

On the other hand, the data obtained for the $7/2^+ \to 3/2^+$ transitions by Fogelberg *et al.*[2] could not be well reproduced by the model.

In all these transitions we can observe, as general trend, the importance of contributions that include one phonon in the initial state or one phonon in the final state, indicated in the table as (0,1)+(1,0). The contributions (1,2) + (2,1) are small in general but are relevant in the case of the $5/2^+ \rightarrow 1/2^+$ transition. The contribution from (1,1) and (2,2) are always very small.

In table 3, organized in a way similar to table 2 the static quadrupole moments of the $3/2^+$, $1/2^+$ and $11/2^-$ levels are presented. Also here the remarks made in the case of transition probabilities about the influence of pairing and collective effects are pertinent. It is interesting to note that the present calculation can reproduce the change of sign that occurs for the $3/2^+$ level between A = 119 and A = 121. This can be understood by examining the factor $(u^2_{3/2} - v^2_{3/2})$ that appears in the (0,0) contributions to Q_{theor}. When the occupation probability is close to 0.5 its sign can change from one nucleus for the next neighbor isotope.

The magnetic dipole reduced transition probabilities $B(M1)$ are listed in table 4. For $3/2^+ \rightarrow 1/2^+$ and $5/2^+ \rightarrow 7/2^+$ transitions, the contributions of $B(M1)_{sp}$ and (0,0) are zero because of selection rules for magnetic transitions. The contributions (0,1) + (1,0) and (1,2) + (2,1) are zero in all transitions because the nuclear model used does not include states of angular momentum $J^\pi = 1^+$. The inclusion of these states in the calculation would certainly improve the agreement with experimental data.

Table 3 - Electric quadrupole moments. Similar to table 2; experimental data compiled by Van Gunsteren *et al.*, ref.7.

Level	A	Q_{exp} ($e\,fm^2$)	Q_{sp} ($e\,fm^2$)	Q_{theor} ($e\,fm^2$)					
				(0,0)	(0,1)+(1,0)	(1,1)	(1,2)+(2,1)	(2,2)	Total
$\frac{3}{2}^+$	119	-6	-10.9	-1.76	-5.74	0.102	-0.246	$\sim 10^{-3}$	-7.64
	121	±8	-11.0	0.922	3.66	0.437	0.439	0.0186	5.48
$\frac{7}{2}^+$	115	±26	-18.0	10.6	31.9	0.519	3.79	0.0240	46.7
$\frac{11}{2}^-$	113	(-)140	-24.4	-15.2	-49.2	-3.81	-19.7	-0.788	-88.7
	115	±40	-24.6	-15.3	-46.3	-3.01	-13.9	-0.445	-78.8
	119	-20	-24.9	-11.8	-36.2	-1.17	-5.94	-0.101	-55.2

Table 4 - Magnetic dipole transitions. Similar to table 2.

Transition	A	$B(M1)_{exp}$ (μ_N^2)	$B(M1)_{sp}$ (μ_N^2)	$B(M1)_{theor}$ (μ_N^2) (0,0)	(0,1)+(1,0)	(1,1)	(1,2)+(2,1)	(2,2)	Total
$\frac{3}{2}^+ \to \frac{1}{2}^+$	115	0.0272[c] / 0.000133	0	0	0	0.00244	0	$\sim 10^{-7}$	0.00239
	117	0.027[a]	0	0	0	0.00112	0	$\sim 10^{-7}$	0.00118
	119	0.027[a]	0	0	0	$\sim 10^{-4}$	0	$\sim 10^{-6}$	$\sim 10^{-4}$
$\frac{5}{2}^+ \to \frac{3}{2}^+$	115	0.114[c] / 0.0067	1.40	0.398	0	0.0170	0	$\sim 10^{-5}$	0.256
	117	0.112[a]	1.40	0.361	0	0.00303	0	$\sim 10^{-5}$	0.306
	119	[0.035][a]	1.40	0.317	0	0.00361	0	$\sim 10^{-5}$	0.397
$\frac{5}{2}^+ \to \frac{7}{2}^+$	115	0.0120[c] / 0.00190	0	0	0	$\sim 10^{-4}$	0	$\sim 10^{-5}$	0.00126

a = ref. (1) ; b = ref. (2) ; c = ref. (3)

In table 5 the magnetic dipole moments are presented, The influente of 2^+ phonons on the final results in the present calculation is given only for (1,1) and (2,2) contributions. Also here the absence of $J^\pi = 1^+$ states in the nuclear model is felt. As is well known this absence is equivalent to neglecting important core polarization effects[9].

Table 5 - Magnetic dipole rnoments. Similar to table 2; experimental data compiled by Van Gunsteren et *al.*, ref.7.

Level	A	μ_{exp} (μ_N)	μ_{sp} (μ_N)	μ_{theor} (μ_N) (0,0)	(0,1)+(1,0)	(1,1)	(1,2)+(2,1)	(2,2)	Total
$\frac{1}{2}^+$	113	±0.88	-1.91	-1.69	0	0.0169	0	-0.00324	-1.68
	115	-0.92	-1.91	-1.73	0	0.0108	0	-0.0126	-1.73
	117	-1.00	-1.91	-1.69	0	-0.0640	0	-0.0302	-1.79
	119	-1.05	-1.91	-1.66	0	0.189	0	-0.0425	-1.89
4'	117	0.,	1.15	1.08	0	-0.00662	0	0.00282	1.08
	119	0.66	1.15	1.10	0	0.0126	0	0.00218	1.12
	121	10.70	1.15	1.08	0	0.0657	0	0.00554	1.15
$\frac{7}{2}^+$	115	0.68	1.49	1.32	0	0.123	0	0.00806	1.45
$\frac{11}{2}^-$	113	-1.29	-1.91	-1.38	0	-0.421	0	-0.106	-1.91
	115	-1.37	-1.91	-1.48	0	-0.355	0	-0.0636	-1.90
	119	-1.4	-1.91	-1.69	0	-0.203	0	-0.0217	-1.92

An enlargement of model space, taking into account also the neighboring major shells, is expected to be important essentially forthe magnetic moment and magnetic transition probabilities. Ontheother hand, a more realistic treatment of the interaction, including non-central terms, may have influence on both electric and magnetic properties and is a subject under investigation by the authors.

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Resumo

Este trabalho estuda sistematicamente as transições E2 e M1 e os momentos de quadrupolo elétrico e de dipolo magnéticos nos isótopos ímpares de estanho usando um modelo do tipo quasipartícula vibração.