Tensor Forms of a Redefined Two-Pion Exchange Three-Nucleon Potential

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Abstract We present here a summary of arguments concerning the two-pion exchange three-nucleon potentials ($\pi\pi\pi$-3NP), derived by means of chiral symmetry. This sets earlier developments into a single framework, avoiding confusion among readers. We also derive tensor forms of $\pi\pi\pi$-3NP, especially an irreducible tensor representation of those forces in such a way that spin-isospin terms are not mixed with those of configuration space. This form facilitates calculations and emphasizes the underlying physics.

1. INTRODUCTION

It is now well recognized\(^1\) that the fine features of trinucleon systems cannot be entirely described by the two-nucleon interaction alone. This conclusion arises from various works\(^1\) done in recent years, based on different calculation techniques and realistic two-nucleon potentials. Strong circumstantial evidence exists for three-nucleon potentials (3NP) in the trinucleons. Recent works\(^1\) to\(^5\) conjecture that the inclusion of the two-pion-exchange ($\pi\pi\pi$) component of this force should account for a substantial part of the discrepancy in the data. This $\pi\pi\pi$ component is so far the best studied and it gives a significant contribution to the three-nucleon binding energy\(^1\), which is greater for the São Paulo-Recife\(^2\) and Urbana\(^4\),\(^5\) forces, and smaller for the Tucson-Melbourne\(^3\) force. On the other hand, most of these calculations find\(^1\) a small effect on the charge form factors, possibly coming from the use of perturbation theory\(^6\). At present technical problems in the theory, combined with a lack of experimental data as well as difficulties in

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more rigorous calculations recommend a deeper study of the problem.

The final forms of the $\pi\pi E$ derived recently have become the subject of some controversy, which has motivated an effort towards a better understanding of the problem. One of the important implications of a critical assessment of the $\pi\pi E-3NP$ is that the expressions originally derived by the Tucson-Melbourne and by the São Paulo-Recife groups have to be modified in order to be used in realistic calculations. In this article we discuss very briefly the most important points that have led to the form of the potential to be adopted in this work.

One purpose of this paper is to present a summary of arguments concerning the construction of a $\pi\pi E-3NP$ in a way to set earlier developments into a single framework, thereby avoiding confusion among readers. Another purpose is to derive that potential in tensor forms. An Irreducible Tensor Operator (ITO) representation is derived as well. The motivations for this effort are manifold. Calculations of the complete $\pi\pi E-3NP$ effect on trinucleon systems have largely depended on perturbation theory or variational methods. However for observables very sensitive to the wavefunction, such as charge form factors, very precise calculations of the wavefunction are needed. In this way, $\pi\pi E-3NP$ as such should be considered on the same footing as the two-nucleon potential, $V$, (which contains tensor parts of rank 0 and 2) in the Schrodinger equation. That means that the $\pi\pi E-3NP$ should be treated non-perturbatively. For practical and physical purposes it is important to write $\pi\pi E-3NP$ in the ITO form, since it allows us to analyze their various contribution terms, as in the corresponding two-nucleon potential. The tensor parts of $\pi\pi E-3NP$ deserve particular attention, especially those of rank 2, since they can significantly contribute to observables, in particular to the binding energy.

In section 2 we give a single redefined version of $\pi\pi E-3NP$; in section 3 we derive the tensor forms of that potential. Finally in section 4 we give the conclusions.

2. THE REDEFINED THREENUCLEON POTENTIAL

The $\pi\pi E-3NP$ originally derived in references 2 and 3 is here
denoted by \( W \) and has the following generic form

\[
W = \left( \frac{C}{\mu^2} \right) (\hat{t}(i), \hat{t}(j), \hat{t}(k)) (\hat{\sigma}(i), \hat{\sigma}(j), \hat{\sigma}(k)) U_0(r_{kt}) U_0(r_{jk})
+ \left( \frac{C'}{\mu^2} \right) (\hat{t}(i), \hat{t}(j), \hat{t}(k)) (\hat{\sigma}(i), \hat{\sigma}(j), \hat{\sigma}(k)) U_0(r_{kt}) U_0(r_{jk})
+ \left( \frac{C'}{\mu^2} \right) (\hat{t}(i), \hat{t}(j), \hat{t}(k)) (\hat{\sigma}(i), \hat{\sigma}(j), \hat{\sigma}(k)) U_0(r_{kt}) U_0(r_{jk})
+ \text{C.P.}
\]

(2.1)

where the subscripts \( s \) and \( p \) indicate partial waves in the intermediate pion-nucleon amplitude, \( \text{c.p.} \) indicates cyclic permutations of the indices \( (i,j,k) = (1,2,3) \), and \( U_0(r) \) is a Yukawa function modified by the \( \pi\text{NN} \) form factor

\[
U_0(r) = \frac{4\pi}{\mu} \int \frac{d^3k}{(2\pi)^3} \frac{e^{-\mu k}}{k^2 + \mu^2} \left( \frac{\Lambda^2 - \mu^2}{\Lambda^2 + k^2} \right)^2 \left( \frac{\Lambda}{\mu} \right) e^{-\Lambda r}
- \frac{1}{2} \frac{\mu}{\Lambda} \left( \frac{\Lambda^2}{\mu^2} - 1 \right) e^{-\Lambda r}
\]

(2.2)

(In reference 8, \( U_0 \) has no zero subscript).

The coefficients \( C_\pi, C_\pi' \), and \( C_\pi'' \) are known in terms of the \( \pi\text{N} \) scattering amplitude and their numerical values are \( C_\pi = 0.92 \text{ MeV}, C_\pi' = -1.99 \text{ MeV}, C_\pi'' = -0.67 \text{ MeV}. \)

The terms proportional to \( C_\pi \) and \( C_\pi' \) are the same as those of reference 2. The term with coefficient \( C_\pi'' \), on the other hand, has to be modified by inserting the operator

\[
\left[ 1 + \frac{1}{\mu^2} \left( \hat{\psi}_{k}^2 - \mu^2 \right) + \frac{1}{\mu^2} \left( \hat{\psi}_{j}^2 - \mu^2 \right) \right]
\]

just before \( U_0(r_{kt}) U_0(r_{jk}) \). This modification is needed, as pointed out in ref. 10, in order to make the \( a \)-contribution to the intermediate \( \pi\text{N} \) scattering amplitude compatible with the Adler consistency condition. \[12\]
After having introduced the factor $(\bar{\phi}^2 - \mu^2)$ into the potential $\mathcal{H}$, it has to be redefined before being used in realistic calculations. An interesting characteristic of this redefinitions of $\mathcal{W}$ is that it prescribes the exclusion of factors $(\bar{\phi}^2 - \mu^2)^6$.

Our final form for the $\pi\piE-3NP$ is obtained by eliminating all contact interactions and their derivatives from the potential obtained by means of chiral symmetry. The motivation for this procedure can be summarized as follows. In the absence of form factors, the potential contains $\delta$-functions, arising from the derivatives of the Yukawa functions, and describing a contact interaction between two of the nucleons. Form factors, on the other hand, are related to the distribution of hadronic matter within the nucleons. Hence, when they are introduced into the potential, the $\delta$-functions turn into terms that describe contact interactions between extended nucleons. These terms cannot be associated with the propagation of pions and do not correspond to a $\pi\piE-3NP$. Instead, they could be called a $\pi\piE-3NP$, since the parameter of the form factor does not let us know the type of particles being exchanged.

Hence, following the above prescriptions, the redefined version of the potential, indicated by a caret on $\mathcal{W}$, is given by

$$
\widehat{\mathcal{W}}(k) = C_1 \left( \left( \bar{\phi}(\bar{z}) \cdot \bar{\tau}(\bar{j}) \right) (\sigma(\bar{z}) \cdot \bar{\tau}(\bar{j}) \cdot \bar{r}_{kj}) U_1(r_{kj}) U_1(r_{jk}) + \frac{C_2}{3} (\bar{\phi}(\bar{z}) \cdot \bar{\tau}(\bar{j}) \cdot \bar{r}_{kj}) U_2(r_{kj}) U_2(r_{jk}) \times \left[ \delta(\bar{z}) \cdot \bar{r}_{kj} \right] U_0(r_{kj}) U_0(r_{jk}) + \left[ 3 (\sigma(\bar{z}) \cdot \bar{r}_{kj}) (\sigma(\bar{j}) \cdot \bar{r}_{jk}) - \delta(\bar{z}) \cdot \sigma(\bar{j}) \cdot \bar{r}_{jk} \right] U_2(r_{kj}) U_2(r_{jk}) + \left[ 3 (\sigma(\bar{z}) \cdot \bar{r}_{kj}) (\sigma(\bar{j}) \cdot \bar{r}_{jk}) - \delta(\bar{z}) \cdot \sigma(\bar{j}) \cdot \bar{r}_{jk} \right] U_0(r_{kj}) U_0(r_{jk}) + \left[ 3 \cos(\bar{z}) \cdot \bar{r}_{kj} \right] U_2(r_{kj}) U_2(r_{jk}) \right) \left\{ - \frac{C_1}{3} (\bar{\phi}(\bar{z}) \cdot \bar{\tau}(\bar{j}) \cdot \bar{r}_{kj}) U_0(r_{kj}) U_0(r_{jk}) \times \left[ \delta(\bar{z}) \cdot \bar{r}_{jk} \right] U_0(r_{kj}) U_0(r_{jk}) + \left[ 3 (\sigma(\bar{z}) \cdot \bar{r}_{kj}) (\sigma(\bar{j}) \cdot \bar{r}_{jk}) - \delta(\bar{z}) \cdot \sigma(\bar{j}) \cdot \bar{r}_{jk} \right] U_2(r_{kj}) U_2(r_{jk}) \right\}
$$

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where \( U_0 \) is given by (2.2) and

\[
\cos \theta_k = \frac{\tilde{r}_{\bar{k}i} \tilde{r}_{\bar{j}k}}{r_{ki} r_{jk}} ,
\]

(2.4)

and

\[
U_1 (x) = - \frac{e^{-ix}}{ix} \left[ 1 + \frac{1}{ix} \right] + \frac{\Lambda^2}{\mu^2} \left[ 1 + \frac{1}{ix} \right] e^{-\frac{\Lambda x}{\mu^2}} - \frac{1}{2} \frac{\Lambda^2}{\mu^2} \left[ 1 + \frac{1}{ix} \right] e^{-\frac{\Lambda x}{\mu^2}} ,
\]

(2.5)

\[
U_2 (x) = \frac{e^{-ix}}{ix} \left[ 1 + \frac{3}{ix} + \frac{3}{\mu^2 x^2} \right] - \frac{\Lambda^3}{\mu^3} \frac{e^{-\frac{\Lambda x}{\mu^2}}}{ix} \left[ 1 + \frac{3}{\Lambda x} + \frac{3}{\Lambda^2 x^2} \right] - \frac{1}{2} \frac{\Lambda^2}{\mu^2} \left[ 1 + \frac{1}{ix} \right] e^{-\frac{\Lambda x}{\mu^2}} \left[ 1 + \frac{1}{ix} \right] .
\]

(2.6)

The expression for \( \hat{H}(k) \) is the same as that of eq. (67) of ref. 2, except for an unfortunate misprint in the sign of the term proportional to \( C'_p \) in that equation.

3. TENSOR OPERATOR FORMS OF THE THREE-NUCLEON FORCE

In order to obtain tensor forms of the three-nucleon force we take eq. (2.3) as the starting point. A very convenient form of eq. (2.3) is obtained by recombining their different terms. We obtain, after some straightforward algebra, a tensor form of eq. (2.3):

\[
\hat{W}_s (k) = \frac{G}{3} \tilde{r}_{\bar{i}} (\bar{i}) . \tilde{r}_{\bar{j}} (\bar{j}) . \tilde{r}_{\bar{k}} (\bar{k}) . \omega_{\bar{k}i} (\bar{i}) \theta_{\bar{k}j} (\bar{j}) . \omega_{\bar{k}k} (\bar{k}) . \hat{H} (\bar{k}) + S_{\bar{i} \bar{j}} \omega_{\bar{i} j} (\bar{i}) \omega_{\bar{j} k} (\bar{j}) . \omega_{\bar{k} i} (\bar{k}) . \hat{H} (\bar{k}) ,
\]

(3.1)
\[ \hat{V}_p(k) = \frac{c}{9} \frac{\dot{\tau}(i) \cdot \dot{\tau}(j)}{2 \cos \theta(k)} U_2(r_{ki}) U_2(r_{jk}) \sigma(i) \sigma(j) \]

\[ + \left[ U_0(r_{ki}) - U_2(r_{ki}) \right] U_2(r_{jk}) S_{ij}(\tilde{r}_{jk}, \tilde{r}_{jk}) \]

\[ + U_2(r_{ki}) \left[ U_0(r_{jk}) - U_2(r_{jk}) \right] S_{ij}(\tilde{r}_{ki}, \tilde{r}_{ki}) \]

\[ + 3 \cos \theta(k) U_2(r_{ki}) U_2(r_{jk}) S_{ij}(\tilde{r}_{ki}, \tilde{r}_{jk}) \]

\[ \hat{V}_p(k) = \frac{c}{9} \left[ \frac{\dot{\tau}(k) \cdot \dot{\tau}(i) \cdot \dot{\tau}(j)}{2 \cos \theta(k)} \sigma(i) \sigma(j) \right] \]

\[ + U_0(r_{ki}) U_2(r_{jk}) \left[ \frac{1}{2} \left[ S_{ij}(\tilde{r}_{jk}, \tilde{r}_{jk}) \sigma(i) \sigma(j) - \sigma(k) \sigma(\tilde{k}) \right] S_{ij}(\tilde{r}_{jk}, \tilde{r}_{jk}) \]

\[ + U_2(r_{ki}) U_0(r_{jk}) \left[ \frac{1}{2} \left[ S_{ij}(\tilde{r}_{ki}, \tilde{r}_{ki}) \sigma(i) \sigma(j) + \sigma(k) \sigma(\tilde{k}) \right] S_{ij}(\tilde{r}_{ki}, \tilde{r}_{ki}) \]

\[ + U_2(r_{ki}) U_2(r_{jk}) \left[ \frac{1}{2} \left[ S_{ij}(\tilde{r}_{ki}, \tilde{r}_{ji}) S_{jk}(\tilde{r}_{jk}, \tilde{r}_{jk}) - S_{jk}(\tilde{r}_{jk}, \tilde{r}_{jk}) S_{ij}(\tilde{r}_{ki}, \tilde{r}_{ki}) \right] \right] \]

where the tensor \( S_{ij} \) for two unit vectors \( \tilde{u} \) and \( \tilde{v} \) is defined as

\[ S_{ij}(\tilde{u}, \tilde{v}) = 3 \left( \sigma(i) \cdot \tilde{u} \right) \left( \sigma(j) \cdot \tilde{v} \right) - \left( \sigma(i) \cdot \tilde{v} \right) \left( \sigma(j) \cdot \tilde{u} \right) \].

The above equations are not written in an ITO format. We should also notice that spins are mixed with terms in configuration space. Our final objective is to write eq. (2.3) in the ITO form in such a way that spin-isospin terms are not mixed with those of configuration space. That will make the calculation of the matrix elements of \( \hat{V} \) much simpler and direct; only terms allowed by the triangle selection rule of the tensor ranks of the configuration space operators involved will be picked up.

In eq. (2.3), terms appear which are of the types

\[ \left( \sigma(i) \cdot \tilde{r}_{ki} \right) \left( \sigma(j) \cdot \tilde{r}_{jk} \right) , \left( \sigma(i) \cdot \tilde{r}_{ki} \right) \left( \sigma(j) \cdot \tilde{r}_{ki} \right) \]

and

\[ \sigma(k) \cdot \left( \tilde{r}_{ki} \cdot \tilde{r}_{jk} \right) \left( \sigma(i) \cdot \tilde{r}_{ki} \right) \left( \sigma(j) \cdot \tilde{r}_{jk} \right) \].

Using eq. (A.5) the first two equations above can be written as
The third term can also be written as

\[ \langle \sigma^-(\mathbf{z}_j) \sigma^+(\mathbf{z}_j) \sigma^-\sigma^- \rangle = \frac{1}{\sqrt{\beta}} T_0(\sigma^-,\sigma^-) + \sum_{\ell=0}^{2} (-\ell^2) T_\ell(\sigma^-,\sigma^-) T\langle \sigma^-\sigma^- \rangle \cdot \]

(3.4)

and

\[ \langle \sigma^-\sigma^- \rangle = \langle \sigma^-\sigma^- \rangle \]

\[ \langle \sigma^-\sigma^- \rangle = \frac{1}{\sqrt{\beta}} T_0(\sigma^-,\sigma^-) + \sum_{\ell=0}^{2} (-\ell^2) T_\ell(\sigma^-,\sigma^-) T\langle \sigma^-\sigma^- \rangle \cdot \]

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(3.6)

where we have used eq. (A.3).

Hence we can obtain

\[ \langle \sigma^-(\mathbf{z}_j) \sigma^+(\mathbf{z}_j) \sigma^-\sigma^- \rangle = \frac{1}{\sqrt{\beta}} T_0(\sigma^-,\sigma^-) + \sum_{\ell=0}^{2} (-\ell^2) T_\ell(\sigma^-,\sigma^-) T\langle \sigma^-\sigma^- \rangle \cdot \]

(3.7)

With eqs. (3.4)-(3.7) in mind we can obtain the ITO form of \( \tilde{\nu}(k) \)

\[ \tilde{\nu}(k) = \tilde{\nu}_S(k) + \tilde{\nu}_P(k) + \tilde{\nu}^I(k) , \]

(3.8)

where
\[
\hat{\theta}_g(k) = -\sqrt{3}C_0 \mathcal{T}_0 \left( \hat{T}(\hat{\tau}), \hat{T}(\hat{\tau}) \right) \left[ \mathcal{T}_0 \left( \hat{\sigma}(\hat{\tau}), \hat{\sigma}(\hat{\tau}) \right) \mathcal{T}_0 \left( \hat{\nu}_{k\ell} \hat{\nu}_{\ell j} \right) - T_1 \left( \hat{\sigma}(\hat{\tau}), \hat{\sigma}(\hat{\tau}) \right) T_1 \left( \hat{\nu}_{k\ell} \hat{\nu}_{\ell j} \right) + T_2 \left( \hat{\sigma}(\hat{\tau}), \hat{\sigma}(\hat{\tau}) \right) T_2 \left( \hat{\nu}_{k\ell} \hat{\nu}_{\ell j} \right) \right] U_{11} \left( \hat{\varepsilon} \right), \tag{3.9}
\]

\[
\hat{\theta}_p(k) = \frac{C}{3} \left[ T_0 \left( \hat{T}(\hat{\tau}), \hat{T}(\hat{\tau}) \right) \left[ \mathcal{T}_0 \left( \hat{\sigma}(\hat{\tau}), \hat{\sigma}(\hat{\tau}) \right) \mathcal{T}_0 \left( \hat{\nu}_{k\ell} \hat{\nu}_{\ell j} \right) - T_0 \left( \hat{T}(\hat{\tau}), \hat{T}(\hat{\tau}) \right) U_{00} \left( \hat{\varepsilon} \right) + \mathcal{T}_0 \left( \hat{\nu}_{k\ell} \hat{\nu}_{\ell j} \right) \right] \times T_0 \left( \hat{\nu}_{k\ell} \hat{\nu}_{\ell j} \right) - 1 \right] U_{22} \left( \hat{\varepsilon} \right) + \mathcal{T}_0 \left( \hat{\nu}_{k\ell} \hat{\nu}_{\ell j} \right) U_{12} \left( \hat{\varepsilon} \right) - \mathcal{T}_0 \left( \hat{\nu}_{k\ell} \hat{\nu}_{\ell j} \right) T_0 \left( \hat{T}(\hat{\tau}), \hat{T}(\hat{\tau}) \right) U_{22} \left( \hat{\varepsilon} \right) - \mathcal{T}_0 \left( \hat{\nu}_{k\ell} \hat{\nu}_{\ell j} \right) \times T_2 \left( \hat{\nu}_{k\ell} \hat{\nu}_{\ell j} \right) U_{22} \left( \hat{\varepsilon} \right) + \mathcal{T}_0 \left( \hat{\nu}_{k\ell} \hat{\nu}_{\ell j} \right) T_2 \left( \hat{\nu}_{k\ell} \hat{\nu}_{\ell j} \right) U_{22} \left( \hat{\varepsilon} \right) \times T_1 \left( \hat{\sigma}(\hat{\tau}), \hat{\sigma}(\hat{\tau}) \right) T_1 \left( \hat{\nu}_{k\ell} \hat{\nu}_{\ell j} \right) U_{22} \left( \hat{\varepsilon} \right) \right], \tag{3.10}
\]

and finally

\[
\hat{\theta}_P(k) = \frac{2C_1}{3} \left[ T_0 \left( \hat{T}(\hat{\tau}), \hat{T}(\hat{\tau}) \right) \left[ \mathcal{T}_0 \left( \hat{\sigma}(\hat{\tau}), \hat{\sigma}(\hat{\tau}) \right) \mathcal{T}_0 \left( \hat{\nu}_{k\ell} \hat{\nu}_{\ell j} \right) - T_0 \left( \hat{T}(\hat{\tau}), \hat{T}(\hat{\tau}) \right) U_{00} \left( \hat{\varepsilon} \right) + \mathcal{T}_0 \left( \hat{\nu}_{k\ell} \hat{\nu}_{\ell j} \right) \right] \times T_0 \left( \hat{\nu}_{k\ell} \hat{\nu}_{\ell j} \right) - 1 \right] U_{22} \left( \hat{\varepsilon} \right) + \mathcal{T}_0 \left( \hat{\nu}_{k\ell} \hat{\nu}_{\ell j} \right) U_{12} \left( \hat{\varepsilon} \right) - \mathcal{T}_0 \left( \hat{\nu}_{k\ell} \hat{\nu}_{\ell j} \right) T_0 \left( \hat{T}(\hat{\tau}), \hat{T}(\hat{\tau}) \right) U_{22} \left( \hat{\varepsilon} \right) - \mathcal{T}_0 \left( \hat{\nu}_{k\ell} \hat{\nu}_{\ell j} \right) \times T_2 \left( \hat{\nu}_{k\ell} \hat{\nu}_{\ell j} \right) U_{22} \left( \hat{\varepsilon} \right) + \mathcal{T}_0 \left( \hat{\nu}_{k\ell} \hat{\nu}_{\ell j} \right) T_2 \left( \hat{\nu}_{k\ell} \hat{\nu}_{\ell j} \right) U_{22} \left( \hat{\varepsilon} \right) \times T_1 \left( \hat{\sigma}(\hat{\tau}), \hat{\sigma}(\hat{\tau}) \right) T_1 \left( \hat{\nu}_{k\ell} \hat{\nu}_{\ell j} \right) U_{22} \left( \hat{\varepsilon} \right) \right] \tag{3.11}
\]
Notice that

\[ U_{\alpha\beta}(j\ell) \equiv U_{\alpha}(r_{j\ell})U_{\beta}(r_{j\ell}) . \]

(Each single latin index \((i, j, k)\) in the argument of the left-hand side equation \(U_{\alpha\beta}\) means one missing index in the corresponding subscripts of \(r\) in the right-hand side equations; e.g. \(i \leftrightarrow jk, \quad j \leftrightarrow k\ell\).

We can see from the above equations that \(\hat{\mathbf{W}}\) is now written in the ITO form (besides having the configuration space operators factored out from the spin-isospin part).

4. CONCLUSIONS

We have presented here a summary of arguments concerning the derivation of a single expression of \(\pi\piE-3NP\), since originally two slightly different forms appeared in the literature, known as the São Paulo-Recife\(^2\) (also known as the Brazilian potential) and the Tucson-Melbourne\(^3\) potentials. The Brazilian potential used an effective Lagrangian approach while the Tucson-Melbourne potential used current algebra, both based on chiral symmetry. It is shown that both approaches produce essentially the same results. As far as the case of extended nucleons is concerned, we have argued that the expressions derived by the above groups\(^2,\)\(^3\) need to be modified in order to be used in realistic calculations. The problem with these potentials is that they become strongly distorted when form factors are present. These distortions are incompatible with our expectation that form factors should modify the potential for point-like nucleons at short distances. This odd behaviour is associated with terms in the potential that can be interpreted as contact interactions between extended nucleons. We have argued that the elimination of those contact terms from the potential makes it much better behaved. Therefore we have redefined the \(\pi\piE-3NP\) as the potential associated only with the propagation of pions described by an Yukawa function regularized at the origin. This redefined version of the potential is given by eq.\((2.3)\).

We would like to point out that \(\hat{\mathbf{W}}\), the redefined potential, is not the outcome of indisputable formal derivations. Instead, it is closely
associated with a particular way of interpreting a physical picture allowed by the mathematical formulation.

In this paper we have also given two tensor forms for the redefined potential, $\hat{\mu}$. The first one is given by eqs. (3.1) - (3.3) and the second one, eqs. (3.8) - (3.11), is written in the ITO form in such a way that the spin-isospin terms are not mixed with those of configuration space. Both versions are very useful in practical calculations. The ITO version, in particular, is extremely suitable for the hyperspherical harmonic (HH) approach\textsuperscript{9,20}. Thus in the evaluation of matrix elements in the HH approach we can apply the Wigner-Eckart\textsuperscript{17} theorem in a straightforward way.

The separation of $\hat{\mu}$ in different tensor ranks, helps to compare the relative importance of $\pi\pi\text{E-3NP}$ with the corresponding two-nucleon interaction, $V$. In a separate paper\textsuperscript{8} we have studied it extensively.

**APPENDIX A: USEFUL EXPRESSIONS FOR ITO**

The tensor direct product of two spherical tensor operators $T_{k_1}$ and $T_{k_2}$ (forming an ITO of rank $k$) is defined\textsuperscript{17} as

$$T_k(T_{k_1},T_{k_2}) = (T_{k_1} \otimes T_{k_2})^Q = \sum_{q_1,q_2} \langle k_1,k_2,q_1,q_2 | kQ \rangle T_{k_1}^q T_{k_2}^{q_2} \quad \text{(A.1)}$$

The scalar product or contraction of two commuting tensor operators is defined as

$$T_k \cdot U_k = \sum_{q=-k}^k (-)^q T_k^{-q} U_k^q \quad \text{(A.2)}$$

where one can easily see that

$$T_0(T_k,U_k) = (T_k \otimes U_k)_0 = \frac{(-)^k}{\sqrt{2k+1}} T_k \cdot U_k \quad \text{(A.3)}$$

In a similar way one can show\textsuperscript{18} that

$$T_1(\vec{a},\vec{b}) = (\vec{a} \otimes \vec{b}) = \frac{i}{\sqrt{2}} \vec{a} \times \vec{b} \quad \text{(A.4)}$$
Also very useful are the expressions\footnote{\textsuperscript{19}}:

\[
\left( T_{k_1} \otimes T_{k_2} \right) \cdot \left( T_{k_3} \otimes T_{k_4} \right) = (2k+1)(-)^{k_1+k_3} 
\]

\[
\times \sum_{k'} (-)^{k_1+k_2+k_3+k_4} \left\{ \begin{array}{ccc} k_1 & k_2 & k' \\ k_4 & k_3 & k \end{array} \right\} \left( T_{k_1} \otimes T_{k_3} \right) \cdot \left( T_{k_2} \otimes T_{k_4} \right) , \quad (A.5)
\]

where $\left[ T_{k_2}, T_{k_3} \right] = 0$. and

\[
\left( \sigma(k) \otimes \tilde{\alpha} \right) \cdot \left( \sigma'(l) \otimes \sigma'(j) \right) \otimes (\tilde{b} \otimes \tilde{c}) = \frac{1}{\sqrt{3}(2k+1)} \sum_{l'=1}^{1+2k} (-)^{l'} \left( \sigma(k) \otimes \tilde{\alpha} \right) \cdot \left( \sigma'(l) \otimes \sigma'(j) \right) \l', \quad (A.6)
\]

where $\tilde{\alpha}$, $\tilde{b}$ and $\tilde{c}$ are vectors.

REFERENCES

Apresentamos um resumo dos resultados referentes aos potenciais de três nucleons (ππE-3NP), obtidos através da simetria chiral. Estes resultados colocam dentro de um esquema Único os resultados anteriormen-
te obtidos, evitando confusão para os leitores. Também derivamos formas
tensoriais de ππ-3NP, especialmente uma representação tensorial irredu-
tível das forças de 3 corpos, de modo tal que termos de spin-isospin não
são misturados com os do espaço de configuração. Esta forma facilita os
cálculos e dá destaque à física subjacente.