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Non Linear Sigma Models Probing the String Structure

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Abstract We discuss the introduction of a term depending on the extrinsic curvature to the string action, and related non linear sigmamodels defined on a symmetric space SO(D)/SO(2) x SO(D-2). Coupling to fermions are also treated.

String theories have been used to understand low energy phenomenology the so called old dual modes (which must actually be described using the Liouville theory to avoid the critical dimension), as well as critical phenomena. However, the most exciting possibility is the description of all elementary interactions from a unified (super) string point of view.

The starting point of string theory (in the bosonic case) is the Nambu-Goto action

$$S = \frac{1}{2} T \int d^2\xi \sqrt{-g} \tag{1}$$

where

$$g_{ab} = \partial_a X_\mu \partial_b X^\mu$$
$$a,b = 1,2$$
$$\mu = 1,\ldots,D$$

We have adopted an euclidean formulation which is more suitable to the construction which will follow. The construction of this first part is due to Polyakov[1].

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The equation[2]

$$\partial_a\partial_b X^\mu = \Gamma^c_{ab}\partial_c X^\mu + K^i_{ab}\eta^\mu_i \tag{2}$$

defines K^i_{ab}, where Γ is the Christoffel symbol, and $\{\eta_i\}$, $i=1,\ldots,D-2$ is a set of D-2 vectors perpendicular to the world sheet:

$$\eta^\mu_i\eta^\mu_j = \delta_{ij}$$
$$\eta^\mu_i\partial_a X^\mu = 0$$

The intrinsic curvature is given by

$$R = (K^{ia}_a)^2 - K^{ib}_a K^{i\ a}_{\ b} \tag{3}$$

The Gauss-Bonnet theorem states that the Einstein action in two dimensions

$$S = \int d^2\xi\sqrt{g}\, R \tag{4}$$

is a topological quantity, the integrand being a total divergence. As a consequence it does not contribute to the partition function (at least in perturbation theory). However, the Nambu-Goto action can be modified to

$$S = \frac{1}{2}T\int d^2\xi\,\sqrt{g} + \frac{1}{2f}\int d^2\xi\, K^{ib}_a K^{ia}_b\,\sqrt{g} \tag{5}$$

since the second term is conformally invariant in D dimensions.

Let us discuss the new term in more detail. We shall use the orthogonal gauge

$$\partial_a X^\mu\partial_b X_\mu = g_{ab} = \rho\delta_{ab} \tag{6}$$

It is possible to rewrite the new term in other forms. We compute

$$L = \sqrt{g}\, g^{ab}\partial_a t_{\mu\nu}\partial_b t_{\mu\nu}$$

where

$$t_{\mu\nu} = \frac{\varepsilon^{ab}}{\sqrt{g}}\partial_a X_\mu\partial_b X_\nu$$

Substituting $t_{\mu\nu}$ back we have

$$L = \sqrt{g}\, g^{ab}\, \partial_a t_{\mu\nu} \partial_b t_{\mu\nu}$$

$$= \frac{1}{\sqrt{g}} g^{ab} \varepsilon^{cd} \varepsilon^{ef} \{\partial_a \partial_c X_\mu \partial_d X_\nu \partial_e \partial_b X_\mu \partial_f X_\nu + \partial_c X_\mu \partial_a \partial_d X_\nu \partial_b \partial_e X_\mu \partial_f X_\nu$$

$$+ \partial_a \partial_c X_\mu \partial_d X_\nu \partial_e X_\mu \partial_b \partial_f X_\nu + \partial_c X_\mu \partial_a \partial_d X_\nu \partial_e X_\mu \partial_b \partial_f X_\nu \}$$

$$= \frac{1}{\sqrt{g}} g^{ab} \varepsilon^{cd} \varepsilon^{ef} \{ (K^i_{ab} \eta^i_\mu + \Gamma^h_{ac} \partial_h X_\mu) \partial_d X_\nu (K^j_{be} \eta^j_\mu + \Gamma^k_{be} \partial_k X_\mu) \partial_f X_\nu$$

$$+ \partial_c X_\mu (K^i_{ad} \eta^i_\nu + \Gamma^h_{ad} \partial_h X_\nu)(K^j_{bf} \eta^j_\nu + \Gamma^k_{bf} \partial_k X_\nu) \partial_e X_\mu)$$

$$+ \partial_c X_\mu (K^i_{ad} \eta^i_\nu + \Gamma^h_{ad} \partial_h X_\nu)(K^j_{be} \eta^j_\mu + \Gamma^k_{be} \partial_k X_\mu) \partial_f X_\nu$$

$$+ (K^i_{ac} \eta^i_\mu + \Gamma^h_{ac} \partial_h X_\mu) \partial_d X_\nu \partial_e X_\mu (K^j_{bf} \eta^j_\nu + \Gamma^k_{bf} \partial_k X_\nu)\} \qquad (7)$$

where eq. (2) has been used.

The Γ's join in a term proportional to

$$(\Gamma_{abc})^2 - (\Gamma_{a\ b}^{\ b})^2$$

which is zero in the conformal gauge. For the K's we have

$$L = \frac{1}{\sqrt{g}} g^{ab}\, \varepsilon^{cd}\, \varepsilon^{ef}\, 2 \cdot g_{df}\, K_a{}^i{}_e\, K_b{}^i{}_e$$

$$= 2\sqrt{g}\, K^{i\ b}_{\ a}\, K^{i\ a}_{\ b} \qquad (8)$$

We conclude with the equality

$$\int d^2\xi \sqrt{g}\, K^{ib}_a\, K^{i\ a}_{\ b} = \frac{1}{2} \int d^2\xi \sqrt{g}\, g^{ab}\, \partial_a t_{\mu\nu}\, \partial_b t_{\mu\nu} \qquad (9)$$

We can also prove equivalence of the previously defined action to a sigma model action. If we define a gauge field A_a^{ik} by

$$D_a\eta_i^\mu = \partial_a\eta_i^\mu + A^{i\ k}_{\ a}\eta_k^\mu = -K^{i\ b}_{\ a}\partial_b X^\mu \tag{10}$$

then we have

$$\int d^2\xi\ \sqrt{g}\ K^{i\ b}_{\ a}K^{i\ a}_{\ b} = \int d^2\xi\ \sqrt{g}\ g^{ab}\ D_a\eta_i^\mu D_b\ \eta_i^\mu \tag{11}$$

This model is defined on a symmetric space $SO(D)/\{SO(2)\otimes SO(D-2)\}$. It is not difficult to see that we have the equality

$$\int d^2\xi\ \sqrt{g}\ K^{i\ b}_{\ a}K^{i\ a}_{\ b} = \int d^2\xi\ \sqrt{g}\ \{\frac{1}{\sqrt{g}}\partial_a\ \sqrt{g}\ g^{ab}\partial_b X^\mu\ \}^2$$

At the conformal gauge, the action turns out to be:

$$S = \frac{1}{2f_0}\int d^2\xi\ \left\{\rho^{-1}(\partial^2\vec{x})^2 + \lambda^{ab}(\partial_a\vec{X}.\partial_b\vec{X} - \rho\delta_{ab}) + T_0\int d^2\xi\ \rho\right. \tag{12}$$

We shall compute the one loop renormalization constants of the above theory.

If we split the fields into slow and fast components, we can integrate out the fast components, analogously to the background field method:

$$\begin{aligned} X_\mu &= X_{0\mu} + X_{1\mu} \\ \rho &= \rho_0 + \rho_1 \\ \lambda^{ab} &= \lambda_0^{ab} + \lambda_1^{ab} \end{aligned} \tag{13}$$

where

$$\partial_a X_0^\mu \partial_b X_0^\mu = \rho_0\ \delta_{ab} \tag{14}$$

Consider

$$S = S_0 + S_{\text{II}}^{(a)} + S_{\text{II}}^{(b)} \tag{15}$$

where

$$S_{\mathrm{II}}^{(a)} = \frac{1}{2f_0}\int d^2\xi\ \{\rho_0^{-1}(\partial^2 X_1)^2 + 2\ \lambda_1^{ab}\partial_a X_0^\mu \partial_b X_1^\mu - \lambda_1^{aa}\rho_1\}$$

$$S_{\mathrm{II}}^{(b)} = \frac{1}{2f_0}\int d^2\xi\ \{\lambda_0^{ab}\partial_a X_1^\mu \partial_b X_1^\mu - 2\ \rho_1\rho_0^{-2}\partial^2 X_0^\mu \partial^2 X_1^\mu + \rho_1^2\rho_0^{-3}(\partial^2 X_0)^2\} \tag{16}$$

We find the propagators.

At zeroth loop order, we read off eq. (16) that

$$<X_{1\mu}(p)X_{1\nu}(-p)> = \frac{f_0\rho_0}{p^4}\ \delta_{\mu\nu} \tag{17}$$

We must find the λ_1-two point function. To do so, we integrate out X_1:

$$\frac{1}{2}\left\{X_1\ \frac{1}{f_0\rho_0}\ (\partial^2)^2 X_1 + \frac{2}{f_0}\ \lambda_1^{ab}\partial_a X_0 \partial_b X_1\right\} =$$

$$= \frac{1}{2}\left\{X_1 + \frac{1}{f_0}\ \lambda_1\partial_a X_0\partial_b\ \frac{f_0\rho_0}{(\partial^2)^2}\right\}\frac{1}{f_0\rho_0}\ (\partial^2)^2\left\{X_1 + \frac{f_0\rho_0}{(\partial^2)^2}\frac{1}{f_0}\ \lambda_1^{ab}\partial_a X_0\partial_b\right\} - \frac{1}{2f_0}\ \lambda_1^{ab}\partial_a X_0\partial_b\ \frac{\rho_0}{(\partial^2)^2}\ \lambda_1^{cd}\partial_c X_0\partial_d \tag{18}$$

The last term contributes, in momentum space, as

$$-\frac{1}{2f_0}\int d^2k\ \frac{(k_a\ \lambda_1^{ab}\ )^2}{(k^2)^2}\ \rho_0^2 \tag{19}$$

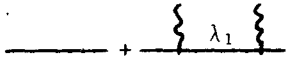
where $\partial_a X_0^\mu \partial_b X_0^\mu = \rho_0\ \delta_{ab}$ has been used.

In order to compute the λ_1-propagator we just take the inverse (see eq. (23) for further details);

$$<k_b\lambda_1^{ab}(k)k_d\lambda_1^{cd}(-k)> = -\frac{(k^2)^2}{\rho_0^2}\ \delta^{ac}\ f_0 \tag{20}$$

· We can compute the 1-loop X_1-propagator

$$<X_{1\mu}(p)X_{1\nu}(-p)> =$$



$$= \frac{f_0\rho_0}{p^4}\,\delta_{\mu\nu} + \frac{1}{2}\,\frac{f_0\rho_0}{p^4}\,p^2\,\frac{\partial_a X_{0\mu}}{f_0^2}\,\partial^a X_{0\nu}\,\frac{(-p)^2 f_0 2}{\rho_0^2}\,\frac{f_0\rho_0}{p^4} \tag{21}$$

$$= \frac{f_0\rho_0}{p^4}\,\delta_{\mu\nu} - \frac{1}{p^4}\,f_0\,\partial_a X_{0\mu}\partial^a X_{0\nu} = \frac{f_0}{p^4}\left[\delta_{\mu\nu}\rho_0 - \partial_a X_{0\mu}\partial_a X_{0\nu}\right]$$

Relation (21) provides a first counterterm to S_0, when substituted in $S_{\text{II}}^{(b)}$ [$S_{\pi}^{(b)}$ is treated as a perturbative mass, in the sense that $S_{\pi}^{(a)}$ has higher order derivatives].

$$\frac{1}{2f_0}\int d^2\xi\;\lambda_0^{ab}\,<\partial_a X_1^\mu \partial_b X_1^\mu> =$$

$$= \frac{1}{2f_0}\;f_0(D-2)\int d^2\xi\;\lambda_0^{ab}\;\rho_0\int\frac{d^2p}{(2\pi)^2}\,\frac{p_a p_b}{(p^2)^2} = \frac{D-2}{8\pi}\,\log\frac{\Lambda}{\tilde{\Lambda}}\int d^2\xi\;\lambda_0^{aa}\;\rho_0 \tag{22}$$

which is a first counterterm.

We come back to

$$L = -\int d^2k\left\{\frac{(k_a\,\lambda_1^{ab})}{k^4} + \lambda_1^{aa}\rho_1\right\}\frac{1}{2f_0} \tag{23}$$

Using the decornposition

$$\lambda_1^{ab} = \xi\left(\delta_{ab} - \frac{k_a k_b}{k^2}\right) + \left(k_a f_b + k_b f_a - (k.f)\,\delta_{ab}\right) \tag{24}$$

We can read off the propagators

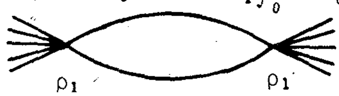
$$<f^a(k)f^b(-k)> = -\,\delta^{ab}$$

$$<\xi(k)\;\xi(-k)> = k \tag{25}$$

$$<\rho_1(k)\;\rho_1(-k)> = -\frac{1}{k^2}\,f_0\;\rho_0^2$$

and from the diagram

$$-\frac{1}{f_0}\,\partial^2 X_0\rho_0^{-2}\partial^2 X_1 \qquad -\,\partial^2 X_1\frac{1}{f_0}\,\partial^2 X_0\rho_0^{-2}$$



we find the integral (at each vertex there ir a factor $S_{\mathrm{II}}^{(b)}$ depending on background fields)

$$-\frac{1}{2}\frac{1}{f_0^2\rho_0^4}(\partial^2 X_0)^2\int\frac{d^2p}{(2\pi)^2}\left\{p^2\frac{f_0\rho_0}{p^4}p^2\right\}\frac{f_0\rho_0^2}{p^2} =$$

$$= -\frac{(\partial^2 X_0)^2}{2\rho_0}\frac{2}{4\pi}\log\frac{\Lambda}{\tilde{\Lambda}} \tag{26}$$

The counterterm reads

$$-\frac{2}{4\pi}\log\frac{\Lambda}{\tilde{\Lambda}}\int d^2\xi\,\frac{1}{2}\rho_0^{-1}(\partial^2 X_0)^2 \tag{27}$$

The one loop effective action is given by the expression

$$S = \frac{1}{2f_0}\int d^2\xi\left\{\left(1-\frac{f_0}{2\pi}\log\frac{\Lambda}{\tilde{\Lambda}}\right)\rho_0^{-1}(\partial^2 X_0)^2 + \right.$$

$$\left. + \lambda_0^{ab}\left[\partial_a X_0\partial_b X_0 - \delta_{ab}\rho_0\left(1-\frac{D-2}{4\pi}f_0\log\frac{\Lambda}{\tilde{\Lambda}}\right)\right]\right\} \tag{28}$$

Renorrnalization is achieved by

$$X_0 = Z^{1/2}X\ ;\ \lambda_0 = Z^{-1}\lambda$$

$$Z = 1-\frac{D-2}{\pi}f_0\log\frac{\Lambda}{\tilde{\Lambda}} \tag{29}$$

$$\frac{1}{f} = \frac{1}{f_0}-\frac{D}{2}\frac{1}{2\pi}\log\frac{\Lambda}{\tilde{\Lambda}}$$

Therefore

$$S = \frac{1}{2f}\int d^2\xi\ \{\rho^{-1}(\partial^2 X)^2 + \lambda^{ab}[\partial_a X\partial_b X - \delta_{ab}\rho]\} \tag{30}$$

It is important to note that

$$\frac{1}{f} = \frac{1}{f_0}-\frac{D}{4\pi}\log\frac{\Lambda}{\tilde{\Lambda}} \tag{31}$$

Physical Interpretation

If there is no (non trivial) fixed point, then $\langle\lambda^{ab}\rangle = \bar{\lambda}\,\delta^{ab}$, meaning that the increase of the coupling constant will be stopped by the forrnation of the average Lagrange rnultiplier. As it turns out, $\bar{\lambda}^{-1/2}$ is a correlation length for the norrnals to the string surface.

The effective action in the infrared region is given by

$$S = \frac{\bar{\lambda}}{2f_0} \int d^2\xi \ (\partial_a X^\mu)^2 + \mu_0 \int d^2\xi \ \rho \tag{32}$$

which leads to the Liouville theory[3]. The string tension is given by $\bar{\lambda}$, which cannot be made zero by adjusting μ_0. This is the creased string, and the Regge trajectories are straight lines.

QCD and Ising strings do not belong to the above class, since they must present critical behavior, and $\bar{\lambda}$ must go to zero, with a critical exponent defined by theanomalous dimension. Thecorresponding string must be smooth.

It is conceivable that a θ term could induce non perturbatibe contributions to the β-function through instantons, generating a non trivial zero.

Related sigma models

Suppose that an $SO(D)$ valued field g is given, transforrning under a gauge group H = $SP(D-P) \otimes SO(P)$. That field can be written under the following form (Z and Y are rectangular matrices)

$$g = (Z_{D\otimes P} \ Y_{D\otimes D-P}) \tag{33}$$

In the case of interest P = 2. The previous field n^i_μ will correspond to Y above. The gauge symmetry is implemented by the gauge field

$$A_\mu = \begin{bmatrix} Z^+\partial_\mu Z & 0 \\ 0 & Y^+\partial_\mu Y \end{bmatrix} \tag{34}$$

and the action is defined by

$$S = \frac{1}{2} \operatorname{tr} \int d^2\xi \ \overline{D_\mu g} \ D_\mu g \tag{35}$$

where

$$D_\mu g = \partial_\mu g - gA_\mu$$

Using the symmetric space constraints

$$Z^+ Z = 1_P \qquad Y^+ Y = 1_{D-P}$$
$$Z^+ Y = 0 = Y^+ Z \quad ; \quad ZZ^+ + YY^+ = 1_D \tag{36}$$

we have identity

$$S = \mathrm{tr} \int d^2\xi \, D_\mu Z^+ D_\mu Z = \mathrm{tr} \int d^2\xi \, D_\mu Y^+ \, D_\mu Y$$

where

$$D_\mu Z = \partial_\mu Z - ZZ^+ \, \partial_\mu Z$$
$$D_\mu Y = \partial_\mu Y - YY^+ \, \partial_\mu Y \tag{37}$$

It follows that the theory containing n_i (namely Y_i) can be replaced by the theory containing Z_i, $i = 1,2$.

We shall work with the complex fields Z, and $\bar{Z}$ defined by

$$Z = Z_1 + i \, Z_2$$
$$\bar{Z} = Z_1 - i \, Z_2 \tag{38}$$

in terms of which the following constraints hold

$$Z^2 = 0 = \bar{Z}^2$$
$$\bar{Z}Z = \frac{D}{f_0} \tag{39}$$

where we introduced a coupling constant f_0/D. The partition function can be defined by

$$Z = \int \mathcal{D}Z\mathcal{D}\bar{Z}\mathcal{D}\alpha\mathcal{D}\beta\mathcal{D}\bar{\beta}\mathcal{D}\lambda_\mu \exp - \int d^2x \left\{ \bar{Z}\left[-D^\mu D_\mu + m^2 + \frac{i}{\sqrt{D}}\,\alpha\right] Z \right.$$
$$\left. + \frac{i\beta}{\sqrt{D}} Z^2 - \frac{i\bar{\beta}}{\sqrt{D}} \bar{Z}^2 - \frac{i\sqrt{D}}{f_0}\,\alpha(x) \right\} \tag{40}$$

where $D_\mu = \partial_\mu + i/\sqrt{D} \;\; \lambda_\mu$, and the fields α, β, $\bar{\beta}$ implement the constraints eq, (39), and λ_μ linearizes the Z-field equation.

Imposing that the vacuum expectation value of vanishes (or equivalently, gives the mass m^2) we have'

$$m^2 = \Lambda^2 e^{-4\pi/f} \tag{41}$$

or

$$\frac{1}{f_0} = \frac{1}{f} + \frac{1}{2\pi} \log \frac{\Lambda}{\mu} \quad . \tag{42}$$

Rewriting $f \to f\ D/2$ to compare with the previous result, we have (compare with eq.(31)):

$$\frac{1}{f_0} = \frac{1}{f} + \frac{D}{4\pi} \log \frac{\Lambda}{\mu} \tag{43}$$

Moreover we can compute the λ_μ propagator by summing the one loop, Z-diagrams, which is equivalent to taking $D \to \infty$:

$$<\lambda_\mu(p)\lambda_\nu(-p)> = (\delta_{\mu\nu} - \frac{p_\mu p_\nu}{p^2}) \{(p^2 + 4m^2) A(p) - \frac{1}{\pi}\}^{-1} \tag{44}$$

where

$$A(p) = \frac{1}{2\pi\sqrt{p^2(p^2 + 4m^2)}} \log \frac{\sqrt{p^2 + 4m^2} + \sqrt{p^2}}{\sqrt{p^2 + 4m^2} - \sqrt{p^2}}$$

The above impliesin a massless pole. It follows that Z is confined, and a normal to the string world sheet cannot be defined at finite separation. The string is severely creased, confirming previous results[4].

Fermionic models

A simple toy model (non supersymmetric) is given by (seecomment before eq. (48), and ref.(5)).

$$L = \overline{DZ_M}\, DZ_M \oplus i\, \bar{\psi}_M \not{\partial}\, \psi_M \quad ; \quad M = 1,\ldots,D$$

Using the same techniques, we can compute the λ_μ two-point function. The fermion contribution is given by a constant $(1/\pi)$, which is the usual anomaly

$$\begin{aligned} <\lambda_\mu(p)\lambda_\nu(-p)> &= (\delta_{\mu\nu} - \frac{p_\mu p_\nu}{p^2})\{(p^2 + 4m^2) A(p)\}^{-1} \\ &\simeq (\delta_{\mu\nu} - \frac{p_\mu p_\nu}{p^2})\ \pi \end{aligned} \tag{45}$$

implying disappearance of the massless pole.

However we have yet similar conclusions

$$1)\ \beta(f) = -\frac{f^2}{4\pi} < 0 \quad \left[\text{or}\ \frac{1}{f} = \frac{1}{f_0} - \frac{D}{4\pi}\log\frac{\Lambda}{\mu}\right] \tag{46}$$

$$2)\ \langle Z_M(p) Z_M(-p)\rangle = \frac{\delta_{MN}}{p^2+m^2} \tag{47}$$

with a non zero mass gap (mass transmutation), implying short distance correlation, and the string is creased, implying Regge behavior.

It can be proved that the σ-model is integrable[5], and the S--matrix is known up to bound state poles.

Supersymmetric coupling

Generally, non'linear sigma models interacting with fermions can be defined in a geometrical way, considering the fermion as belonging to some representation of the gauge group H[5]. In the previous case ψ belongs to the direct representation. It has been proved that if χ belongs to the adjoint representation, then there is a supersymmetry transformation leaving the action invariant. The usual supersymmetric partner is'defined by

$$\psi = g\chi \tag{48}$$

where

$$g = (Z\ Y)$$

obeys the previous constraints eq.(36).

However, we have now extra constraints defining the fermionic interaction, which arise as follows.

The field χ is related to ψ as

$$\chi = g^+\chi = \begin{bmatrix} Z^+\psi^z & Z^+\psi^y \\ Y^+\psi^z & Y^+\psi^y \end{bmatrix} \tag{49}$$

χ is the adjoint representation of the gauge group $SO(P)\otimes SO(D-P)$, and we impose the constraints[5]

$$Z^+\psi^z = 0 = Y^+\psi^y$$

also

$$Z^{+}\psi^{y} = -\psi^{z+}Y$$
$$Y^{+}\psi^{z} = -\psi^{y+}Z \tag{50}$$

The supersymmetric lagrangian reads

$$L = D_\mu g^{+} D_\mu g + \frac{i}{2}\bar{\chi}\not{\partial}\chi - \frac{1}{4}\bar{\chi}\gamma^\mu\chi\bar{\chi}\gamma_\mu\chi \tag{51}$$

with the supersymmetry transformations given by

$$\delta g = g\bar{\varepsilon}\chi$$
$$\delta\chi = -i\, g^{+}D^\mu g\gamma_\mu\varepsilon \tag{52}$$

The following set of identities can be deduced using eqs.(36) and (50):

$$\frac{1}{2}(\bar{\chi}\gamma_\mu\chi)^2 = \frac{1}{2}\bar{\psi}^{z^i}\gamma^\mu\psi^{z^j}\bar{\psi}^{z^j}\gamma_\mu\psi^{z^i} + (\psi^z \to \psi^y)$$
$$\bar{\chi}\overleftrightarrow{\not{\partial}}\chi = \frac{1}{2}\bar{\psi}^{z}\overleftrightarrow{\not{\partial}}\psi^{z} + \frac{1}{2}\bar{\psi}^{y}\overleftrightarrow{\not{\partial}}\psi^{y} + \bar{\psi}^{z}\gamma^\mu J_\mu\psi^{z} + \bar{\psi}^{y}\gamma^\mu J_\mu\psi^{y} \tag{53}$$

where

$$J_\mu = Z\overleftrightarrow{D}_\mu Z^{+} = Y\overleftrightarrow{D}_\mu Y^{+}$$

Using the identities

$$\psi^{y} = (ZZ^{+} + YY^{+})\psi^{y} = ZZ^{+}\psi^{y} = -Z\psi^{z+}Y \tag{54a}$$

$$D_\mu Y^{+}\psi^{z} = -D_\mu Y^{+}Y\psi^{+y}Z = 0 \tag{54b}$$

we have

$$\psi^{z+}J_\mu\psi^{z} = 0 = \psi^{y+}J_\mu\psi^{y} \tag{55a}$$

$$\bar{\chi}\overleftrightarrow{\not{\partial}}\chi = \frac{1}{2}\bar{\psi}^{z}\overleftrightarrow{\not{\partial}}\psi^{z} + \frac{1}{2}\bar{\psi}^{y}\overleftrightarrow{\not{\partial}}\psi^{y} \tag{55b}$$

Also

$$\bar{\psi}_M^{y^i}\gamma_\mu\psi_M^{y^j} = -Y_M^j\bar{\psi}_M^{z^k}\gamma_\mu\psi_N^{z^k}\bar{Y}_N^i \tag{56}$$

and it follows that

$$\frac{1}{2}(\bar{\chi}\gamma_\mu\chi)^2 = \frac{1}{2}\,\bar{\psi}^{z^i}_M\,\gamma^\mu\psi^{z^j}_M\,\bar{\psi}^{z^j}_N\,\gamma_\mu\psi^{z^i}_N + \frac{1}{2}\,\bar{\psi}^{z^i}_M\gamma_\mu\psi^{z^i}_N\;\bar{\psi}^{z^j}_N\,\gamma_\mu\psi^{z^j}_M \tag{57}$$

$$\bar{\psi}\overleftrightarrow{\not{D}}\,\psi = \bar{\psi}^z\overleftrightarrow{\not{D}}\,\psi^z = \bar{\psi}^y\overleftrightarrow{\not{D}}\,\psi^y \tag{58}$$

The conclusion is that the full action may be written completely in terms of Zand \$, or equivalently Y and @ as below:

$$\begin{aligned} L &= \overline{D_\mu Y}D_\mu Y + i\,\bar{\psi}^y\not{D}\psi^y + \frac{1}{4}\left[(\bar{\psi}^y\psi^y)^2 - (\bar{\psi}^y\gamma_5\psi^y)^2\right] \\ &= \overline{D_\mu Z}D_\mu Z + i\,\bar{\psi}^z\not{D}\psi^z + \frac{1}{4}\left[(\bar{\psi}^z\psi^z)^2 - (\bar{\psi}^z\gamma_5\psi^z)^2\right] \end{aligned} \tag{59}$$

with the constraints

$$Z^2 = \bar{Z}^2 = 0\;;\;\bar{Z}Z = 2\;;\;\bar{\psi}Z = \bar{Z}\psi = Z\psi = \bar{Z}\bar{\psi} = 0$$

and similarly for Y and ψ^y respectively.

The forces in this model are all short ranged:

$$\langle Z_M(p)\;\bar{Z}_N(-p)\rangle = \frac{\delta_{MN}}{p^2+m^2} \tag{60}$$

$$\langle\psi_M(p)\;\bar{\psi}_N(-p)\rangle = \frac{\delta_{MN}}{i\not{p}-m} \tag{61}$$

$$\begin{aligned} \langle\lambda_\mu(p)\;\lambda_\nu(-p)\rangle &= \left(\delta_{\mu\nu} - \frac{p_\mu p_\nu}{p^2}\right)\{(p^2+4m^2)A(p)\}^{-1} \\ &\simeq \left(\delta_{\mu\nu} - \frac{p_\mu p_\nu}{p^2}\right)\pi, \end{aligned} \tag{62}$$

The supersymmetric string is also creased, with Regge behavior. On shell scattering can be defined, and computed (up to bound state poles)[5].

The β-function has the same previous value

$$\beta = -\frac{f^2}{4\pi} \tag{63}$$

The model can be coupled to local supersymmetry, with similar conclusions, the only difference being a long ranged graviton and gravitino fields.

The only difference between the models interactings with fermions and the purely bosonic model is the presence of the long range force in the bosonic case, which is presumably hiding an infinite string tension.

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Resumo

discutimos a introdução de um termo que depende da curvatura extrínsica à ação da corda, e os modelos sigma não lineares associados, que são definidos no espaço simétrico $SO(D)/SO(2) \times SO(D-2)$. O acoplamento a férmions é também tratado.