Revista Brasileira de Física, Vol. 16, nº 4, 1986

Transverse Susceptibility of the 1D Isotropic XY-Model at Zero Temperature

L.L. GONÇALVES
Department of Theoretical Physics, 1 Keble Road, Oxford OX1 3NP, England

Recebido em 22 de abril de 1986

Abstract An exact expression is obtained for the dynamic transverse susceptibility $\chi^{xx}(\omega,i,j)$ of the one-dimensional isotropic XY-model both on an open and on a closed chain with arbitrary number of sites at zero temperature, when the transverse field is greater than the absolute value of the exchange constant. The dynamic transverse wave-vector-dependent susceptibility for the closed infinite chain is also determined, and it is shown that in all cases the isothermal susceptibility is identical to the static susceptibility.

The one-dimensional XY-model in a transverse field $(s = 1/2)$ is among the few non-trivial many-body problems which can be solved exactly[1]. Although a great deal of work has been done there are still some unknown results related to the transverse correlations. A complete list of references on the results obtained in the model can be found in the works by Perk[2] and Taylor and Müller[3]. The most recent results have been obtained by Müller and Shrock[4] and are related to the anisotropic XY-model. In that work, by exploring the simple form presented by the ground-state of the system for a particular set of values of the transverse field, they have been able to calculate the static transverse susceptibilities $\chi^{xx}(q)$ and $\chi^{yy}(q)$ at zero temperature.

In this note we are going to consider the isotropic XY-model in a transverse field, whose Hamiltonian, for an open chain with N sites, is given by

$$H = \sum_{j=N}^{N-1} J(S_j^x S_{j+1}^x + S_j^y S_{j+1}^y) - \sum_{j=1}^{N} hS_j^z \qquad (1)$$

For fields satisfying the condition $h \geqslant |J|$, the ground state consists of all spins pointing in the field direction, and it decomposes into a

Present Address: Departamento de Física da UFCe, Campus do Pici, Caixa Postal 3004, Fortaleza, 60000, Ceará, Brasil direct product of single-site spin states which is shown in eq.(20) of reference 5. This also happens in the cases analysed by Müller and Shrock[4], and constitutes the main reason why it is possible to evaluate the transverse correlations in closed form. Although we have also pointed out this fact in our paper[5], we did not use it explicitly when evaluating the tranrverse time-dependent correlation $<S_j^x(t)S_\ell^x(0)>$ at zero temperature, which is given by

$$<S_j^x(t)S_\ell^x(0)> = \frac{2(-1)^{j+\ell}}{N+1}\sum_k \sin k_j \sin k_\ell \, e^{iE_k t} \quad , \tag{2}$$

where

$$E_k = J\cos k - h \quad \text{with} \quad k = \frac{n\pi}{N+1} \quad n = 1,2,\ldots, N \ , \tag{3}$$

provided the condition $h \geqslant |J|$ is satisfied. In passing, we would like to note that in the isotropic case the correlation $<S_j^x(t)S_\ell^x(0)>$ is equal to $<S_j^y(t)S_\ell^y(0)>$, and aiso that the phase factor $(-1)^{j+\ell}$ was missing in our original result. The Fourier transform of $<S_j^x(t)S_\ell^x(0)>$, defined as

$$C^{xx}(\omega,j,\ell) = \int_{-\infty}^{\infty} <S_j^x(t)S_\ell^x(0)> e^{i\omega t}\, dt \tag{4}$$

is then obtained from eq.(2) and given by

$$C^{xx}(\omega,j,\ell) = \frac{4\pi(-1)^{j+\ell}}{N+1}\sum_k \sin kj \sin k\ell \ \delta(\omega + E_k) \ . \tag{5}$$

The dynamic susceptibility $\chi^{xx}(\omega,j,\ell)$ and the isothermal susceptibility $\chi^{xx}(\omega,j,\ell)$ can be determined through the relations[6,7]

$$\chi^{xx}(\omega,j,\ell) = \lim_{\beta\to\infty} \left\{ -\frac{1}{2\pi}\int_{-\infty}^{\infty} \frac{(1-e^{-\beta\omega'})C^{xx}(\omega',j,\ell)\,d\omega'}{\omega-\omega'} \right\}$$

$$\chi_T^{xx}(j,\ell) = \lim_{\beta\to\infty} \left\{ \int_0^\beta <S_j^x(-i\lambda)\ S_\ell^x(0)>d\lambda \right\} \ , \tag{6}$$

where β as usual is $1/k_BT$, and by using eqs. (2) and (5) we obtain

$$\chi^{xx}(\omega,j,\ell) = - \frac{2(-1)^{j+\ell}}{N+1} \sum_k \frac{\sin i \sim \sin k}{\omega + J\cos k - h}$$
$$\chi_T^{xx}(j,\ell) = - \frac{2(-1)^{j+\ell}}{N+1} \sum_k \frac{\sin kj \sin k}{J\cos k - h} \qquad (7)$$

From the above result we can conclude immediately that the isothermal susceptibility and the static susceptibility are identical.

The dynamic susceptibility for the closed chain is obtained from eq. (7) by considering just the term that depends on $j-\ell$, and it is obtained by taking the limit $j\to\infty$ keeping j- R constant. This gives imnediately the result

$$\chi^{xx}(\omega,j-1) = - \frac{(-1)^{j-\ell}}{N+1} \sum_k \frac{\cos k(j-\ell)}{\omega + J\cos k - h} , \qquad (8)$$

and from the previous equations we can obtain the wave-vector-dependent susceptibility $\chi^{xx}(\omega,q)$, defined as

$$\chi^{xx}(\omega,q) = \sum_{n=-N}^{N} e^{-iqn} \chi^{xx}(\omega,n) \quad , n = j - \ell , \qquad (9)$$

which is given by

$$\chi^{xx}(\omega,q) = - \frac{1}{2(\omega - J\cos q - h)} , \qquad (10)$$

where in its calculation we have explicitly considered N large in eq. (9).

The static susceptibility obtained from eq. (10) is, in the limit $J = h$, given by

$$\chi^{xx}(0,q) = \frac{1}{2J(\cos q + 1)} , \qquad (11)$$

which reproduces the result recently obtained by Müller and Shrock[4] in the zero anisotropy limit, apart from a factor of 1/4 which is missing in their result[a].

We have benefited from some correspondence with Dr. G. Muller whose remarks are gratefully acknowledged

REFERENCES

1. E. Lieb, T. Schultz, and D.C. Mattis, Ann. Phys. (N.Y.) *16*,407(1961).
2. J.H.H. Perk, PhD Thesis, University of Leiden (1979).
3. J.H. Taylor and G. Muller, Physica *130*A, 1 (1985).
4. G. Müller and R.E. Shrock, Phys. Rev. *32*, 5845 (1985).
5. H.B. Cruz and L.L. Gonçalves, J. Phys. C*14*, 2785 (1981).
6. T. Horiguchi and T. Morita, Can. J. Phys. *53*, 2375 (1975).
7. W.E. Parry, *The Many-Body Problem* (Clarendon Press, Oxford, 1973).
8. G. Muller and R.E. Shrock, private communication.

Resumo

Uma expressão exata é obtida para a susceptibilidade dinâmica transversa $\chi^{xx}(\omega,i,j)$ do modelo XY isotrópico uni-dimensional em uma cadeia aberta e em uma cadeia fechada com número arbitrário de sítios, a temperatura zero, quando o campo transverso é maior do que o valor absoluto da constante de troca. A susceptibilidade dinâmica transversa dependente do vetor de onda para uma cadeia infinita fechada é também determinada, e é mostrado que em todos os casos a susceptibilidade isotérmica é idêntica a susceptibilidade estática.