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# Hermitian Metric in Gravitation

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**Abstract** A gravitational model, with an Hermitian metric, is proposed here. This approach leads to Einstein-Cartan equations, for non symmetric fields. By means of a minimum limit, stated for the radius coordinate, the singularity at the origin is eliminated. The redshift, the bending of light and the shift of the perihelium of Mercury are examined.

## 1. INTRODUCTION

The idea of considering a non symmetric metric in gravitation is not new. Einstein[1] proposed and analized this subject before. Papapetrou[2] dealed later with such a metric, for a statically and spherically symmetric case. Recently Moffat[3] developed a gravitational model, with a Hermitian metric, and pointed out the existence of a minimum limit for the radius, which helps to clear up singularity problems, at the origin.

Here we deal with an approach based on Moffat's idea; however the geometrical aspects we take are established on fiber bundle techniques. Einstein-Cartan equations, for non symmetric fields, appear naturally, and the Poincaré group is taken as a symmetry group. The assumption of an assymmetrical connection and solder form leads to curvature and torsion. A total Lagrangian is proposed and, by means of Euler-Lagrange equations, Einstein-Cartan equations are derived. An additional condition is required: the signature of the metric cannot be changed, and this points out the existence of a core centered at the origin. This eliminates the singularity at $r=0$.

At the end, redshifts of electromagnetic radiation are calculated, and the results given agree with experimental data. A small correction term to the prediction of General Relativity (GR) arises. The bending of light by the sun and the shift of the perihelium of Mercury are also examined.

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The compact language of differential forms will be used when necessary.

## 2. GEOMETRICAL ASPECTS

We consider initially a $P$ bundle of linear frames, where Minkowski space-time is the base manifold M, and the Poincaré group $G = SO(3,1) \otimes T_4$ is the symmetry group. Let $\{dx^\mu\}$ be a holonomic basis on M, and

$$g = g_{\alpha\beta}\, dx^\alpha \otimes dx^\beta \ , \ (\alpha,\beta = 1,\dots,4) \tag{2.1}$$

the metric tensor.

Now we assume that the components $g_{\alpha\beta}$ may be decomposed into a symmetrical part $s_{\alpha\beta}$, and into a skew-symmetrical part $a_{\alpha\beta}$, by means of

$$g_{\alpha\beta} = \frac{1}{2}\,(g_{\alpha\beta} + g_{\beta\alpha}) + \frac{i}{2}\,(g_{\alpha\beta} - g_{\beta\alpha}) = s_{\alpha\beta} + ia_{\alpha\beta} \ . \tag{2.2}$$

To generalize the assumption of symmetric components $g_{\alpha\beta}$ in GR, we require that $g_{\alpha\beta}$ have Hermitian symmetry, i.e.

$$g_{\alpha\beta} = g^*_{\beta\alpha} \ . \tag{2.3}$$

So, the matrix representation for the tensor g is

$$(g_{\alpha\beta}) = \begin{bmatrix} s_{11} & s_{12}+ia_{12} & s_{13}+ia_{13} & s_{14}+ia_{14} \\ s_{12}-ia_{12} & s_{22} & s_{23}+ia_{23} & s_{24}+ia_{24} \\ s_{13}-ia_{13} & s_{23}-ia_{23} & s_{33} & s_{34}+ia_{34} \\ s_{14}-ia_{14} & s_{24}-ia_{24} & s_{34}-ia_{34} & s_{44} \end{bmatrix} \tag{2.4}$$

and the spur of $(g_{\alpha\beta})$ is real,

$$\mathrm{tr}(g_{\alpha\beta}) = \sum_{\alpha=1}^{4} s_{\alpha\alpha} \ . \tag{2.5}$$

The signature $s$ of $(g_{\alpha\beta})$ may be negative, null or positive, depending on the signs of $s_{\alpha\alpha}$. The components $g^{\alpha\beta}$ are given by

$$g^{\sigma\nu} g_{\lambda\nu} = g^{\mu\sigma} g_{\mu\lambda} = \delta^{\sigma}_{\lambda} \quad , \tag{2.6}$$

where the order of the indexes must be taken into account.

A connection form $W$, on the $P$ bundle, is a 1-form valued on the Lie algebra of G [4]. Considering W as a linear connection, it may be given by[5]

$$W = \Gamma + S \ . \tag{2.7}$$

Here the connection $\Gamma$ is valued on the algebra of the Lorentz group,

$$\Gamma = J_a{}^b \, \Gamma^a{}_{b\mu} \, dx^\mu \ , \tag{2.8}$$

and $S$ is the solder form[6], valued on the algebra of the $T_4$, group,

$$S = I_a \, S^a_\mu \, dx^\mu \quad . \tag{2.9}$$

Both connections are written on the basis $\{dx^\mu\}$ mentioned before. In expressions (2.8) and (2.9) $J_a{}^b$ and $I_a$ are the generatorç of the Lorentz group and the group of translations, respectively. These generators satisfy the commutation rules below[7], which establish the Poincaré group algebra

$$\begin{aligned}
[J_a{}^b, J_c{}^d] &= \frac{1}{2} (\eta_a{}^d J_c{}^b - \eta_{ac} J^{bd} + \eta_c{}^d J_a{}^d - \eta^{bd} J_{ac}) \ , \\
[J_a{}^b, I_c] &= \frac{1}{2} (\eta_c{}^b I_a - \eta_{ac} I^b) \quad , \\
[I_a, I_b] &= 0 \quad ,
\end{aligned} \tag{2.10}$$

where a, $b$, ... = 1, ..., 4 and $\eta$ is the Minkowski metric.

The curvature $F$, of the connection $\Gamma$, is[4]

$$F = d\Gamma + \Gamma \wedge \Gamma \ , \tag{2.11}$$

and has the components

$$F^{a}{}_{b\mu\nu} = \partial_\mu \Gamma^{a}{}_{b} - \partial_\nu \Gamma^{a}{}_{b} + \Gamma^{a}{}_{c\mu}\Gamma^{c}{}_{b\nu} - \Gamma^{a}{}_{c\nu}\Gamma^{c}{}_{b\mu} \qquad (2.12)$$

on the holonomic basis $\{dx^\mu\}$.

These components may be written with space-time indexes, by means of the four-legs $h^{a}_{\alpha}$ [5]

$$F^{\alpha}_{\beta\mu\nu} = h^{\alpha}_{a} h^{b}_{\beta} F^{a}_{b\mu\nu} = \partial_\mu \Gamma^{\alpha}_{\beta\nu} - \partial_\nu \Gamma^{\alpha}_{\beta\mu} + \Gamma^{\alpha}_{\sigma\mu}\Gamma^{\sigma}_{\beta\nu} - \Gamma^{\sigma}_{\sigma\nu}\Gamma^{\sigma}_{\beta\mu} \quad , \qquad (2.13)$$

which presupposes the connection $\Gamma$ as already projected onto the base manifold. From expression (2.13) we can obtain the contracted components of the curvature

$$F_{\beta\mu} = \partial_\mu \Gamma^{\alpha}_{\beta\alpha} - \partial_\nu \Gamma^{\nu}_{\beta\mu} + \Gamma^{\alpha}_{\sigma\mu}\Gamma^{\sigma}_{\beta\alpha} - \Gamma^{\alpha}_{\sigma\alpha}\Gamma^{\sigma}_{\beta\mu} \quad . \qquad (2.14)$$

In GR theory the $F^{\alpha}_{\beta\mu\nu}$ are the components of the Riemann tensor and $F_{\beta\mu}$ the components of the Ricci tensor, because in that case $\Gamma$ is a symmetrical connection. However, in our case, the $\Gamma^{\alpha}_{\mu\nu}$ are asymmetrical.

The curvature $F$ satisfies Bianchi's identity

$$dF + [\Gamma, F] = 0 \quad , \qquad (2.15)$$

where $d$ is the exterior derivative operator. Interpreting $\Gamma$ as a gauge potential, then $F$ is the corresponding gauge field, and the field equations are Yang-Mills (YM) equations. In the sourceless case, YM equations are the Bianchi identities, but written for the dual of $F$ [6]:

$$d * F + [\Gamma, *F] = 0 \quad . \qquad (2.16)$$

When projected onto the base manifold, YM equations become[5]

$$F^{\alpha}{}_{\beta\mu\nu}{}^{;\mu} = 0 \quad . \qquad (2.17)$$

Using Bianchi identity for $F^{\alpha}_{\beta\mu\nu}$, and lowering indexes with $g_{\alpha\mu}$, eq. (2.17) reduces to

$$F_{\alpha\beta;\lambda} - F_{\alpha\lambda;\beta} = 0 \quad . \qquad (2.18)$$

In the particular case of a Levi-Civita connection, and for a real and symmetrical metric, eq. (2.18) reduces to Yang's gravitational equation

$$R_{\alpha\beta;\lambda} - R_{\alpha\lambda;\beta} = 0 \quad , \tag{2.19}$$

for the Ricci tensor[7]. It has Einstein sourceless equation

$$R_{\alpha\beta} = 0 \ , \tag{2.20}$$

as a very particular solution. So, eq. (2.18) is a generalization of Yang's equation.

YM equations may be written in a equivalent way to eq. (2.16):

$$\delta F + *^{-1} \left[\Gamma, *F\right] = 0 \ , \tag{2.21}$$

where

$$\delta = *^{-1} d* = (-1)^{p(n-p) + \frac{n-s}{2}} *d* \ , \tag{2.22}$$

is the coderivative exterior operator[a]; $p$ is the degree of the differential form to be used, $n$ is the dimension of the base manifold and $s$ is the signature of space-time metric. In the above case $p=3$ and $n=4$. The signature may assume the values 0, $\pm 2$ and $\pm 4$. In order to be coherent to GR theory, we will choose either the values +2 or -2.

The torsion T, of the connection $\Gamma$, is given by the covariant derivative of the solder form, related to $\Gamma$ itself[9]:

$$T = D_\Gamma S = ds + \Gamma \wedge S + S \wedge \Gamma + S \wedge S \tag{2.23}$$

and has the components

$$T^a_{\mu\nu} = \partial_\mu S^a_\nu - \partial_\nu S^a_\mu + \Gamma^a_{c\mu} S^c_\nu - \Gamma^a_{c\nu} S^c_\mu \ , \tag{2.24}$$

in the holonomic basir $\{dx^\mu\}$. By choosing a basis where $S^a_\nu = \delta^a_\nu$ and supposing the connection $\Gamma$ as already projected onto the base manifold

M, the components of T are

$$T^{\alpha}_{\mu\nu} = \Gamma^{\alpha}_{\nu\mu} - \Gamma^{\alpha}_{\mu\nu} \; . \tag{2.25}$$

## 3. LAGRANGIAN FORMALISM

The Lagrangian density, which leads to the field equations, is

$$L = \tilde{g}^{\mu\nu}\left[F_{\mu\nu}(\Gamma) + kE_{\mu\nu}\right] \; , \tag{3.1}$$

where

$$\tilde{g}^{\mu\nu} = \sqrt{-g}\; g^{\mu\nu} \; , \tag{3.2}$$

$E_{\mu\nu}$ is the stress energy and k is a constant.

The action integral is

$$A = \int d^4x \; L \; , \tag{3.3}$$

and the external condition $\delta A = 0$ gives Euler-Lagrange equations. To make up for this variation we have to consider that[10]

$$\begin{aligned} \delta\tilde{g}^{\mu\nu} &= \sqrt{-g}\;(1 - \tfrac{1}{2}\, g_{\mu\nu}\;\; g^{\mu\nu})\,\delta g^{\mu\nu} \; , \\ \delta F_{\mu\nu} &= (\delta\Gamma^{\sigma}_{\mu\sigma})_{;\nu} - (\delta\Gamma^{\sigma}_{\mu\nu})_{;\sigma} + (\Gamma^{\sigma}_{\alpha\nu} - \Gamma^{\sigma}_{\nu\alpha})\,\delta\Gamma^{\alpha}_{\mu\sigma} \; . \end{aligned} \tag{3.4}$$

After a straighforward calculation we are led to the equations

$$F_{\mu\nu} - \frac{1}{2}\, g_{\mu\nu}\, F = kE_{\mu\nu} \; , \tag{3.5}$$

$$g^{\mu\nu}\, T^{\alpha}_{\nu\sigma} + \delta^{\alpha}_{\sigma}\, g^{\mu\nu}\, T^{\beta}_{\beta\nu} + g^{\mu\alpha}\, T^{\lambda}_{\sigma\lambda} \; , \tag{3.6}$$

$$g^{\mu\alpha}{}_{;\sigma} = 0 \tag{3.7}$$

$$g^{\mu\nu}{}_{;\nu} = 0 \tag{3.8}$$

The former is similar to Einstein's gravitational equation, and the second is Cartan's equation, for the torsion[11]. Eqs. (3.7) and (3.8) state conditions for the paralel displacement, i.e. the covariantderivative of the metric tensor, related to $\Gamma$, is identically null. In the Riemannian case eq. (3.5) reduces to Einstein equàtion of GR and eq. (3.6) disappears naturally, because the $\Gamma^{\alpha}_{\beta\mu}$ are symmetrical. Even so, eqs. (3.7) and (3.8) are still valid.

## 4. CONSIDERATIONS ABOUT THE METRIC

We will start with the metric proposed by Papapetrou[2], for the static case, with spherical symmetry and imposing signature -2.

$$(g_{\mu\nu}) = \begin{bmatrix} -\alpha & 0 & 0 & \omega \\ 0 & - & 0 & 0 \\ 0 & 0 & -r^2\ \mathrm{sen}^2\theta & 0 \\ -\omega & 0 & 0 & \gamma \end{bmatrix} \tag{4.1}$$

Here a, y and $\omega$ arbitrary functions of the coordinate $r$. For this case

$$\sqrt{-g} = r^2\ \mathrm{sen}^2\theta\ \sqrt{\alpha\gamma-\omega^2} \quad , \tag{4.2}$$

and so, the only components $g^{[\mu\nu]}$ different from zero are

$$g^{[14]} = -g^{[41]} = -\frac{\omega r^2\ \mathrm{sen}\theta}{\sqrt{\alpha\gamma-\omega^2}} \quad . \tag{4.3}$$

From eq. (3.8) we conclude that all equations $\bar{g}^{[\mu\nu]}_{\ \ :\nu} = 0$ are satisfied for $\mu = 1,2,3$, except for the case $\mu = 4$, which is

$$\partial_r(\omega r^2\ \mathrm{sen}\ \theta/\sqrt{\alpha\gamma-\omega^2}) = 0 \ . \tag{4.4}$$

The solution of eq. (4.4) is

$$\omega^2 = \frac{C\ \alpha\ \gamma}{C + r^4} \quad , \tag{4.5}$$

where C is a constant. The components of the connection $\Gamma$, in the holonomic basis $\{dx^{\mu}\}$ are

$$\Gamma^1_{11} = \frac{\alpha'}{2\alpha} \;,\; \Gamma^1_{22} = -\frac{r}{\alpha} \;,\; \Gamma^1_{33} = \frac{r\,\mathrm{sen}^2\theta}{\alpha} \;,\; \Gamma^1_{44} = \frac{\gamma'}{2\alpha} + \frac{4\omega^2}{\alpha^2 r} \;,\; \Gamma^2_{12} = \Gamma^2_{21} = \frac{1}{r}$$

$$\Gamma^2_{33} = -\mathrm{sen}\theta\cos\theta \;,\; \Gamma^3_{13} = \Gamma^3_{31} = \frac{1}{r} \;,\; \Gamma^3_{23} = \Gamma^3_{32} = \tan\theta \;,\; \Gamma^4_{14} = \Gamma^4_{41} = \frac{\gamma'}{2\gamma} + \frac{2\omega^2}{2r\gamma}$$

$$\Gamma^2_{42} = -\Gamma^2_{24} = \Gamma^3_{43} = -\Gamma^3_{34} = \frac{\omega}{\alpha r} \;,\; \Gamma^1_{14} = -\Gamma^1_{41} = \frac{2\omega}{\alpha r} \tag{4.6}$$

where the prime means $d/dr$. The remainning components of $\Gamma$ are null.

With the components given in eq. (4.6) and using expression (2.13), we find the components $F_{\alpha\beta}$ which are not null:

$$F_{11} = \frac{1}{2}\frac{\gamma'}{\gamma} + \frac{\gamma'}{4\gamma}\left(\frac{\gamma'}{\gamma} - \frac{\alpha'}{\alpha}\right) - \frac{\alpha'}{\alpha r} + 2\left(\frac{\omega^2}{\alpha r\gamma}\right) + \frac{2\omega^2}{\alpha r\gamma}\left(\frac{\gamma'}{\gamma} - \frac{\alpha'}{2\alpha} + \frac{2\omega^2}{\alpha r\gamma}\right)$$

$$F_{22} = \frac{1}{\mathrm{sen}^2\theta} F_{33} = \frac{r}{2\alpha}\left(\frac{\gamma'}{\gamma} - \frac{\alpha'}{\alpha}\right) + \frac{1-\alpha}{\alpha} + \frac{2\omega^2}{\gamma\alpha^2} \tag{4.7}$$

$$F_{44} = -\frac{1}{2}\left(\frac{\gamma'}{\alpha}\right) + \frac{\gamma'}{4\alpha}\left(\frac{\gamma'}{\gamma} - \frac{\alpha'}{\alpha} - \frac{4}{r}\right) - \frac{4\omega^2}{\alpha^2 r} + \frac{\omega^2}{\alpha^2}\left(\frac{3\gamma'}{\gamma} - \frac{2\alpha'}{\alpha} - \frac{8\omega^2}{\alpha r\gamma} - \frac{14}{r}\right)$$

$$F_{14} = -F_{41} = -2\left(\frac{\omega}{\alpha r}\right)' - \frac{4\omega}{\alpha r^2} \quad .$$

For the sourceless case, eq.(3.5) becomes

$$F_{\mu\nu} = 0 \;, \tag{4.8}$$

and assuming that

$$a = \exp\left[f(r)\right] \quad \text{and} \quad \gamma = \exp\left[g(r)\right] \;, \tag{4.9}$$

with f and g arbitrary functions of r, we find, after solving the differential equations obtained

$$\alpha = \left(1 - \frac{2m}{r}\right)^{-1} , \quad \omega = \pm\frac{C^2}{r^2} \;, \quad \gamma = \left(1 + \frac{C^4}{r^4}\right)\left(1 - \frac{2m}{r}\right) \;, \tag{4.10}$$

where $C$ is the same constant considered in the solution (4.5) and $m$ is another constant.

Since we want a Hermitian metric, we impose that $C^2 = i\ell^2$, where $\ell$ is a constant with the dimension of length. With solutions (4.10) the metric (4.1) is

$$(g_{\mu\nu}) = \begin{bmatrix} -(1-\frac{2m}{r})^{-1} & 0 & 0 & \frac{i\ell^2}{r^2} \\ 0 & -r^2 & 0 & 0 \\ 0 & 0 & -r^2 \operatorname{sen}\theta & 0 \\ -\frac{i\ell^2}{r^2} & 0 & & (1-\frac{2m}{r})(1-\frac{\ell^4}{r^4}) \end{bmatrix} \tag{4.11}$$

From the matrix (4.11) we conclude that the only possibility to have signature -2 is for $r>\ell$ and $r>2m$. The other possibility $\ell<r<2m$ is meaningless, for the weak fields considered here.

## 5. EXPERIMENTAL TESTS

### 5.1 - Redshift

The redshift of electromagnetic radiation is derived here with the same procedure as in GR. However, we have to take into account the correction term, which modifies the metric. The shift in frequency $\Delta\nu$ related to the original frequency $\nu$, is now given by

$$\frac{\Delta\nu}{\nu} = \sqrt{\frac{(1-\frac{2M}{R})(1-\frac{\ell^4}{R^4})}{(1-\frac{M}{r})(1-\frac{\ell^4}{r^4})}} - 1 , \tag{5.1}$$

where R is the radius of the celestial body (earth, sun, etc), M its mass and $r$ the distance between the receiver and the center of the body. Let $r = R + h$, where h is the distance between the receiver and the nearest point of the surface of the body.

Earth experiments give[12]

$$\frac{\Delta\nu}{\nu} = (2.4500 \pm 0.019)\times 10^{-15} \qquad M = 0.4438 \text{ cm}$$
$$h = 2.2600 \times 10^{3} \text{ cm} \qquad R = 6.3710 \times 10^{8} \text{ cm} \tag{5.2}$$

Expanding the expression (5.1) in a power series up to $h^2/R^2$, we find for R

$$\ell \approx \frac{R\left[\frac{M}{R}\left(\frac{h}{R} - \frac{h^2}{R^2}\right) - \frac{\Delta\nu}{\nu} + \frac{M^2}{R^2}\left(\frac{1}{2} - \frac{h}{R} + \frac{h^2}{R^2}\right)\right]^{1/4}}{\left(\frac{5h^2}{R^2} - \frac{2h}{R}\right)^{1/4}}, \tag{5.3}$$

which gives, with the values of eq. (5.2)

$$\ell_{\oplus} \approx 6.1599 \text{ km.}$$

For the sun the expression for R is

$$\ell \approx R_{\odot}\left[\frac{\left(\frac{\Delta\nu}{\nu} + 1\right)^2\left(1 - \frac{2M_{\odot}}{R_{\odot}}\right)\left(1 - \frac{\ell_{\oplus}^4}{R_{\oplus}^4}\right)}{1 - \frac{2M}{R_{\odot}}}\right] \tag{5.4}$$

and with the data

$$M_{\odot} = 1.4766 \times 10^{5} \text{ cm} \qquad R_{\odot} = 6.9605 \times 10^{10} \text{ cm}$$
$$\frac{\Delta\nu}{\nu} = -2 \times 10^{6} \qquad R_{\oplus} = 6{,}37103 \times 10^{8} \text{ cm}$$

we find for R

$$\ell_{\odot} \approx 15{,}438.1153 \text{ km.}$$

The above results show the existence of a core, with a radius R stating a minimum limit for r and eliminating the singularity at $r = 0$. There is no singularity at the point $r = R$, because in this case $\det g = -R^4 \operatorname{sen}^2\theta$. Moreover, the cases $r = 2m$ and $r < 2m$ refer to strong fields, and we are dealing with weak fields only. The case $r<\ell$ suggests a metric with signature different from +2, which is not in accordance with GR.

### 5.2 - Light Bending

The procedure used here to derive the light bending by the sun, is the same developed in GR theory. We will follow, for instance, the technique of A.B.S.[13]. For photons $ds^2 = 0$, so we assume the extremal condition

$$\delta\int g_{\alpha\beta}\frac{dx^{\alpha}}{dq}\frac{dx^{\beta}}{dq}\,dq = 0\ , \tag{5.4}$$

where $q$ is a parameter. For the metric given in eq.(4.11), in spherical coordinates $r,\ \theta,\ \phi$, we have

$$r^2\dot{\phi} = h \qquad \text{and} \qquad \left(1-\frac{m}{r}\right)\left(1-\frac{\ell^4}{r^4}\right)\dot{t} = C \tag{5.5}$$

where dot "." denotes differentiation with respect to $q$, and $h$ and $C$ are constants. For $\theta = \pi/2$ and assuming $r = 1/u$, the condition

$$g_{\alpha\beta}\,dx^{\alpha}\,dx^{\beta} = 0\ , \tag{5.6}$$

becomes

$$h^2 - (1-\ell^4u^4)h^2u'^2 - h^2u^2(1-\ell^4u^4)(1-2mu) = 0\ . \tag{5.7}$$

If we differentiate this expression with respect to $\phi$ and assume that $h\neq 0$, we are led to

$$u'' + u = 3mu^2 + \frac{2\ell^4u^3u'^2}{1-\ell^4u^4} + \frac{2u^5\ell^4}{1-\ell^4\ ^4} - \frac{4mu^6\ell^4}{1-\ell^4u^4}\ . \tag{5.8}$$

This last equation is analogous to equation 6.125 given in A.B.S.:

$$u'' + u = 3mu^2\ , \tag{5.9}$$

except for the correction terms. In the case of the sun

$m = 1.47666\times 10^5$ cm , $r = 6.96050\times 10^{10}$ cm , $\ell = 1{,}543{,}811{,}527$ cm

Expanding $(1-\ell^4u^4)^{-1}$ in a power series, eq. (5.8) becomes

$$u'' + u = 3mu^2 + 2\ell^4u^3u'^2 + 2\ell^8u^7u'^2 + \ldots \tag{5.10}$$

We notice that eq. (5.9) of GR is taken up to order $r^{-3}$, and we see that the first correction term in eq.(5.10) is proportional to $r^{-7}$, which gives a correction of the order of $10^{-39}$. So, this term and the next are negligible. Then eq. (5.10) reduces to eq. (5.9) and the result for the light bending by the sun in is the same predicted by GR theory, i.e. 1.75''.

## 5.3 - Shift of the Perihelium

By the same procedure of ABS[13] we obtain the differential equation

$$u'^2 = \frac{C^2-1+\ell^4u^4}{h^2(1-\ell^4u^4)} + \frac{2mu}{h^2} - u^2 + 2mu^3 , \tag{5.11}$$

where the dash denotes $d/d\phi$, $u = 1/r$ and C is given in expression (5.5).

Expanding $(1-\ell^4u^4)^{-1}$ in a power series up to the order of $u^4$, we get

$$u'^2 = \frac{C^2-1}{h^2} + \frac{2mu}{h^2} - u^2 + 2mu^2 + \frac{C^2\ell^4u^4}{h^2} . \tag{5.12}$$

Now, differentiating eq. (5.12) with respect to $\phi$ and eliminating the particular solution $u' = 0$ (circular orbits), we find

$$u'' + u = \frac{m}{h^2} + 3mu^2 + \frac{2C^2\ell^4u^3}{h^2} , \tag{5.13}$$

and by means of expressions (5.5) the constant $C$ may be determined:

$$C^2 = (1-2mu)^2(1-\ell^4u^4)^2 \left(\frac{dt}{d\phi}\right)^2 \dot{\phi}^2 = (1-2mu)^2(1-\ell^4u^4)^2 h^2u^4\dot{t}^2 . \tag{5.14}$$

This constant inserted in eq. (5.13) gives, at least, a correction term of order $r^{-7}$, which is negligible. So, eq. (5.13) reduces to

$$u'' + u = \frac{m}{h^2} + 3mu^2 , \tag{5.15}$$

leading to the same result predicted by GR theory: the shift of the perihelium of Mercury is 42.6'' per century.

## 6. CONCLUSION

The approach developed here exhibits the Einstein-Cartan model, in a bundle of linear frames and considering the Poincaré group. With the assumption of a Hermitian metric for space-time, we have pointed out the existence of a core, embedding the origin, which avoids the singularity at $r = 0$. The redshift has a small correction, compared to GR theory, while the bending of light and the shift of the perihelium of Mercury have no significant corrections.

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**Resumo**

Um modelo gravitacional com uma métrica hermitiana é proposto aqui. Este modelo leva às equações de Einstein-Cartan, para campos não simétricos. Por meio de um limite mínimo, estabelecido para a coordenada radial, a sinqularidade na origem é eliminada. O *redshift*, o desvio do raio de luz e o do perihélio do Mercúrio são examinados.