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Tensor Forms of a Redefined Two-Pion Exchange Three-Nucleon Potential

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Abstract We present here a summary of arguments concerning the two-pion -exchange three-nucleon potentials (ππE-3NP), derived by means of chiral symmetry. This sets earlier developments into a single framework, avoiding confusion among readers. We also derive tensor forms of ππE-3NP, especially an irreducible tensor representation of those forces in such a way that spin-isospin terms are not mixed with those of configuration space. This form facilitates calculations and emphasizes the underlying physics.

1. INTRODUCTION

It is now well recognized[1] that the fine features of trinucleon systems cannot be entirely described by the two nucleon interaction alone. This conclusion arises from various works[1] done in recent years, based on different calculation techniques and realistic two-nucleon potentials. Strong circumstantial evidence exists for three-nucleon potentials (3NP) in the trinucleons. Recent works[1-5] conjecture that the inclusion of the two-pion-exchange (ππE) component of this force should account for a substantial part of the discrepancy in the data. This ππE component is so far the best studied and it gives a significant contribution to the three-nucleon binding energy[1], which is greater for the São Paulo-Recife[2] and Urbana[4,5] forces, and smaller for the Tucson--Melbourne[3] force. On the other hand, most of these calculations find[1] a small effect on the charge form factors, possibly coming from the use of perturbation theory[6]. At present technical problems in the theory, combined with a lack of experimental data as well as difficulties in

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more rigorous calculations recommend a deeper study of the problem.

The final forms[2-3] of the $\pi\pi E$ derived recently have become the subject of some controversy[10,11,7,9], which has motivated an effort towards a better understanding of the problem. One of the important implications of a critical assessment of the $\pi\pi E$-3NP is that the expressions originally derived by the Tucson-Melbourne[3] and by the São Paulo-Recife[2] groups have to be modified in order to be used in realistic calculations. In this article we discuss very briefly the most important points that have led to the form of the potential to be adopted in this work.

One purpose of this paper is to present a summary of arguments concerning the construction of a $\pi\pi E$-3NP in a way to set earlier developments into a single framework, thereby avoiding confusion among readers. Another purpose is to write that potential in tensor forms. An Irreducible Tensor Operator (ITO) representation is derived as well. The motivations for this effort are manifold. Calculations of the complete $\pi\pi E$-3NP effect on trinucleon systems have largely depended on perturbation theory[13-15] or variational methods[16]. However for observables very sensitive to the wavefunction, such as charge form factors, very precise calculations of the wavefunction are needed. In this way, $\pi\pi E$--3NP as such should be considered on the same footing as the two-nucleon potential, V, (which contains tensor parts of rank 0 and 2) in the Schrodinger equation. That means that the $\pi\pi E$-3NP shold be treated non-perturbatively[6]. For practical and physical purposes it is important to write $\pi\pi E$-3NP in the ITO form, since it allows us to analyze their various contribution terms, as in the corresponding two-nucleon potential. The tensor parts of $\pi\pi E$-3NP deserve particular attention, especially those of rank 2, since they can significantly contribute to observables, in particular to the binding energy[1].

In section 2 we give a single redefined version of $\pi\pi E$-3NP; in section 3 we derive the tensor forms of that potential. Finally in section 4 we give the conclusions.

2. THE REDEFINED THREENUCLEON POTENTIAL

The $\pi\pi E$-3NP originally derived in references 2 and 3 is here denoted by W and has the following generic form

$$W = \left(\frac{C_s}{\mu^2}\right)(\vec{\tau}^{(i)}.\vec{\tau}^{(j)})(\vec{\sigma}^{(i)}.\vec{\nabla}_{ki})(\vec{\sigma}^{(j)}.\vec{\nabla}_{jk})U_0(r_{ki})U_0(r_{jk})$$

$$+ \left(\frac{C_p}{\mu^4}\right)(\vec{\tau}^{(i)}.\vec{\tau}^{(j)})(\vec{\sigma}^{(i)}.\vec{\nabla}_{ki})(\vec{\sigma}^{(j)}.\vec{\nabla}_{jk})(\vec{\nabla}_{ki}.\vec{\nabla}_{jk})U_0(r_{ki})U_0(r_{jk})$$

$$+ \left(\frac{C'_p}{\mu^4}\right)(i\vec{\tau}^{(i)}\times\vec{\tau}^{(j)}.\vec{\tau}^{(k)})(\vec{\sigma}^{(i)}.\vec{\nabla}_{ki})(\vec{\sigma}^{(j)}.\vec{\nabla}_{jk})(i\vec{\sigma}^{(k)}.\vec{\nabla}_{ki}\times\vec{\nabla}_{jk})U_0(r_{ki})U_0(r_{jk})$$

$$+ \text{C.P.}, \tag{2.1}$$

where the subscripts s and p indicate partial waves in the intermediate pion-nucleon amplitude, c.p. indicates cyclic permutations of the indices $(i,j,k) = (1,2,3)$, and $U_0(r)$ is a Yukawa function modified by the πNN form factor

$$U_0(r) = \frac{4\pi}{\mu}\int \frac{d\vec{k}}{(2\pi)^3}\frac{e^{-i\vec{k}.\vec{r}}}{\vec{k}^2+\mu^2}\left(\frac{\Lambda^2-\mu^2}{\Lambda^2+\vec{k}^2}\right)^2 = \frac{e^{-\mu r}}{\mu r} - \frac{\Lambda}{\mu}\frac{e^{-\Lambda r}}{\Lambda r}$$

$$- \frac{1}{2}\frac{\mu}{\Lambda}\left(\frac{\Lambda^2}{\mu^2} - 1\right)e^{-\Lambda r} \quad . \tag{2.2}$$

(In reference (8), U_0 has no zero subscript).

The coefficients C_s, C_p and C'_p are known[2,4] in terms of the πN scattering amplitude and their numerical values are C_s = 0.92 MeV, C_p =-1.99 MeV, C'_p =-0.67 MeV.

The terms proportional to C_p and C'_p are the same as those of reference 2. The term with coefficient C_s, on the other hand, has to be modified by inserting the operator

$$\left[1 + \frac{1}{\mu^2}(\vec{\nabla}^2_{ki} - \mu^2) + \frac{1}{\mu^2}(\vec{\nabla}^2_{jk} - \mu^2)\right]$$

just before $U_0(r_{ki})U_0(r_{jk})$. This modification is needed, as pointed out in ref. 10, in order to make the a-contribution to the intermediate πN scattering amplitude compatible with the Adler consistency condition[12].

After having introduced the factor $(\vec{\nabla}^2-\mu^2)$ into the potential W, it has to be redefined before being used in realistic calculations. An interesting characteristic of this redefinitions of W is that it prescribes the exclusion of factors $(\vec{\nabla}^2-\mu^2)$ [6-8].

Our final form for the $\pi\pi$E-3NP is obtained by eliminating all contact interactions and their derivatives from the potential obtained by means of chiral symmetry[7]. The motivation for this procedure can be summarized as follows. In the absence of form factors, the potential contains δ-functions, arising from the derivatives of the Yukawa functions, and describing a contact interaction between two of the nucleons. Form factors, on the other hand, are related to the distribution of hadronic matter within the nucleons. Hence, when they are introduced into the potential, the δ-functions turn into terms that describe *contact* interactions between extended nucleons. These terms cannot be associated with the propagation of pions and do not correspond to a $\pi\pi$E-3NP. Instead, they could be called a π?E-3NP, since the parameter of the form factor does not let us know the type of particles being exchanged.

Hence, following the above prescriptions, the redefined version of the potential, indicated by a caret on W, is given by

$$
\begin{aligned}
\hat{W}(k) = {} & C_s(\vec{\tau}^{(i)}.\vec{\tau}^{(j)})(\vec{\sigma}^{(i)}.\hat{r}_{ki})(\vec{\sigma}^{(j)}.\hat{r}_{jk})U_1(r_{ki})U_1(r_{jk}) + \frac{C_p}{9}(\vec{\tau}^{(i)}.\vec{\tau}^{(j)}) \\
& \times\{\vec{\sigma}^{(i)}.\vec{\sigma}^{(j)}U_0(r_{ki})U_0(r_{jk}) + [3(\vec{\sigma}^{(i)}.\hat{r}_{jk})(\vec{\sigma}^{(j)}.\hat{r}_{jk})-\vec{\sigma}^{(i)}.\vec{\sigma}^{(j)}]U_0(r_{ki})U_2(r_{jk}) \\
& +[3(\vec{\sigma}^{(i)}.\hat{r}_{ki})(\vec{\sigma}^{(j)}.\hat{r}_{ki})-\vec{\sigma}^{(i)}.\vec{\sigma}^{(j)}]U_2(r_{ki})U_0(r_{jk}) \\
& +[9\cos\theta_k(\vec{\sigma}^{(i)}.\hat{r}_{ki})(\vec{\sigma}^{(j)}.\hat{r}_{jk})-3(\vec{\sigma}^{(i)}.\hat{r}_{jk})(\vec{\sigma}^{(j)}.\hat{r}_{jk})-3(\vec{\sigma}^{(i)}.\hat{r}_{ki})(\vec{\sigma}^{(j)}.\hat{r}_{ki}) \\
& +\vec{\sigma}^{(i)}.\vec{\sigma}^{(j)}]U_2(r_{ki})U_2(r_{jk})\} \\
& -\frac{C'_p}{9}(\vec{\tau}^{(i)}\times\vec{\tau}^{(j)}.\vec{\tau}^{(k)})\{\vec{\sigma}^{(i)}\times\vec{\sigma}^{(j)}.\vec{\sigma}^{(k)}U_0(r_{ki})U_0(r_{jk}) \\
& +[3(\vec{\sigma}^{(k)}\times\vec{\sigma}^{(i)}.\hat{r}_{jk})(\vec{\sigma}^{(j)}.\hat{r}_{jk})-\vec{\sigma}^{(i)}\times\vec{\sigma}^{(j)}.\vec{\sigma}^{(k)}]U_2(r_{ki})U_0(r_{jk})
\end{aligned}
$$

$$+\ \left[3(\vec{\sigma}^{(j)}\times\vec{\sigma}^{(k)}\cdot\hat{r}_{ki})(\vec{\sigma}^{(i)}\cdot\hat{r}_{ki})-\vec{\sigma}^{(i)}\times\vec{\sigma}^{(j)}\cdot\vec{\sigma}^{(k)}\right]U_2(r_{ki})U_0(r_{jk})$$

$$+\ \left[9\vec{\sigma}^{(k)}\cdot(\hat{r}_{ki}\times\hat{r}_{jk})(\vec{\sigma}^{(i)}\cdot\hat{r}_{ki})(\vec{\sigma}^{(j)}\cdot\hat{r}_{jk})-3(\vec{\sigma}^{(k)}\times\vec{\sigma}^{(i)}\cdot\hat{r}_{jk})(\vec{\sigma}^{(j)}\cdot\hat{r}_{jk})\right.$$

$$\left.-\ 3(\vec{\sigma}^{(j)}\times\vec{\sigma}^{(k)}\cdot\hat{r}_{ki})(\vec{\sigma}^{(i)}\cdot\hat{r}_{ki})+\vec{\sigma}^{(i)}\times\vec{\sigma}^{(j)}\cdot\vec{\sigma}^{(k)}\right]U_2(r_{ki})U_2(r_{jk})\},\qquad(2.3)$$

where U_0 is given by eq. (2.2) and

$$\cos\theta_k = \frac{\vec{r}_{ki}\cdot\vec{r}_{jk}}{r_{ki}r_{jk}}\ ,\qquad(2.4)$$

$$U_1(x) = -\frac{e^{-\mu x}}{\mu x}\left[1+\frac{1}{\mu x}\right]+\frac{\Lambda^2}{\mu^2}\left[1+\frac{1}{\Lambda x}\right]\frac{e^{-\Lambda x}}{\Lambda x}+\frac{1}{2}\left[\frac{\Lambda^2}{\mu^2}-1\right]e^{-\Lambda x},$$

$$(2.5)$$

and

$$U_2(x) = \frac{e^{-\mu x}}{\mu x}\left[1+\frac{3}{\mu x}+\frac{3}{\mu^2x^2}\right]-\frac{\Lambda^3}{\mu^3}\frac{e^{-\Lambda x}}{\Lambda x}\left[1+\frac{3}{\Lambda x}+\frac{3}{\Lambda^2x^2}\right]$$

$$-\ \frac{1}{2}\frac{\Lambda}{\mu}\left[\frac{\Lambda^2}{\mu^2}-1\right]e^{-\Lambda x}\left[1+\frac{1}{\Lambda x}\right]\ .\qquad(2.6)$$

The expression for $\hat{W}(k)$ is the same as that of eq.(67) of ref.2, except for an unfortunate misprint in the sign of the term proportional to C'_p in that equation.

3. TENSOR OPERATOR FORMS OF M E THREE-NUCLEON FORCE

In order to obtain tensor forrns of the three-nucleon force we take eq.(2.3) as the starting point. A very convenient form of eq. (2.3) is obtained by recombining their different terms. We obtain, after some straightforward algebra, a tensor form of *eq.* (2.3) :

$$W_s(k) = \frac{C_s}{3}\vec{\tau}^{(i)}\cdot\vec{\tau}^{(j)} U_1(r_{ki}) i\sim(r_{jk}) \sim\ \sim\theta_k q(^{i})s\ .\$(i) + S_{ij}\ ?(\sim\ \bar{\sim}\ \bar{\sim},\ r\ \sim\ \sim)\ ]\ ,$$

$$(3.1)$$

$$\hat{W}_p(k) = \frac{C_p}{9}\,\vec{\tau}^{(i)}.\vec{\tau}^{(j)}\{[U_0(r_{ki})U_0(r_{jk}) + 2P_2(\cos\theta_k)U_2(r_{ki})U_2(r_{jk})]\vec{\sigma}^{(i)}.\vec{\sigma}^{(j)}$$

$$+ [U_0(r_{ki}) - U_2(r_{ki})]U_2(r_{jk})S_{ij}(\hat{r}_{jk},\hat{r}_{jk})$$

$$+ U_2(r_{ki})[U_0(r_{jk}) - U_2(r_{jk})]\,S_{ij}(\hat{r}_{ki},\hat{r}_{ki})$$

$$+ 3\cos\theta_k U_2(r_{ki})U_2(r_{jk})S_{ij}(\hat{r}_{ki},\hat{r}_{jk})\}\ , \qquad (3.2)$$

$$\hat{W}'_p(k) = \frac{C'_p}{9}\,(i\vec{\tau}^{(k)}.\vec{\tau}^{(i)}\times\vec{\tau}^{(j)})\{[U_0(r_{ki})U_0(r_{jk})i\vec{\sigma}^{(k)}.\vec{\sigma}^{(i)}\times\vec{\sigma}^{(j)}]$$

$$+ U_0(r_{ki})U_2(r_{jk})\,\frac{1}{2}[S_{ij}(\hat{r}_{jk},\hat{r}_{jk})\vec{\sigma}^{(k)}.\vec{\sigma}^{(i)} - \vec{\sigma}^{(k)}.\vec{\sigma}^{(i)})S_{ij}(\hat{r}_{jk},\hat{r}_{jk})]$$

$$+ U_2(r_{ki})U_0(r_{jk})\,\frac{1}{2}\,[-S_{ij}(\hat{r}_{ki},\hat{r}_{ki})\vec{\sigma}^{(j)}.\vec{\sigma}^{(k)} + \vec{\sigma}^{(j)}.\vec{\sigma}^{(k)}S_{ij}(\hat{r}_{ki},\hat{r}_{ki})]$$

$$+U_2(r_{ki})U_2(r_{jk})\,\frac{1}{2}[S_{ki}(\hat{r}_{ki},\hat{r}_{ki})S_{jk}(\hat{r}_{jk},\hat{r}_{jk})-S_{jk}(\hat{r}_{jk},\hat{r}_{jk})S_{ki}(\hat{r}_{ki},\hat{r}_{ki})]\ , \qquad (3.3)$$

where the tensor S_{ij} for two unit vectors $\hat{u}$ and $\hat{v}$ is defined as

$$S_{ij}(\hat{u},\hat{v}) = 3(\vec{\sigma}^{(i)}.\hat{u})(\vec{\sigma}^{(j)}.\hat{v})-(\hat{u}.\hat{v})(\vec{\sigma}^{(i)}.\vec{\sigma}^{(j)}).$$

The above equations are not written in an ITO format. We should also notice that spins are mixed with terms in configuration space. Our final objective is to write eq. (2.3) in the ITO form in such a way that spin-isospin terms are not mixed with those of configuration space. That will make the calculation of the matrix elements of $\hat{W}$ much simpler and direct; only terms allowed by the triangle selection rule of the tensor ranks of the configuration space operators involved will be picked up.

In eq. (2.3), terms appear which are of the types

$$(\vec{\sigma}^{(i)}.\hat{r}_{ki})(\vec{\sigma}^{(j)}.\hat{r}_{jk})\quad ,\quad (\vec{\sigma}^{(i)}.\hat{r}_{ki})(\vec{\sigma}^{(j)}.\hat{r}_{ki})$$

and

$$\vec{\sigma}^{(k)}.(\hat{r}_{ki}\ \hat{r}_{jk})(\vec{\sigma}^{(i)}.\hat{r}_{ki})(\vec{\sigma}^{(j)}.\hat{r}_{jk})\ .$$

Using eq. (A.5) the first two equations above can be written as

$$(\vec{\sigma}^{(i)}.\hat{r}_{ki})(\vec{\sigma}^{(j)}.\hat{r}_{jk}) = T_0(\vec{\sigma}^{(i)},\vec{\sigma}^{(j)})T_0(\hat{r}_{ki},\hat{r}_{jk})-T_1(\vec{\sigma}^{(i)},\vec{\sigma}^{(j)})$$
$$\times T_1(\hat{r}_{ki},\hat{r}_{jk})+T_2(\vec{\sigma}^{(i)},\vec{\sigma}^{(j)})T_2(\hat{r}_{ki},\hat{r}_{jk}) = \sum_{\ell=0}^{2}(-)^{\ell}T_{\ell}(\vec{\sigma}^{(i)},\vec{\sigma}^{(j)}).T_{\ell}(\hat{r}_{ki},\hat{r}_{jk}) \tag{3.4}$$

and

$$(\vec{\sigma}^{(i)}\ _{ki})(\vec{\sigma}^{(j)}.\hat{r}_{ki}) = \frac{-1}{\sqrt{3}}T_0(\vec{\sigma}^{(i)},\vec{\sigma}^{(j)}) + T_2(\vec{\sigma}^{(i)},\vec{\sigma}^{(j)})T_2(\hat{r}_{ki},\hat{r}_{ki}) \ . \tag{3.5}$$

The third term can also be written as

$$\vec{\sigma}^{(k)}.(\hat{r}_{ki}\ \hat{r}_{jk})(\vec{\sigma}^{(i)}.\hat{r}_{ki})(\vec{\sigma}^{(j)}.\hat{r}_{jk}) = i\sqrt{6}$$
$$T_0(T_1(\sigma^{k}),T_1(\hat{r}_{ki},\hat{r}_{jk}))\ \sum_{\ell=0}^{2}(-)^{\ell}\ T_{\ell}(\vec{\sigma}^{(i)},\vec{\sigma}^{(j)})T_{\ell}(\hat{r}_{ki},\hat{r}_{jk}) =$$
$$= i\sqrt{6}\sum_{\ell=0}^{2}\sqrt{2\ell+1}\ T_0(T_1(\vec{\sigma}^{(k)}),T_1(\hat{r}_{ki},\hat{r}_{jk}))\ .\ T_0(T_{\ell}(\vec{\sigma}^{(i)},\vec{\sigma}^{(j)}),T_{\ell}(\hat{r}_{ki},\hat{r}_{jk})) \tag{3.6}$$

where we have used eq. *(A.3)*.

Hence we can obtain

$$(\vec{\sigma}^{(k)}.\hat{r}_{ki}\ \hat{r}_{jk})(\vec{\sigma}^{(i)}.\hat{r}_{ki})(\vec{\sigma}^{(j)}.\hat{r}_{jk}) = i\sqrt{2}\ \sum_{\ell=0}^{2}\ \sum_{\ell'=|1-\ell|}^{1+\ell}(-)^{\ell'}$$
$$\times\ T_{\ell'}(T_1(\vec{\sigma}^{(k)}),T_{\ell}(\vec{\sigma}^{(i)},\vec{\sigma}^{(j)})).T_{\ell'}(T_1(\hat{r}_{ki},\hat{r}_{jk}),T_{\ell}(\hat{r}_{ki},\hat{r}_{jk}))\ . \tag{3.7}$$

With eqs. *(3.4)*-*(3.7)* in mind we can obtain the ITO form of $\hat{W}(k)$

$$\hat{W}(k) = \hat{W}_s(k) + \hat{W}_p(k) + \hat{W}'_p(k)\ , \tag{3.8}$$

where

$$\hat{W}_s(k) = -\sqrt{3}\, C_s T_0(\vec{\tau}^{(i)},\vec{\tau}^{(j)})\,[T_0(\vec{\sigma}^{(i)},\vec{\sigma}^{(j)}).T_0(\hat{r}_{ki},\hat{r}_{jk}) -$$

$$- T_1(\vec{\sigma}^{(i)},\vec{\sigma}^{(j)})T_1(\hat{r}_{ki},\hat{r}_{jk})+T_2(\vec{\sigma}^{(i)},\vec{\sigma}^{(j)})T_2(\hat{r}_{ki},\hat{r}_{jk})]U_{11}(ji), \qquad (3.9)$$

$$\hat{W}_p(k) = \frac{C_p}{3} T_0(\vec{\tau}^{(i)},\vec{\tau}^{(j)})\{T_0(\vec{\sigma}^{(i)},\vec{\sigma}^{(j)})[U_{00}(ji)+(9T_0(\hat{r}_{ki},\hat{r}_{jk})$$

$$\times T_0(\hat{r}_{ki},\hat{r}_{jk})-1)U_{22}(ji)] + \sqrt{3}T_2(\vec{\sigma}^{(i)},\vec{\sigma}^{(j)})[T_2(\hat{r}_{ki},\hat{r}_{ki})$$

$$\times(U_{22}(ji)-U_{02}(ji)) + T_2(\hat{r}_{jk},\hat{r}_{jk})(U_{22}(ji)-U_{20}(ji)) +$$

$$+ 3\sqrt{3}\ T_0(\hat{r}_{ki},\hat{r}_{jk})T_2(\hat{r}_{ki},\hat{r}_{jk})U_{22}(ji)]-9T_0(\hat{r}_{ki},\hat{r}_{jk})$$

$$\times T_1(\vec{\sigma}^{(i)},\vec{\sigma}^{(j)})T_1(\hat{r}_{ki},\hat{r}_{jk})U_{22}(ji)\}\ , \qquad (3.10)$$

and finally

$$\hat{W}'_p(k) = \frac{2C'_p}{3}\ T_0(T_1(\vec{\tau}^{(i)},\vec{\tau}^{(j)}),\vec{\tau}^{(k)})\{T_0(T_1(\vec{\sigma}^{(i)},\vec{\sigma}^{(j)}),\vec{\sigma}^{(k)})$$

$$\times[U_{00}(ji)-U_{22}(ji)] + \sqrt{3}\ T_2(T_1(\vec{\sigma}^{(k)},\vec{\sigma}^{(i)}),\vec{\sigma}^{(j)}).T_2(\hat{r}_{jk},\hat{r}_{jk})$$

$$\times[U_{22}(ji)-U_{20}(ji)] + \sqrt{3}\ T_2(T_1(\vec{\sigma}^{(k)},\vec{\sigma}^{(i)}),\vec{\sigma}^{(j)}).T_2(\hat{r}_{ki},\hat{r}_{ki})$$

$$\times[U_{22}(ji)-U_{00}(ji)] + 3\sqrt{3}[-T_1(T_0(\sigma \qquad ,\vec{\sigma}^{(k)})\ T_1(T_0(\hat{r}_{ki},\hat{r}_{jk}),T_1(\hat{r}_{ki},\hat{r}_{jk}))$$

$$+ T_0(T_1(\vec{\sigma}^{(i)},\vec{\sigma}^{(j)}),\vec{\sigma}^{(k)}).T_0(T_1(\hat{r}_{ki},\hat{r}_{jk}),T_1(\hat{r}_{ki},\hat{r}_{jk}))$$

$$- T_1(T_1(\vec{\sigma}^{(i)},\vec{\sigma}^{(j)}),\vec{\sigma}^{(k)}).T_1(T_1(\hat{r}_{ki},\hat{r}_{jk}),T_1(\hat{r}_{ki},\hat{r}_{jk}))$$

$$+ T_2(T_1(\vec{\sigma}^{(i)},\vec{\sigma}^{(j)}),\vec{\sigma}^{(k)}).T_2(T_1(\hat{r}_{ki},\hat{r}_{jk}),T_1(\hat{r}_{ki},\hat{r}_{jk}))$$

$$- T_1(T_2(\sigma \qquad ,\vec{\sigma}^{(j)}),\vec{\sigma}^{(k)}).T_1(T_2(\hat{r}_{ki},\hat{r}_{jk}),T_1(\hat{r}_{ki},\hat{r}_{jk})$$

$$+ T_2(T_2(\vec{\sigma}^{(i)},\vec{\sigma}^{(j)}),\vec{\sigma}^{(k)}).T_2(T_2(\hat{r}_{ki},\hat{r}_{jk}),T_1(\hat{r}_{ki},\hat{r}_{jk}))$$

$$- T_3(T_2(\vec{\sigma}^{(i)},\vec{\sigma}^{(j)}),\vec{\sigma}^{(k)}).T_3(T_2(r_{ki},r_{jk}),T_1(r_{ki},r_{jk}))]U_{22}(ji). \qquad (3.11)$$

Notice that

$$U_{\alpha\beta}(ji) \equiv U_{\alpha}(r_{ki})U_{\beta}(r_{jk}) \quad .$$

(Each single latin index (i,j,k) in the argument of the left-hand side equation $U_{\alpha\beta}$ means one missing index in the corresponding subscripts of r in the right-hand side equations; e.g. $i \leftrightarrow jk$, $j \leftrightarrow ki$).

We can see from the above equations that $\hat{W}$ is now written in the ITO form (besides having the configuration space operators factored out from the spin-isospin part).

4. CONCLUSIONS

We have presented here a summary of arguments concerning the derivation of a single expression of ππE-3NP, since originally two sligthly different forms appeared in the literature, known as the São Paulo-Recife[2] (also known as the Brazilian potential) and the Tucson--Melbourne[3] potentials. The Brazilian potential used an effective Lagrangian approach while the Tucson-Melbourne potential used current algebra, both based on chiral symmetry. It is shown that both approaches produce essentially the same results. As far as the case of extended nucleons is concerned, we have argued that the expressions derived by the above groups[2,3] need to be modified in order to be used in realistic calculations. The problem with these potentials is that they become strongly distorted when form factors are present. These distorsions are incompatible with our expectation that form factors should modify the potential for point-like nucleons at short distances. This odd behaviour is associated with terms in the potential that can be interpreted as *contact* interactions between extended nucleons. We have argued that the elimination of those contact terms from the potential makes it much better behaved. Therefore we have redefined the ππE-3NP as the potential associated only with the propagation of pions described by an Yukawa function regularized at the origin. This redefined version of the potential is given by eq.(2.3).

We would like to point out that $\hat{W}$, the redefined potential, is not the outcome of indisputable formal derivations. Instead, it is closely associated with a particular way of interpreting a physical picture allowed by the mathematical formulation.

In this paper we have also given two tensor forms for the redefined potential, $\hat{W}$. The first one is given by eqs. (3.1) - (3.3) and the second one, eqs. (3.8)-(3.11), is written in the ITO form in such a way that the spin-isospin terms are not mixed with those of configuration space. Both versions are very useful in practical calculations. The ITO version, in particular, is extremely suitable f6r the hyperspherical harmonic (HH) approach[9,20]. Thus in the evaluation of matrix elernents in the HH approach we can apply the Wigner-Eckart[17] theorern in a straightforward way.

The separation of $\hat{W}$ in different tensor ranks, helps to compare the relative importance of ππE-3NP with the corresponding two-nucleon interaction, V. In a separate paper[8] we have studied it extensively.

APPENDIX A: USEFUL EXPRESSIONS FOR ITO

The tensor direct product of two spherical tensor operators T_{k_1} and T_{k_2} (forming an ITO of rank k) is defined[17] as

$$T_k(T_{k_1},T_{k_2}) = (T_{k_1} \otimes T_{k_2})^q_k = \sum_{q_1 q_2} <k_1 k_2 q_1 q_2|kq> T^{q_1}_{k_1} T^{q_2}_{k_2} \quad . \tag{A.1}$$

The scalar product or contraction of two commuting tensor operators is defined as

$$T_k . U_k = \sum_{q=-k}^{k} (-)^q \; T^{-q}_k \; U^q_k \quad , \tag{A.2}$$

where one can easily see that

$$T_0(T_k,U_k) = (T_k \otimes U_k)_0 = \frac{(-)^k}{\sqrt{2k+1}} T_k . U_k \quad . \tag{A.3}$$

In a similar way one can show[18] that

$$T_1(\vec{a},\vec{b}) = (a \otimes b) = \frac{i}{\sqrt{2}} \; \vec{a}\times\vec{b} \quad . \tag{A.4}$$

Also very useful are the expressions[19]:

$$((T_{k_1} \otimes T_{k_2})_k \cdot (T_{k_3} \otimes T_{k_4})_k)_0 = (2k+1)(-)^{k_1+k_4}$$

$$\times \sum_{k'} (-)^{k_1+k_2+k_3+k_4} \begin{Bmatrix} k_1 & k_2 & k \\ k_4 & k_3 & k' \end{Bmatrix} ((T_{k_1} \otimes T_{k_3})_{k'} \cdot (T_{k_2} \otimes T_{k_4})_{k'})_0 \qquad (A.5)$$

where $[T_{k_2}, T_{k_3}] = 0$, and

$$(\vec{\sigma}^{(k)} \otimes \vec{a}) . ((\vec{\sigma}^{(i)} \otimes \vec{\sigma}^{(j)})) \otimes (\vec{b} \otimes \vec{c}) = \frac{1}{\sqrt{3(2\ell+1)}} \sum_{\ell'=|1-\ell|}^{1+\ell} (-)^{\ell'}$$

$$(\vec{\sigma}^{(k)} \otimes (\vec{\sigma}^{(i)} \otimes \vec{\sigma}^{j}))_\ell)_{\ell'} \; (\vec{a} \otimes (\vec{b} \otimes \vec{c})_\ell)_{\ell'} \; , \qquad (A.6)$$

where $\vec{a}$, $\vec{b}$ and $\vec{c}$ are vectors.

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Resumo

Apresentamos um resumo dos resultados referentes aos potenciais de três nucleons (ππE-3NP), obtidos através da simetria chiral. Estes resultados colocam dentro de um esquema Único os resultados anteriormente obtidos, evitando confusão para os leitores. Também derivamos formas tensoriais de ππ-3NP, especialmente uma representação tensorial irredutível das forças de 3 corpos, de modo tal que termos de spin-isospin não são misturados com os do espaço de configuraçao. Esta forma facilita os cálculos e dá destaque à física subjacente.