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A Test of the Equivalence Principle with Polarized Light

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Abstract The behaviour of electromagnetic radiation in the Schwarzschild metric of the sun is studied with the presence of a non-minimal coupling term of electromagnetism and gravity. An anomalous deflection is predicted, as well as a polarization effect. These effects can be used to obtain experimental upper bounds for the coupling constant of the non-minimal coupling. The strength of these upper bounds is discussed.

1. INTRODUCTION

A. R. Prasanna[1] suggested a non-minimal coupling of electromagnestism and gravity by writing the total Lagrangian

$$L = \sqrt{-g}\ \{R + \alpha F_{ab}F^{ab} + \beta R_{abcd}F^{ab}F^{cd}\} \qquad (1.1)$$

which yields the field equations

$$\partial_3\left[\alpha\sqrt{-g}\ F^{ij} + \beta\sqrt{-g}\ R^{pqij}F_{pq}\right] = 0 \qquad (1.2)$$

and

$$R_{ij} - \frac{1}{2} g_{ij} R = -2\alpha\left[F_{ia}F_j^{\ a} - \frac{1}{4} g_{ij}F_{ab}F^{ab}\right] + \beta\left[2\, g_{ij}R_{amsq}F^{am}F^{sq} - 3R_{(i}^{\ \ msq}F_{j)m}F_{sq} + 2\left(F^m{}_{(i}F_{j)}{}^{s}\right)_{;sm}\right] \qquad (1.3)$$

The constant a is related to Einstein's constant

$$\alpha = -\frac{1}{2}\kappa = -1.04.10^{-43}\ \mathrm{Kg}^{-1}\mathrm{m}^{-1}\mathrm{s}^2$$

(or in natural units with $h = c = 1$, a = $-3.3.10^{-69}$ m^2), and β is a new coupling constant with dimension m^4 (in natural units). As β is a dimensional quantity, whose dimension is different from that of a, there is no *a-priori* criterion to estimate its value. The only natural value would be $\beta=0$, invoking the *equivalence principle*. However, the equivalence principle is a physical one and hence it has to be checked by experiments. The main value of a model like the one given by eq. (1.1) is to provide us with specific experimental tests of the equivalence principle. The non-minimal coupling term in eq. (1.1) represents an interesting deviation from the equivalence principle because it respects the geometrical nature of gravity and the gauge symmetry of electromagnetism. Moreover, the classical experiments testing the equivalence principle such as the Eötvös experiment are performed in space time regions of small curvature and they are not specific for couplings of this kind.

For other possible couplings[4,5] proportional to the Ricci tensor R_{ab} or the scalar curvature R there is almost no hope to make significant experiments, because such couplings cause effects only inside the matter distribution that produces the curvature; thus it would be extremely difficult to distinguish between gravitational effects and other interactions.

Although the curvatures' available in experiments are quite moderate, there exist effects that permit interesting limitations on β. In this paper we study the behaviour of electromagnetic radiation from some distant stelar object in the Schwarzschild rnetric of the sun for $\beta \neq 0$, of course neglecting the right-hand side of eq. (1.3). Our aim is to find some measurable effect suitable to give an experimental upper bound for $|\beta|$. In section 3 two such effects are described and in section 4 the corresponding upper bounds are discussed in terms of the right-hand side of eq. (1.3).

The upper bounds that we will obtain will be much bigger than what might be expected theoretically, namely the 4^{th} power of Planck's length $\sim 10^{-137}$ m^4. However, this expectation would be based on a theoretical prejudice concerning length scales, which is essentially the same as accepting the equivalence principle right away.

2. GEOMETRICAL OPTICS WITH $\beta \neq 0$

Usually propagation of light in a weak gravitational background field is discussed in terms of light rays, rather than in terms of wave solutions of Maxwell's equations. However, with $\beta\neq 0$ one has to handle the transition from wave optics to geometrical optics with some care. For $\beta=0$, this transition is usually made by writing for the potential

$$A_\ell(x) = \hat{A}_\ell(x)e^{i\omega S(x)} \tag{2.1}$$

where $\hat{A}_\ell$ may be complex and S is real (see for instance ref.2). One then equates to zero separately the factors of ω^2 and ω in the equation

$$(\sqrt{-g}\, g^{ma}\, g^{nb}(A_{b,a} - A_{a,b}))_{,n} = 0 \tag{2.2}$$

neglecting terms of order w. Looking for a solution such that F_{ab} is of order w one concludes from the ω^2 terms that

$$S^{,b}\hat{A}_b = 0 \tag{2.3}$$

and

$$S^{,b}S_{,b} = 0 \tag{2.4}$$

Eq. (2.4) together with $S_{a;b} = S_{b;a}$ gives $S^{,b}S_{a;b} = 0$. Thus eq. (2.4) tells us that light rays are null-geodesics.

For $\beta\neq 0$, wièh eq. (1.2) instead of eq. (2.2), this would lead to

$$(S^{,b}\hat{A}_b)S^{,a} - (S^{,b}S_{,b})\hat{A}^a + 2\,\frac{\beta}{\alpha}\,R^{pqab}S_{,b}\hat{A}_q S_{,p} = 0 \tag{2.5}$$

One expects that S and $\hat{A}$ can be expanded in a Taylor series in β around $\beta=0$

$$S = \overset{0}{S} + \beta\overset{1}{S} + \ldots$$

$$\hat{A}_\ell = \overset{0}{A}_\ell + \beta\overset{1}{A}_\ell + \ldots$$

Then $\overset{0}{S}$ and $\overset{0}{A}_\ell$ would still fulfill eqs. (2.3), (2.4) and for $\overset{1}{A}_\ell$ and $\overset{1}{S}$ one gets from eq. (2.5)

$$(\overset{1}{S}{}^{,b}\overset{0}{A}_b + \overset{0}{S}{}^{,b}\overset{1}{A}_b)\overset{0}{S}{}^{,a} - 2(\overset{0}{S}{}^{,b}\overset{1}{S}_{,b})\overset{0}{A}{}^a + \frac{2}{\alpha}\,R^{pqab}\overset{0}{S}_{,b}\overset{0}{A}_q\overset{0}{S}_{,p} = 0 \tag{2.6}$$

The vector involving the curvature tensor will in general not be in the plane of $\overset{0}{S}{}^{,a}$ and $\overset{0}{A}{}^{a}$. Therefore eq. (2.6) will in general not have a solution. Physically this means that the interaction will generate a second wave, with different polarization, which cannot be described by the ansatz (2.1).

Despite this failure one can still talk of single light rays, if the geometry of the experiment is chosen appropriately. In our case this geometry is determined by the incident wave, the sun, and the observer. The ansatz of a single locally plane wave will still be successful for special polarizations of the incoming wave relative to the plane defined by the wave vector of incident radiation and the radius between sun and observer. We therefore restrict ourselves to consider eq. (2.5) only in one plane and we look for polarizations relative to this plane so that eqs. (2.5) and (2.6) have a solution.

3. ANOMALOUS DEFLECTION AND POLARIZATION EFFECTS

Let t, r, θ, ϕ be the usual Schwarzschild coordinates, in which the metric takes the diagonal form

$$g_{tt} = -\left(1-\frac{2M}{r}\right), \quad g_{rr} = \left(1-\frac{2M}{r}\right)^{-1}, \quad g_{\theta\theta} = r^2, \quad g_{\phi\phi} = r^2 \sin^2\theta \tag{3.1}$$

where $2M$ is the Schwarzschild radius of the sun. We shall consider a solution of eq. (2.5) in the plane $\phi=0$, in which we suppose the observer to be located. We can make sure that the third term of eq. (2.6) involving R^{pqab} is in the plane formed by $\overset{0}{A}{}^{a}$ and $\overset{0}{S}{}^{,a}$ by choosing a polarization such that $\overset{0}{A}_q$ is an eigenvector of the matrix $R^{pq}{}_a{}^b \overset{0}{S}_{,b} \overset{0}{S}_{,p}$. This is the case if $\overset{0}{A}_q$ is either in the plane $\phi=0$ or orthogonal to that plane. We shall call the corresponding polarizations of the incoming wave radial (R) and tangential (T) respectively. Supposing we have one of these polarizations, we shall now extract information about the phase from eq. (2.6). Multiplying eq. (2.6) with $\overset{0}{A}_a$ and writing $|A|^2$ for $\overset{0}{A}{}^{a} \overset{0}{A}_a$ we get

$$\overset{1}{S}_{,b} \overset{0}{S}{}^{,b} = \frac{1}{|A|^2}\frac{1}{\alpha} R^{pqab} \overset{0}{S}_{,b} \overset{0}{A}_q \overset{0}{S}_{,p} \overset{0}{A}_a \tag{3.2}$$

This equation can be used to get an expression for the integral of $\overset{1}{S}_{,b}$ along the trajectory of the unperturbed light ray

$$\int_{odr} \overset{1}{S}_{,b}\, dx^{b} = -\int_{odr} \overset{1}{S}_{,b}\, \overset{0}{S}{}^{,b}\, dt = -\int_{odr} \frac{1}{\alpha|A|^{2}} R^{pqab}\, \overset{0}{S}_{,b}\, \overset{0}{A}_{q}\, \overset{0}{S}_{,p}\, \overset{0}{A}_{a}\, dt \tag{3.3}$$

where odr means ordinarily deflected ray. As the ordinarily deflected ray and the ray with $\beta\neq 0$ differ only by terms of order β we may replace the first order correction $\overset{1}{S}$ of the phase at a point on the ray with $\beta\neq 0$ by the integral (3.3). Moreover, as the ratio of the solar Schwarzschild radius $2M_{\odot}$ and the solar radius $R_{\odot}$ is about 4.10^{-6} we will discuss S only up to first order in M. The Riemann tensor R^{pqab} is already of order M. Then we can replace $\overset{0}{S}_{,b}$ and $\overset{0}{A}_{q}$ by their values for $M=0$ and we may also simply integrate over the straight line $t = -r\cos\theta$, $r\sin\theta = D = \text{const.}$, $\phi = 0$ which corresponds to an undeflected light ray coming from $z = r\cos\theta = +\infty$ and passing the sun at a distance D from its center. Evaluating eq. (3.3) we obtain

$$\overset{1}{S}_{R/T} = \pm\frac{M}{\alpha}\left\{\cos\theta\left[\frac{2}{r^{2}\sin^{2}\theta} + \frac{1}{r^{2}}\right] - \frac{2}{r^{2}\sin^{2}\theta}\right\} \tag{3.4}$$

where R/T specify the respective polarizations of the incoming wave. Combining this result with the known expression for the phase up to first order in M from ordinary minimal coupling, we get two single wave solutions

$$A_{\ell}^{R} = \hat{A}_{\ell}^{R}\, e^{i\omega(\overset{0}{S}+\beta\overset{1}{S}_{R})}$$
$$A_{\ell}^{T} = \hat{A}_{\ell}^{T}\, e^{i\omega(\overset{0}{S}+\beta\overset{1}{S}_{T})} \tag{3.5}$$

with

$$\overset{0}{S} = t + r\cos\theta + M\left\{2\log\frac{r(1+\cos\theta)}{r_{0}} - \cos\theta\right\}$$

and $\overset{1}{S}$ from eq. (3.4). At the position of the observer, which we suppose to be located at $z = r\cos\theta = -\infty$, with $D = r\sin\theta = \text{const.}$, one gets for the wave vectors of these waves

$$\vec{k}_{R/T} = -\vec{e}_z + \{\frac{4M}{D} \pm \frac{\beta}{\alpha}\frac{8M}{D^3}\}\vec{e}_\theta \qquad (3.6)$$

where $\vec{e}_z$ and $\vec{e}_\theta$ are unit vectors in the directions ∂_z and ∂_θ respectively, ∂_z referring to the coordinates $x = r\sin\theta\cos\phi$, $y = r\sin\theta\sin\phi$, $z = r\cos\theta$. Up to first order in M the parameter D is the minirnal distance between the observed light ray and the center of the sun. In order to avoid solar corona effects D should be chosen considerably larger than the solar radius, taking into account the frequency dependence of the corona effects. We will assume a representative value of $D = 2R_\odot$. For $z \to -\infty$ the unit vectors $\vec{e}_z$ and $\vec{e}_\theta$ become orthogonal. So eq. (3.6) tells us that there will be two deflections of light in direction $\vec{e}_\theta$, with deflection angles $\Delta\theta_{R/T}$ givenby

$$\tan \Delta\theta_{R/T} = \frac{4M}{D} \pm \frac{\beta}{\alpha}\frac{8M}{D^3} \qquad (3.7)$$

The first term $4M/D$ is the ordinary deflection predicted with theequivalence principle. The second term is an anornalous polarizationdependent deflection. For an incoming wave with arbitrary polarization, which will in general be some superposition of the polarizations R and T, $aR + bT$, the outgoing radiation will be the corresponding linear superposition of the two waves of eq. (3.5) : $aA^R + bA^T$. This means that in general – and especially for unpolarized incoming radiation – the incoming light ray gets split into two light rays. As the measurements of deflection are done with unpolarized light one should, for sufficiently big $|\beta|$, expect a doubling of the images of stars. No such effect has been observedwith light passing the sun at'a distance $D \approx 2R_\odot$. Therefore we can at least say that the anomalous deflection is smaller than the ordinary one

$$\left| 8\frac{\beta}{\alpha}\frac{M}{D^3} \right| < \frac{4M}{D} \qquad (3.8)$$

This gives an upper bound for $|\beta|$

$$|\beta| < \frac{1}{2}|\alpha| D^2 \approx 3.10^{-51}\ m^4 \qquad (3.9)$$

Now let us suppose that $|\beta|$ is very much smaller than this value, and suppose that u and the diameter d of the telescope are such that

$$8\,\left|\frac{\beta}{\alpha}\right|\,\frac{M}{D^3}\,\omega d \ll 1 \ .$$

This means that the phase shift due to the 8-coupling is practically constant over the aperture of the telescope. Then we may neglect anomalous defleetion effects and replace the function $\overset{1}{S}$ of eq. (3.4) by its value at the position of the observer

$$\overset{1}{S}_{R/T} = \mp \frac{4M}{\alpha D^2_{obs}} .$$

The two waves A_R and A_T would then travel approximately in the same direction, but with a phase difference of $2\omega\beta\overset{1}{S}$. If the incident light is polarized with a superposition of the polarizations R and T this phase difference causes a change of the polarization state. The corrections of amplitude $\overset{1}{A}_\ell$, which we have not calculated, can alço change the state of polarization. However this change is smaller by one order in ω and we may neglect it. For example, if the incident wave were linearly polarized with $\overset{0}{A}_\ell = \overset{0}{A}{}^R_\ell + \overset{0}{A}{}^T_\ell$, with $|\overset{0}{A}{}^R| = |\overset{0}{A}{}^T|$, the wave in the region of the observer would be eliptically polarized, with a ratio of small to big semi-axis

$$\frac{A_{min}}{A_{max}} = \tan\,|\omega\beta\overset{1}{S}| \quad .$$

Observing the polarization of radiation from pulsars or other sources of polarized radiation when their radiation passes near the sun, one should be able to detect a phase shift of the order of 1. There exist numerous sources of polarized radiation with frequencies ranging from some 1000 MHz up to visible light with degrees of polarization of a few percent up to 50% [3]. If no change of polarization occurs one can conclude that

$$|2\omega\beta\overset{1}{S}| < 1 \qquad (3.10)$$

This would lead to an upper bound for $|\beta|$

which is better than the upper bound (3.9) by a factor $(4\omega M)^{-1}$. For a β of this order of magnitude the necessary condition for neglecting the anomalous deflection is simply $d << D$, which is of course always fulfilled.

For a radiation of about 2000 MHz one has $\omega \approx 42\ m^{-1}$. With $M_{\odot} = 1.48.10^3$ m we get

$$|\beta| < |\alpha|\ D^2.2.10^{-6} \approx 10^{-56}\ m^4 \qquad (3.12)$$

For visible light we would gain another factor of 10^{-5} giving an upper bound $|\beta| < 10^{-61}$ m^4. In principle one could improve this upper bound arbitrarily by using shorter and shorter wave lengths.

4. CONCLUSIONS

We got two upper bounds for $|\beta|$: one is based on present experimental data and the other one may be obtained some day with appropriate experiments. How restrictive are these upper bounds? One can give an interpretation of these results in terms of the right-hand side of eq. (1.3), asking how big must the curvature be in order that the β-term in eq. (1.3) becomes comparable to the ordinary a-contribution. That is, what value of R_{ABCD} is needed to have a and βR_{ABCD} of the same order of magnitude, where A,B,C,D refer to some normalízed tetrad basis. With the inequaiities (3.9) and (3.11) we get $|R_{ABCD}| > 2/D^2$ for the first upper bound and $|R_{ABCD}| > 8\omega M_{\odot}/D^2$ for the second upper bound. On the other hand R_{ABCD} on the surface of the sun can be (depending on the component) $2M_{\odot}/R_{\odot}^3 = 8M_{\odot}/R_{\odot}D^2$. This means that on the surface of the sun, for β of eq. (3.9), the β-contribution to eq. (1.3) is at least 5 orders of magnitude smaller than the ordinary a-contribution; for the second upper bound it would be at 'least by 10 (or 15 for the upper bound from visible light) orders of magnitude smaller. So the upper bounds are not uninterestingly weak. On the other hand, they are not strong enough to banish the nonminimal coupling suggested by Prasanna into an uninteresting region of weakness. After all, thereexistcurvatuces much bigger than the one produced by the sun.

We have discussed only the deflection and polarization effects in the gravitational field of the sun. Considering other masses, which produce a Schwarmschild metric, we observe that the anomalousdeflection only depends on the effective mass density p $\equiv 3M/4\pi D^3$, whereas the polarination effect is proportional to $D.\rho$. If there were a $\beta \neq 0$ one could expect to find measurable anomalous deflection effects with objects of much higher density such as neutron stars or black holes. A signal for such an effect would be the doubling of images of stars, where the two images are linearly polarined with two different, mutually orthogonal, polarization angles.

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Resumo

O comportamento da radiação eletromagnéticada métrica de Schwarzschild do sol é estudado com a presença de um termo de acoplamento não-mínimo entre eletromagnetismo e gravitação. Uma deflexão anômala é predita, assim como um efeito de polarização. Estes efeitos podem ser usados para obter-se limites experimentais para a constante de acoplamento não-mínimo. Discute-se quão restritivos são estes limites.