Revista Brasileira de Física, Vol. 17, nº 1, 1987

Scalar Electrodynamics in Three Dimensions with Topological Mass Terms

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Recebido em 15 de agosto de 1986

Abstract. The interaction between a charged scalar field and a gauge field in a three-dimensional space-time is studied. The topological mass term (the Chern-Simons term) is added to the system and we investigate how this term, odd by P and τ symmetry, modified the corrections to the propagators and vertices of this theory. These corrections are obtained to order e^2 in perturbation theory. In the correction of the linear vertex a new type of term arises. Although this new term, which comes from the topological one, presents an abnormal parity, Ward's identity is still valid.

1. INTRODUCTION

In this work, we study the interaction between a charged scalar field with an abelian gauge field massive topologically in a three-dimensional space-time. It is known that it is possible to add to the free Yang-Mills Lagrangian density the topological mass term,

$$L_\mu = \frac{\mu}{4} \cdot \varepsilon^{\mu\nu\lambda} F_{\mu\nu} A_\lambda \quad ,$$

leaving the action

$$S = \int d^3x\, L$$

invariant. (The Lagrangian density, by a gauge transformation, changes by a total derivative[1]). This topological term implies mass for the photon and it is odd by P and τ symmetry.

The study of the interaction between this topologically massive gauge field and fermions was made by S. Deser *et al*[1]. They showed that in a three-dimensional space-time, the fermion mass term, $L_m = -m\bar{\psi}\psi$, odd by P and τ, and the Chern-Simons term belong together, i.e., when one of them is present the other can be induced by radiative correction. J.F. Schonfeld[2] has shown also that the system of three-dimensional gauge field theory with a topological mass term in presence of a static point source of electric charge "e" presents vortex configurations.

Because of these vesy interesting aspects of this gauge field theory, we present below the study of the interaction between this field and the charge scalar one. We also present, using radiative correction to order e^2, propagators and vertices of this theory and the importance of the gauge field mass in these corrections. Now we would like to poínt out that the study of the three-dimensional theory is motivated by the connection between this theory and a four-dimensional one at high temperature .

The rest of the paper is divided as follows: in section 2, we introduce the Lagrangian density of the system and cite some its classical and quantum properties at the three level. In section 3, we make radiative corrections at one loope to the propagators and vertices of this theory, and we show how these corrections are photon-mass-dependent. We show that in the linear gauge field vertex ($j_\mu A^\mu$) a new term arises from the one due to Chern-Simons, and kow this term behaves by P and τ transformations. In section 4 we present the summary and discussion.

2. THE SYSTEM AND SOME PROPERTIES

In the first part of this section we present the physical system and some of its properties at the classical level. The second part treats the quantum diagrams at tree-level only.

i - General Properties

Scalar Electrodynamics in the three-dimensional space-time is described by the following Lagrangian density

$$L(x) = L_S^0(x) + L_g^0(x) + L_I(x) \tag{2.1}$$

where

$$L_S^0(x) = (\partial_\mu\phi)(\partial^\mu\phi^*) - m^2\phi^*\phi \tag{2.2a}$$

$$L_g(x) = -\frac{1}{4} F_{\mu\nu}F^{\mu\nu} + \frac{\mu}{4} \varepsilon^{\mu\nu\lambda}F_{\mu\nu}A_\lambda \; , \; F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu \tag{2.2b}$$

$$L_I(x) = -ieA_\mu(\phi^*\partial^\mu\phi - \phi\partial^\mu\phi^*) + e^2A_\mu A^\mu \phi^*\phi - \frac{\lambda}{4}(\phi^*\phi)^2 \quad (2.2c)$$

The metric tensor is $g_{\mu\nu}$ = diag (1,-1,-1).

The coupling constants "e" and "λ" have dimensions $(\text{mass})^{1/2}$ and $(\text{mass})^1$ respectively.

Under a gauge transformation

$$\phi \to \phi' = e^{-i\Delta(x)}\phi \quad (2.3a)$$

$$A_\alpha \to A'_\alpha = A_\alpha + \frac{1}{e}\partial_\alpha\Delta \quad (2.3b)$$

the Lagrangian density changes by a total derivative

$$L(x) \to L'(x) = L(x) + \partial_\alpha\left(\frac{\mu}{4e}\varepsilon^{\alpha\mu\nu}F_{\mu\nu}\Delta\right) . \quad (2.3c)$$

By parity P and time-inversion τ transformations, the gauge and scalar field change

$$\begin{aligned} PA^0(t,\vec{r})P^{-1} &= A^0(t,\vec{r}\,') \\ PA^1(t,\vec{r})P^{-1} &= -A^1(t,\vec{r}\,') \\ PA^2(t,\vec{r})P^{-1} &= A^2(t,\vec{r}\,') \\ P\phi(t,\vec{r})P^{-1} &= \phi(t,\vec{r}\,') \\ \vec{r} = (x,y)\ , \ \vec{r}\,' &= (-x,y) \end{aligned} \quad (2.4)$$

and

$$\begin{aligned} \tau A^0(t,\vec{r})\tau^{-1} &= A^0(t',\vec{r}) \\ \tau\vec{A}(t,\vec{r})\tau^{-1} &= -\vec{A}(t',\vec{r}) \\ \tau\phi(t,\vec{r})\tau^{-1} &= \phi^*(t',\vec{r}) \\ t' &= -t \quad . \end{aligned} \quad (2.5)$$

We can see that except for the topological mass term(*)

$$L_\mu = \frac{\mu}{4} \varepsilon^{\mu\nu\lambda} F_{\mu\nu} A_\lambda$$

all terms in eq. (2.2a-c) are invariant by P and τ.

The equations of motion of our system are

$$\partial^\beta F_{\beta\alpha} + \mu\varepsilon_{\alpha\mu\nu}(\partial^\mu A^\nu) = j_\alpha(x) = ie[\phi^*\partial_\alpha\phi - \phi\partial_\alpha\phi^*] - 2e^2 A_\alpha \phi^*\phi \quad . \tag{2.6a}$$

and

$$D_\mu D^\mu \phi = -m^2\phi - \frac{\lambda}{2}\phi(\phi^*\phi) \quad . \tag{2.6b}$$

Applying in eq.(2.6a) the derivative operator $\varepsilon^{\lambda\delta\alpha}\partial_\delta$ we obtain

$$(\Box + \mu^2 + 2e^2\phi^*\phi)^*F^\lambda = \mu j^\lambda + 2ie\varepsilon^{\lambda\rho\alpha}(\partial_\rho\phi^*)(\partial_\alpha\phi) - 2e^2 A_\alpha \varepsilon^{\lambda\rho\alpha}\partial_\rho(\phi^*\phi) \tag{2.7}$$

where $^*F^\lambda$ is the dual field strengh, $^*F^\lambda = \frac{1}{2}\varepsilon^{\lambda\alpha\beta}F_{\alpha\beta}$, which ir a vector in three dimensions. If the system (2.7) presents spontaneous symmetry breakdown, we have

$$\phi(r) = a^{in\theta} f(r) \tag{2.8a}$$

with

$$f(r) \xrightarrow[r\to\infty]{} \sqrt{-2m^2/\lambda} \tag{2.8b}$$

where $m^2 < 0$. In this case, we can see from eq. (2.7) that the gauge field excitation acquires mass by two different processes: by the topological term, μ^2, and by the Higgs field, $-4\frac{e^2m^2}{\lambda}$. This system prerents vortices configurations with electric charge

$$Q = \int d^2x\, j_0(r) = \mu\left(\frac{2\pi n}{e}\right)$$

(*) There is an analogous term, the Chern-Simons term, whichmaybe added to the action of any non-Abelian gauge theory in a space of oddnumberof dimensions. This term is topological is character. Because of its similarity to the Chern-Simons term, the Abelian term L_μ is frequently referred to as Chern-Simons term to ~ .In the three dimensional space this term is quadratic in the gauge field giving mass for it.

and finite energy[4].

ii - Quantization

The quantization of the system (2.1) is given by the following functional generator

$$Z[j,j^*,j_\mu] = \int[dA_\mu][d\phi][d\phi^*]\exp\{i\int d^3x[L + L_{gf} + j_\mu A^\mu + j\phi^* + j^*\phi]\} \quad (2.9)$$

The propagators, to zero order in perturbatíon theory, are

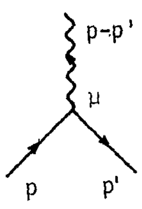
$$D_{\mu\nu}(p) = \frac{-i}{p^2-\mu^2+i\varepsilon}\left(g_{\mu\nu} - \frac{p_\mu p_\nu}{p^2}\right) - i\alpha\frac{p_\mu p_\nu}{p^4} - \frac{\mu\,\varepsilon_{\mu\nu\lambda}\,p^\lambda}{p^2(p^2-\mu^2+i\varepsilon)} \quad (2.10a)$$

and

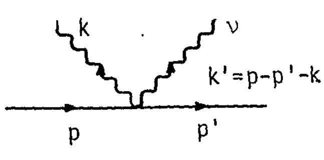
$$G(p) = \frac{i}{p^2 - m^2 + i\varepsilon}\,. \quad (2.10b)$$

(We can see that the vectorial boson field propagator presents a pole at $p^2 = \mu^2$. α is a gauge parameter. For $\alpha=1$ we have the Feynman gauge and for $\alpha=0$ the Landau one).

The system presents three different vertices: one linear in the gaugefield, onequadraticandanotherquartic in thescalar field, as represents by the diagrams below.

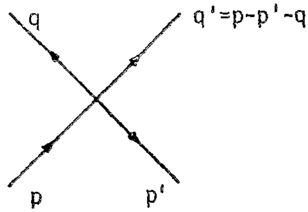


$$-ie(p_\mu + p'_\mu) \quad (2.11a)$$



$$2ie^2 g_{\mu\nu} \quad (2.11b)$$

and



$$- i\lambda \quad . \tag{2.11c}$$

3. ONE LOOP CORRECTION

In this section we calculate the corrections to the propagatorã and vertices of this theory; we do it to order e^2 in perturbation theory. In this work we want to find out how the topological mass term modifies the corrections, the new terms that arise, and how these new terms behave by the discrete transformations P and τ. For this reason we put, on purpose, the coupling constant $\lambda = 0$. We split this section into two parts: in the first we will calculate the corrections ta the propagators (in *this* part *we* will find two diagramã, the vacuum polarization and self-energy), in the second part we will calculate the corsections to the vertices.

I.a) The Photon Propagator

The phaton propagator at tree level is given in eq. (2.10a). And the correction will be given below

$$D^{-1}_{\mu\nu}(p) = D^{-1}_{\mu\nu}(p) - i\ \pi_{\mu\nu}(p) \quad . \tag{3.1}$$

The vacuum polarization tensor, in the one loop approximation, is given by graphs of the fig.1.

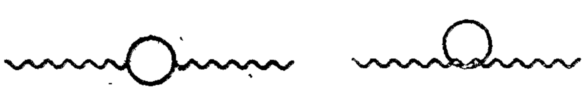


Fig.1 - Vacuum polarization graphs.

$$\pi_{\mu\nu}(p) = \pi^1_{\mu\nu}(p) + \pi^2_{\mu\nu}(p) \tag{3.2}$$

where

$$\pi^1_{\mu\nu}(p) = -ie^2 \int \frac{d^3k}{(2\pi)^2} (2k_\mu+p_\mu)G(k)(2k_\nu+p_\nu)G(p+k)+O(e^4) \qquad (3.3a)$$

and

$$\pi^2_{\mu\nu}(p) = i(2ie^2)g_{\mu\nu} \int \frac{d^3k}{(2\pi)^3} G(k) + O(e^4) \ . \qquad (3.3b)$$

The integrals above are linearly and logarithmically divergent. The logarithrnic divergence does not arise because of symmetric integration, but the linear one rernains and; by dimensional arguments, we can see that it will be proportional to the metric tensor $g_{\mu\nu}$. This divergence, in the vacuum polarization, is not gauge invariant and can be removed by any gauge invariant regularization procedure. We use the Pauli-Villars regularization in this work. The dirnensional regularization can be used too because terms in epsilon, which have not a direct generalization to continuous dirnensions, do not appear here. Evaluating eq. (3.2), and integrals (3.3a,b) we obtain:

$$\pi_{\mu\nu}(p) = -g_{\mu\nu}\frac{e^2\Delta}{3\pi^2} + \frac{e^2}{8\pi}P_{\mu\nu}(p)\left(2|m| + \frac{(p^2-4m^2)}{2\sqrt{p^2}}\log\left[\frac{2\sqrt{p^2m^2}+p^2}{2\sqrt{p^2m^2}-p^2}\right]\right) + O(e^4) \qquad (3.4)$$

where A is a cut-off and

$$P_{\mu\nu}(p) = g_{\mu\nu} - \frac{p_\mu p_\nu}{p^2}$$

the transverse projection operator.

The Pauli-Villars regularized tensor. $\pi_{\mu\nu}(p)$ is given by

$$\pi^{\text{Reg}}_{\mu\nu}(p) = \pi_{\mu\nu}(p,m) - \lim_{M\to\infty} \pi_{\mu\nu}(p,M) \qquad (3.5)$$

which is gauge invariant without the isolated $g_{\mu\nu}$ term.

$$\pi^{\text{Reg}}_{\mu\nu}(p) = \frac{e^2}{8\pi} P_{\mu\nu}(p)\pi(p) \qquad (3.6a)$$

where

$$\pi(p) = 2|m| + \frac{(p^2-4m^2)}{2\sqrt{p^2}} \log\left[\frac{2\sqrt{p^2m^2}+p^2}{2\sqrt{p^2m^2}-p^2}\right] \quad (3.6b)$$

(The same result is also obtained using dimensional regularization).

We can observe that $\pi(p)$ vanishes at $p^2=0$, so the vacuum polarization tensir is non-singular there. In the above approximation, we çee that there does not arise any term is epsilon, which is odd by P and τ transformations. Recently S.Coleman and B.Hill[5] have shown that for a very general form of Electrodynamics there are no more corrections to the topological mass term beyond the one-loop one. For this reason, we conclude that the topological mass term is not modified in our system.

1.b) Scalar Boson Propagator

The corrected scalar boson propagator is

$$G^{-1}(p) = G^{-1}(p) + i\sum(p) \quad (3.7)$$

The self-energy $\sum(p)$ in the one loop approximation can be obtained by the graphs in fig. 2.

Fig.2 - Self-energy graphs.

$$\sum(p) = \sum{}^1(p) + \sum{}^2(p)$$

where

$$\sum{}^1(p) = -ie^2\int\frac{d^3k}{(2\pi)^3}(2p_\mu+k_\mu)G(p+k)(2p_\nu+k_\nu)D^{\mu\nu}(k)+O(e^4) \quad (3.8a)$$

and

and

$$\sum^2(p) = i(2ie^2 g_{\mu\nu}) \int \frac{d^3k}{(2\pi)^3} D^{\mu\nu}(k) + O(e^3) \tag{3.8b}$$

The integral (3.8a) presents linear and logarithmic divergences. The logarithmic one disappears and the linear one is gauge dependent (α--dependent) and can be eliminated using the Landau gauge (a = 0). The integral (3.8b) is linearly divergent; this divergence can be removed by Pauli-Villars regularization procedure.

The scalar boson's self-energy is

$$\sum(p) = \frac{2e^2\Delta}{\pi} + \frac{e^2p^2}{\pi}\frac{1}{\sqrt{p^2}}\log\left(\frac{2\sqrt{p^2m^2}+p^2+m^2-\mu^2}{2\sqrt{p^2\mu^2}+m^2-\mu^2-p^2}\right) - \frac{2}{\pi}\Bigg[|m| - \frac{(m^2-p^2)}{2\sqrt{p^2}}\log\left(\frac{2\sqrt{p^2m^2}+p^2+m^2}{m^2-p^2}\right)\Bigg] + e^2\frac{|(m^2-p^2)-\mu^4|}{16\pi\mu^2} I_1(p,m,\mu)$$

$$- I_2 \frac{(m^2-p^2-\mu^2)}{8\pi\sqrt{\mu^2}} \,.\, I_2(p,m,\mu) - \frac{e^2|\mu|}{\pi} + O(e^4) \quad . \tag{3.9}$$

The terms $I_{\frac{1}{2}}(p,m,\mu)$ are two lengthy expressions given below(*). To regularize eq. (3.8) we have two possibilities: if we use a massive scalar field as regulator one we obtain an expression without divergence andthe linear $|\mu|$ term (these two terms come from eq. (3.3b) which is indepen-

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(*) $$I_1(p,m,\mu) = \frac{1}{\sqrt{p^2}}\log\frac{2\sqrt{p^2m^2}+m^2-p^2}{m^2-p^2} - \frac{a}{ap^2+m^2-p^2}\log\left(\frac{m^2(1+a)+p^2(a-1)+2\sqrt{a(ap^2+m^2-p^2)m^2}}{a(m^2-p^2)}\right)$$

and

$$I_2(p,m,\mu) = 2 + \sqrt{a}\log\left(\frac{\sqrt{a}-1}{\sqrt{a}+1}\right), \quad \text{where} \quad a = \frac{(m^2-\mu^2-p^2)^2}{4p^2\mu^2}$$

dent of the scalar mass field), but a new term arises: $e^2p^2/(4\pi\sqrt{p^2})$. Using A massive vector field, a new divergence arises which is proportional to this mass, $e^2|M|/\pi$. In this second case we must add to the original Lagrangian density a counterterm in srder to remove the divergence in the self-energy. (Again, if we use the dirnensional regularization, we obtain for $\Sigma(p)$ the same expression given in eq. (3.8) without a divergent tem).

2) Vertices

This theory presents two kinds of vertices: linear and quadratic in the gauge field. Here we will treat the linear vertex and afterwards we make some coments about the quadratic one. In the calculation of the diagrams related to this vertex, we will use the Landau gauge again.

The one loop correction to the linear vertex is given by the graphs of fig. 3

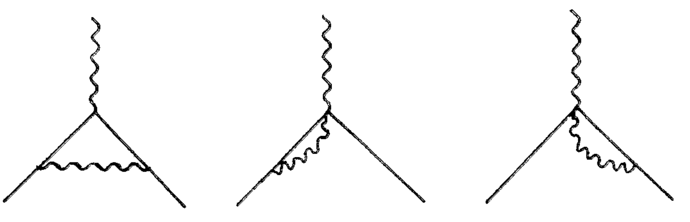

Fig. 3 - Correction graphs to the vertex function A_μ.

$$\Delta_\mu(p,p') = \Delta^1_\mu(p,p') + \Delta^2_\mu(p,p') \qquad (3.10)$$

where

$$\Delta^1_\mu(p,p') =$$

$$=-e^3\int \frac{d^3k}{(2\pi)^3}\,(2p_\alpha+k_\alpha)G(p+k)\,(p_\mu+p'_\mu+2k_\mu)G(p'+k)\,(2p'_\beta+k_\beta)D^{\alpha\beta}(k) + O(e^4)$$

(3.11a)

and

$$\Delta_\mu^2(p,p') = ie^3 \int \frac{d^3k}{(2\pi)^3} (2p_\beta + k_\beta) G(p+k) D^{\beta\alpha}(k) . 2g_{\mu\alpha}$$

$$+ ie^3 \int \frac{d^3k}{(2\pi)^3} . 2g_{\mu\alpha} G(p'+k) D^{\alpha\beta}(k)(2p'_\beta + k_\beta) + O(e^4) \tag{3.11b}$$

The integrals above are free of divergences. The logarithmic one disappears by symmetric integration. Thus, these diagrams do not require a regularization procedure.

In the (proper) vertex function calculation there arises a terms which has an abnormal parity. This term is related to the epsilon termin the photon propagator which, as we know, is associated with an odd -P term in the Lagrangian. From eq. (3.11a) we have

$$\Lambda_\mu(p,p') = \frac{ie^3 \varepsilon_{\mu\alpha\beta} p^\alpha p'^\beta}{2\pi|\mu|\sqrt{(p-p')^2}} \left[I(p,p',m^2,\mu^2) - I(p,p',m^2,\mu^2=0)\right] + \ldots, \tag{3.12a}$$

where

$$I(p,p',m^2,\mu^2) = \int_0^1 dx \, .$$

$$\cdot \log\left[\frac{2\sqrt{(p-p')^2\left[p^2(x^2-x)+m^2x+\mu^2(1-x)\right]} + 2(p-p')^2x + p'^2(x^2-x)+m^2x+\mu^2(1-x)}{2\sqrt{(p-p')^2\left[p'^2(x^2-x)+m^2x+\mu^2(1-x)\right]} + p'^2(x^2-x)+m^2x+\mu^2(1-x)}\right] \tag{3.12b}$$

and the fots represent. the terms of normal parity.

In spite of this anomalous term Ward's identity is mantained, because the term in epsilon vanishes in $\Lambda_\mu(p,p)$

$$\Lambda_\mu(p,p) = -e \frac{\partial\Sigma(p)}{\partial p^\mu} \quad .$$

For the quadratic vertex there are many graphs which are given in fig. 4.

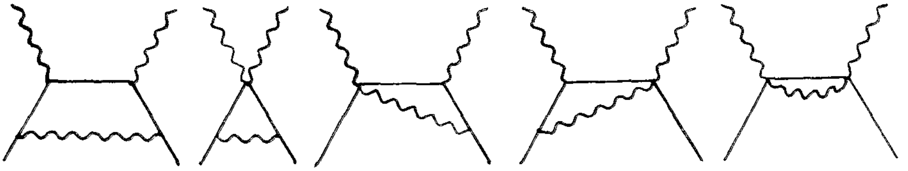

Fig. 4 - Correction graphs to the vertex function $A_{\mu\nu}$.

Initialy we can see that all these graphs are finite and canbe obtained by combinations of the linear and quadratic vertices given in eqs. (2.1 la-b). In their calculation, many contributions arise, some of them, which are proportional to the $g_{\mu\nu}$ tensor correct the quartic vertex in the usual way, but new kind of terms proportional to the epsilon tenâor alss arise. These terms, as we can see, come from the axial term in the pheton propagator. We omit the lengthy expressions for these corrections.

4. SUMMARY AND DISCUSSION

We saw in this work how the topological mass term contributes, to order e", in the correction to the diagrams of scalar electrodynamics. In the vacuum polarization and boson's self-energy, the axial partof the photon propagator does not lead to any new term, but in the vertices functions these do arise. In the specific case of the linear vertex function $\Delta_\mu(p,p')$, it was shown explicity. It was not possible for us toobtain an exact expression for the integral (3.11b). Nevertheless, for a specific choice for the parameters, we can obtain this integral and verify that the contribution to A_μ given in eq.(3.12a) is not zero.

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Resumo

Estudamos a interação entre um campo escalar carregado e um campo de gauge definido num espaço-tempo tri-dimensional. Adicionamos ao sistema o termo de massa topológica (termo de Chern-Simons), o qual é ímpar por transformações P e τ, e investigamos como as correções aos propagadores e vértices da teoria são modificadas por este termo. Estas correções são obtidas até ordem e^2 em teoria de perturbação. Na correção ao vértice linear da teoria, surge um novo tipo de termo. Apesar deste novo termo, que decorre do termo topológico, apresentar paridade anormal, a identidade de Ward é ainda preservada.