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Triton Cluster States in ^{15}N

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Abstract A coupled channels calculation of some high-lying positive parity bands of triton cluster states in ^{15}N based on a ^{12}C core in its 0^+ ground state and 2^+(4.44 MeV) first excited state is presented. A comparison with the experimental data obtained from triton transfer reactions onto ^{12}C and with a recent comprehensive compilation of the properties of all known states in ^{15}N is made. The triton cluster structure in ^{15}N appears to be very similar to that in the neighbouring nuclei ^{17}F, ^{17}O, ^{18}F, ^{18}O and ^{19}F, thus rupporting the hypothesis that such clustering is a stable feature of this mass region.

1. INTRODUCTION

The existence of alpha particle cluster structure in light nuclei (up to A ~ 40) is now widely accepted[1]. Many calculations have been made[2,3] which convincingly reproduce the measured values of the energies, electromagnetic moments, electromagnetic transition strengths and alpha decay widths of the proposed alpha cluster states. In addition, recent work[4] has pointed to the continuing occurrence of alpha particle clustering in heavy nuclei, suggesting that it probably persists across a very large part of the periodic table.

This observation immediately raises the question of whether other cluster structures are also present in atomic nuclei, and if so, to what extent. In fact, there is growing evidence that triton and helion cluster states are not only present but also a stable feature across quite an appreciable range of light nuclei. Bands of these cluster states are selectively excited in triton and helion transfer reactions, display a 'rotational' energy spectrum (sometimes staggered by the effects of non-central forces) and can exhibit strong E1 inter-band transitions because their centres of mass and charge do not coincide. Various triton and helion cluster model calculations have been highly successful in correlating a wide range of these experimentally determined properties in nuclei such as ^{7}Li, ^{7}Be [5,6], ^{17}F, ^{17}O, ^{18}F, ^{18}O and ^{19}F [2,7-11]

It is a particularly opportune moment to continue this systematic sequence of studies by re-examining the question of ^{12}C-triton cluster states in ^{15}N because on the theoretical side the local potential cluster model of Buck, Dover and Vary[12] has recently been extended to include excited states of the core in a coupled channels formalism[13] and on the experimental side Pilt[14] has recently reviewed the triton transfer reaction data onto ^{12}C and Ajzenberg-Selove[15] has updated her comprehensive tabulation of the properties of all known states in ^{15}N. In this paper a coupled channels calculation of some high lying positive parity bands of triton cluster states in ^{15}N, based on a ^{12}C core in its 0^+ ground state and its 2^+ (4.44 MeV) first excited state, is presented and the results are compared with the aforementioned experimental data.

2. THE BUCK-DOVER-VARY CLUSTER MODEL

In its original form the cluster model of Buck, Dover and Vary[12] gave an effective cluster-core potential by folding thedensities of these two constituents with an effective nucleon-nucleon interaction. Both cluster and core were treated as inert, structureless entities and the relative motion of the two components das characterised by the principal quantum number N and the orbital angular momentum R. However, these latter quantities were restricted to correspond tothemicroscopic situation in which the cluster nucleons are completely excluded from the last major shell occupied by the core nucleons. For example, in the case of ^{12}C-triton cluster states, this constraint would be written as

$$N = 2n + \ell \geqslant 6 \qquad (2.1)$$

(where n is the number of interior nodes, in the radial wave function) corresponding to the three cluster nucleons being placed in at leastthe (sd)-shell. In this way the major requirernents of the Pauli exclusion principle are satisfied. Energies and wave functions of the cluster states of interest can then be obtained by a numerical solution of the appropriate single channel single-particle Schrödinger equation and their properties computed for comparison with the available experimental data.

It was found that the folded cluster-core potential could be accurately and conveniently parametrised by a cosh potential $V(r)$ given by

$$V(r) = \frac{V_0(1+\cosh\frac{R}{a})}{\cosh\frac{r}{a}+\cosh\frac{R}{a}} = V_0 f(r) \tag{2.2}$$

and this form was subsequently used consistently in studies of triton and helion clustering in ^{17}F, ^{17}O, ^{18}F, ^{18}O and ^{19}F [7-11]. In all these nuclei the radius parameter R was taken as 2.0 fm and the diffuseness parameter a as 1.3 fm, whilst the depth V_0 was chosen to give the binding energy of one selected cluster state convincingly identified by its strong excitation in the appropriate three-nucleon transfer reaction.

For triton and helion cluster states a spin-orbit force was found to be an important component of the cluster-core interaction. This was written in the usual Thomas form as

$$V_{LS}(r) = V_{SO}\left(\frac{\hbar}{m_\pi c}\right)^2 \left(\frac{1}{r}\left|\frac{df}{dr}\right|\right)\vec{\ell}.\vec{\sigma} \tag{2.3}$$

where $\vec{\sigma}$ is the Pauli spin matrix for the cluster and all other symbols have their usual meaning. The depth of the spin-orbit potential was taken as -2.5 MeV for the N=6 bands of triton/helion cluster states in all five nuclei mentioned above. This equality of parameters for such similar cluster-core combinations is a very satisfying result and lends support to the idea that the triton/helion cluster states in these neighbouring nuclei are indeed closely related. We shall therefore try to use these same values (R=2.0 fm, a=1.3 fm and V_{SO}=-2.5 MeV) to describe an N=6 band of triton cluster states in ^{15}N, leaving only the central potential depth V_0 as a truly free parameter, to be fixed by the energy of one state in the band.

Finally we note that the cluster-core Coulomb potential is dealt with by treating the cluster as a point charge and the core as a uniformly charged sphere of radius R_c. A value of R_c = 3.0 fm was found to be satisfactory for the five cases mentioned above, and will be maintained for the ^{12}C core of the present calculation.

3. THE COUPLED CHANNELS FORMALISM

We follow closely the treatment of Baldock et al[13], where the coupled channels problem for a ^{12}C core in its 0^+ ground state and 2^+ (4.44 MeV) first excited state interacting with an inert alpha particle cluster is discussed, and details are given of a method of solution for states with energies below the cluster-core separation energy. These states are truly bound, with energies $E<0$. Resonant states, whose energies lie above the cluster-core separation threshold (i.e. having $E>0$) can be treated by the methods commonly employed in the calculation of inelastic scattering[16]. The present '' C-triton case is exactly analogous to the ^{12}C-alpha case except that we must include the additional complication of a cluster spin (and the associated spin-orbit force) in the formalism.

The wave function for a given total angular momentum J (projection M) is written as a sum over the channels i of a product of relative radial functions and relative angular/intrinsic functions

$$\Psi_{JM} = \sum_{i} \frac{1}{r} U_i^J(r)\ \phi_i^{JM}(\hat{\vec{r}},\xi) \tag{3.1}$$

where each channel $\vec{i}$ is defined by the core spin $\vec{I}_i$, an intermediate angular momentum $\vec{j}_i$ (formed by coupling the relative orbital angular momentum ℓ_i with the cluster spin s (= 1/2) so that $j_i = \ell_i \otimes s$), internal energy e_i and any other necessary quantum nurnbers. In equation (3.1) r is the magnitude and $\hat{\vec{r}}$ the angular coordinates of the vector $\vec{r}$ between the centres of mass of cluster and core, and ξ represents the internal coordinates. The total angular momentum $\vec{J}$ is formed by coupling the core spin $\vec{I}_i$ with the intermediate angular momentum $\vec{j}_i$ so that $\vec{J} = \vec{I}_i \otimes j_i$.

Substitution of equation (3.1) into the single-particle Schrödinger equation and projection onto the channel states yields, for each channel $\vec{i}$,

$$\left\{ -\frac{\hbar^2}{2\mu}\left[\frac{d^2}{dr^2} - \frac{\ell_i(\ell_i+1)}{r^2}\right] + e_i - E \right\} U_i^J(r) + \sum_k V_{ik}^J(r) U_k^J(r) = 0 \tag{3.2}$$

where μ is the reduced mass of the cluster-core system. The potential matrix is given by

$$V_{ik}^{J}(r) = (\phi_i^{JM}(\hat{r},\xi)\,|V(\vec{r},\xi)\,|\,\phi_k^{JM}(\hat{r},\xi)) \tag{3.3}$$

where the round brackets imply integration over all internal coordinates and over the relative angular coordinates $\hat{r}$. The interaction term $V(\vec{r},\xi)$ contains a spherical direct term, a quadrupole coupling term and a cluster spin-orbit term. Other terms such as a core spin-orbit force or a tensor interaction involving the two spins $\vec{I}_i$ and $\vec{s}$ are allowed by parity and time reversal invariance considerations but are expected to be small (from experience in neighbouring nuclei) and will be ignored in the following.

It is assumed that the 0^+ ground state and the 2^+ (4.44 MeV) first excited state of ^{12}C are the first two members of a $K^{\pi} = 0^+$ rotational band, and the only internal coordinate required is the orientation $\hat{s}$ of the symmetry axis. The potential may thus be written

$$V(\vec{r},\xi) = V_0 f(r) + V_0\left(-r\,\frac{df}{dr}\right)\beta\sum_q D^2_{q0}(\hat{s})\;Y^*_{2q}(\hat{r}) + V_{LS}(r) \tag{3.4}$$

for small deformations (where the usual derivative form has been taken for the radial part of the coupling potential). The geometry of the spherical direct term, as well as the cluster spin-orbit term were discussed in the last section, and the choice of the numerical values of the par-ameters V, and β will be deferred to the next section.

The contributions to the potential matrix elements may now be written as follows. For the monopole term

$$V_{ik}^{J(0)} = V_0\; f(r)\;\delta_{\ell_i\ell_k}\;\delta_{I_iI_k}\;\delta_{j_ij_k} \tag{3.5}$$

For the quadrupole term

$$V_{ik}^{J(2)} = \frac{V_0}{\sqrt{20\pi}}\left(-r\,\frac{df}{dr}\right)\beta\,(-1)^{J-\frac{1}{2}}\;(i)^{\ell_k-\ell_i+I_k-I_i}\;\hat{I}_i\hat{I}_k\hat{j}_i\hat{j}_k\hat{\ell}_i\hat{\ell}_k$$

$$\times <\ell_i0\;R_k\,0|20><I_i0I_k0|20>\;W(j_ij_kR_iR_k;\,2\tfrac{1}{2})W(I_i\,I_kj_ij_k;2J) \tag{3.6}$$

For the cluster spin-orbit term

$$V_{ik}^{LS}(r) = 2V_{SO}\left(\frac{\hbar}{m_\pi c}\right)^2\left(\frac{1}{r}\left|\frac{df}{dr}\right|\right)\delta_{j_ij_k}\delta_{I_iI_k}\delta_{\ell_i\ell_k}\left[j_i(j_i+1)-\ell_i(\ell_i+1)-\frac{3}{4}\right]\times$$
$$\times\frac{[J(J+1)+j_i(j_i+1)-I_i(I_i+1)]}{\sqrt{4J(J+1)j_i(j_i+1)}} \quad (3.7)$$

where δ_{mn} is the Kronecker delta, $\hat{\ell}=\sqrt{(2\ell+1)}$, $\langle j_1m_1j_2m_2|\,j_3m_3\rangle$ is a Clebsch-Gordan coefficient and $W(abcd;ef)$ is a Racah coefficient. The potential matrix elements are given by the sum of these last three expressions

$$V_{ik}^{J}(r) = V_{ik}^{(0)}(r) + V_{ik}^{J(2)}(r) + V_{ik}^{LS}(r) \quad (3.8)$$

4. CALCULATION AND DISCUSSION

To calculate the $N=6$ triton cluster state spectrum and associated wave functions in ^{15}N we must fix the parameters β and V_0. The recently measured quadrupole moment of the 2^+ (4.44 MeV) state in ^{12}C[17] indicates that β should be negatlve and we shall use the value $\beta=-0.22$ of Baldock et al[13] here (even though it should be adjusted slightly to allow for our different potential geometry) to keep the number of variable parameters to a minimum. The only truly free parameter is then the depth of the central potential which defines the energy of the band head relative to the ^{12}C-triton breakup threshold in ^{15}N, and must be chosen by reference to the experimental spectrum.

It is expected that ^{12}C-triton cluster states, with the core in its ground state, should be strongly excited in high energy triton transfer reactions onto ^{12}C. Pilt[14] has recently reviewed the various available data of this type, and suggested the following candidates, $\frac{1}{2}^+$ (8.31 or 9.05), $\frac{3}{2}^+$ (9.93 or 10.07), $\frac{5}{2}^+$ (9.16), $\frac{7}{2}^+$ (13.17), $\frac{9}{2}^+$ (10.69) and $\frac{13}{2}^+$ (15.27) in addition to the speculation that the $\frac{11}{2}^+$ member may have been seen near to 19.5 MeV. The energies quoted here have been taken from Ajzenberg-Selove[15], and we note that that work labels the state at

9.93 MeV as $\frac{3}{2}^-$. We shall therefore use these states (with this latter correction) as a guide to our choice of V_0.

The suggestion that the N=6 ^{12}C-triton cluster state band head should be located around 8-9 MeV in ^{15}N is supported by a tracing through of similar band heads from neighbouring nuclei. In ^{19}F the N=6 ^{16}O-triton band begins with the ground state[7]. In ^{18}F there is an N=6 band viewed as a linear combination of ^{15}N-^{3}He and ^{15}O-triton with its head at 2.5 MeV, while in ^{18}O the N=6 ^{15}N-triton band head is at 5.0 MeV [10]. In ^{17}O an N=6 ^{14}C-^{3}He band has its head at 6.8 MeV [9] and a similar ^{14}N-triton band in ^{17}O and a ^{14}N-^{3}He band in ^{17}F both have their heads at 8.86 MeV [11]. The proposed identification of the N = 6 ^{12}C(0$^+$ g.s.) + triton band in ^{15}N is therefore consistent with the view that this cluster structure persists throughout the mass range $15 \leqslant A \leqslant 19$.

The cluster states based on an excited core are not expected to be so strongly populated in transfer, although there is some hope that they might be seen in resonant particle spectroscopy experiments [18-20] with excited ^{15}N nuclei breaking up into ^{12}C and triton fragments whose excitation energies can be determined. At the present time the only recourse open to us is to compare our predicted J^{π} values and excitation energies with the tabulation of Ajzenberg-Selove in the hope that enough coincidences will act as an incentive to experimentalists to try and locate them more definitively.

Based on these considerations, we therefore adopt a value of V_0 = -99.5 MeV for the depth of the central potential. The resulting energies calculated in the coupled channels formalism of all the cluster states lying below the ^{12}C-triton separation energy in ^{15}N (14.85 MeV) are shown in table 1, together with the predominant values of the cluster-core relative orbital angular momentum ℓ, the intermediate angular momentum j and the core spin I. The mixing of states with equal J but different ℓ, j or I is not generally very great (except when two states are accidentally nearly degenerate) and so the values of these quantum numbers (and also the number of interior radial nodes n) can be easily checked against the corresponding single channel calculation. These and the higher lying state energies are shown in figure 1 as a function of $4J(J+1)$.

Table 1 - ^{12}C-triton bound state spectrum

$2J^{\pi}$	ℓ	$2j$	I^{π}	E_{PER}(MeV)	E_{cc}(MeV)	E_{EXP}(MeV)
1^+	0	1	0^+	8.30	8.32	8.31
5^+	2	5	0^+	9.06	9.04	9.16
3^+	2	3	0^+	10.13	10.14	10.07
9^+	4	9	0^+	11.42	11.35	10.69,12.55
7^+	4	7	0^+	13.16	13.14	13.17
3^+	0	1	2^+	12.54	12.57	-
5^+	0	1	2^+	12.44	12.47	12.49
5^+	2	5	2^+	13.81	13.83	13.61,13.99
7^+	2	5	2^+	13.33	13.33	-
9^+	2	5	2^+	14.21	14.16	14.09

The angular momentum coupling scheme is $\mathbf{j} = \ell \otimes s(=\frac{1}{2})$ and $J = I \otimes j$
E_{PER} - perturbative calculation, E_{cc} - coupled channels calculation, E_{EXP} - experiment.

Six bands of states are produced corresponding to all the possible couplings of the core spin of 0 and 2, and they are labelled by (twice) the angular momenta of the initial and final statas in each of them. The staggering produced by the spin-orbit force is clearly visible, but the effects of the tensor forces associated with the quadrupole deformation are also important and give rise to irregular deviations from the well known $\vec{\ell}\cdot\vec{I}$ splitting pattern. The phenomenon of band crossing is also present, with the higher members ($2J = 11,15,17$) of the second band appearing in or as a continuation of the lowest lying band.

Figure 1 shows that the energies of five of the first six mem-

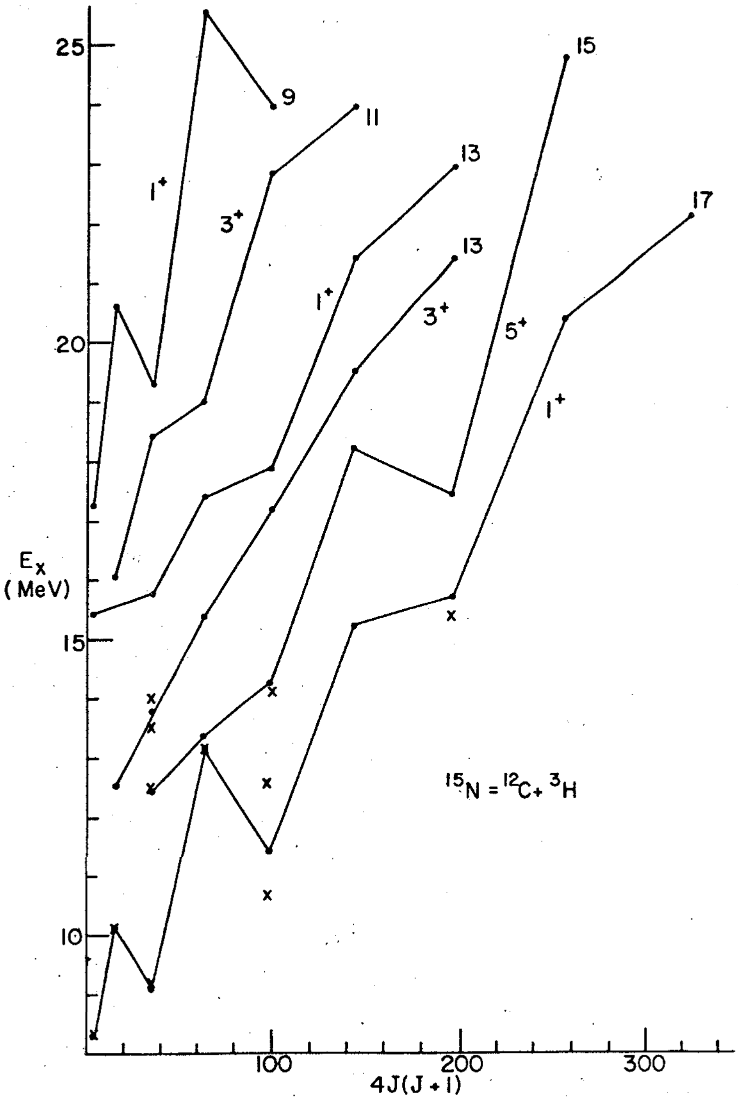

Fig.1 - The energies of the members of the $N=6$ bands of ^{12}C-triton cluster states in ^{15}N as calculated in the coupled channels formalism as a function of $4J(J+1)$ (full circles). Corresponding experimental candidates are marked by crosses.

bers of the $^{12}C(0^+$ g.s.)+triton band are excellently reproduced in the model. The exception is the $\frac{9}{2}^+$ state ($I=0, \ell=4 \otimes s=\frac{1}{2}$). However, we note that in addition to the $\frac{9}{2}^+$ state at 10.69 MeV a second $\frac{9}{2}^+$ state at 12.55 MeV is also strongly excited in the $^{12}C(^7Li,\alpha)$ ^{15}N experiments of Tserruya *et al*[21] and Zeller *et al*[22]. This suggests that the two states may be sharing the cluster strength, and we see that our predicted energy fits their centroid quite closely.

Five other cluster states are calculated below the ^{12}C-triton separation energy in ^{15}N, and the tabulation of Ajzenberg - Selove suggests counterparts for three of them (marked by crosses in figure 1). Clearly, it would be highly desirable to have data on triton decay widths and electromagnetic transitions between the cluster states to test the model more rigorously, but no such measurements appear to have been made. This is unfortunate since the ^{12}C-alpha calculation of Baldock *et al* shows that the real power of the coupled channels calculation lies in the improved wave functions and widths it produces. It would be particularly gratifying to observe the strong E1 inter-band transitions and the large triton decay widths which are expected to be the unmistakable signatures of these states. We can only hope that experimentalists will be stimulated to look for more triton cluster states and measure additional properties in view of the suggestiveness of the model predictions so far.

It is interesting to note that in the present case, where only energies are tested, a perturbative calculation based on a single channel approach[23] is equally satisfactory. By solving the single channel single-particle Schrödinger equation with the monopole potential of equation (3.5) (with V_0 = -99.5 MeV, R = 2.0 fm and a = 1.3 fm) for ℓ= = 0,2,4 and 6 (and 3,2,1 and 0 interior radial nodes respectively), then using the resulting wave functions to calculate matrix elements of the quadrupole and spin-orbit potentials of equations (3.6) and (3.7) and diagonalizing the resulting matrix, close agreemerit with the coupled channels energies is obtained. The results for the bound state energies in this first order perturbative approach are shown in table 1, where the largest discrepancy with the coupled channels calculation is seen to be 0.07 MeV, and is usually less (~0.02-0.03 MeV).

The model can, of course, generate higher bands of cluster states corresponding to values of N = 7,8,... etc. The negative parity N = 7 band is expected to have its band head around 15-16 MeV in ^{15}N, but the currently available data are too sparse to seek corresponding experimental candidates in that region. In addition, mirror $^{12}C-^{3}He$ cluster states are expected in ^{15}O, but again there is not enough experimental evidence to convincingly identify them.

5. CONCLUSION

We have presented a coupled channels calculation of triton cluster states in ^{15}N based on a ^{12}C core in its 0^+ ground and 2^+ (4.44 MeV) first excited states. By using the same potential geometry (and parameters) as for triton/helion cluster states in neighbouring nuclei, and a ^{12}C deformation parameter consistent with the quadrupole moment of the 2^+ (4.44 MeV) state, so that only the central potential depth V_0 was a truly free parameter, we have been able to reproduce the energies of the best known experimental triton cluster state candidates in ^{15}N very well. We predict five additional bands of positive parity triton cluster states which might be accessible to experiment through resonant particle spectroscopy. We strongly urge experimentalists to undertake a systematic study of the nuclei with masses $15 \leqslant A \leqslant 19$ to investigate triton/helion cluster structure across these nuclei and to try and measure electromagnetic properties and, if possible, triton/helion decay widths of the suggested cluster states to confirrn that closely related states exhibiting this structure exist throughout this mass region.

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Resumo

É apresentado um cálculo, usando o formalismo de canais acoplados, de algumas bandas de estados de 'cluster' de trítio com energias elevadas e paridade positiva em ^{15}N. Estas bandas são construidas em cima de um caroço de ^{12}C que se encontra no estado fundamental, 0^+, ou no primeiro estado excitado, 2^+ (4.44 MeV). É feita uma comparação entre o cálculo e os dados experimentais obtidos em reações de transferência de

trítio para um alvo de ^{12}C e também com uma recente compilação abrangente das propriedades de todos os estados conhecidos em ^{15}N. A estrutura dos estados de 'cluster' de trítio em ^{15}N apresenta uma grande semelhança à mesma estrutura nos núcleos vizinhos ^{17}F, [illegible]O, ^{18}F, ^{18}O e ^{19}F, portanto apoiando a hipótese de que este é um fenômeno estável nesta região de massa.