

Baryon Masses and Hyperfine Splittings

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Abstract Assuming that the three quark interaction can be described in terms of pair interactions, and that the quark-quark interaction is related to the quark-antiquark interaction ($V(q\bar{q})=1/2 V(q\bar{q})$), we have calculated the baryon masses as three-quark bound states. We have also calculated the relativistic corrections coming from the spin-spin interaction. Finally, our results have been compared to the available experimental data.

1. INTRODUCTION

Using a phenomenological potential we have calculated the baryon masses and the hyperfine splittings. This model, which is able to reproduce the aspects of confinement and asymptotic freedom of the strong interaction theory (QCD) has been used with success by many authors in hadron spectroscopy. In the present work, we have obtained the baryon spectrum. For this we have used a coordinate transformation made by Flügge et al.^{1,2} and Zickendraht³ that permits us to write the Schrodinger equation in one variable only. This method, which has been used for the ccc and bbb systems by d'Oliveira et al.⁴ has now been applied to all the baryons formed by quark flavors u, d, s, c and b using the parameters calculated by Castro et al.⁵. For the ground states, we have calculated the hyperfine splittings coming from the spin-spin, term of the Breit-Fermi Hamiltonian.

2. NONRELATIVISTIC THREE-BODY PROBLEM

For the three-body system (fig. 1) the Schrodinger equation in the center-of-mass system can be written as

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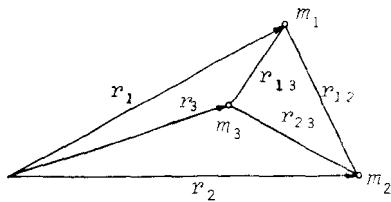


Fig.1 - Three-body system: m_1 , m_2 and m_3 are the masses of the quarks.

$$\left\{ \frac{\hbar^2}{m} (\Delta_1 + \Delta_2) + E - V \right\} \psi = 0 \quad (1)$$

where Δ_1 and Δ_2 refer to the coordinates \vec{x}_1 and \vec{x}_2

$$\vec{x}_1 = \left[\frac{2m_1 m_2}{(m_1 + m_2)m} \right]^{1/2} (\vec{r}_2 - \vec{r}_1) \quad (2)$$

$$\vec{x}_2 = \left[\frac{2m_3(m_1 + m_2)}{(m_1 m_2 + m_2 m_3 + m_3 m_1)m} \right]^{1/2} (\vec{r}_3 - \frac{m_1 \vec{r}_1 + m_2 \vec{r}_2}{m_1 + m_2}) \quad (3)$$

where \vec{r}_1 and \vec{r}_2 are the position vectors, and m is the reduced mass.

Following Flügge *et al.*^{1,2} and Zickendraht³, we can perform a sequence of coordinate transformations and go from the coordinates \vec{x}_1 and \vec{x}_2 to the coordinate $y, \alpha, \beta, \phi, \theta, \psi$.

In this system, the distances are given by

$$r_{12} = \left[\frac{(m_1 + m_2)m}{4m_1 m_2} \right]^{1/2} y (1 - \sin \alpha \sin \beta)^{1/2} \quad (4)$$

$$r_{23} = \left[\frac{(m_2 + m_3)m}{4m_2 m_3} \right]^{1/2} y (1 - \sin \alpha \sin(\beta - \delta_1))^{1/2} \quad (5)$$

$$r_{31} = \left[\frac{(m_3 + m_1)m}{4m_1 m_3} \right]^{1/2} y (1 - \sin \alpha \sin(\beta - \delta_2))^{1/2} \quad (6)$$

where

$$\delta_1 = \arccos \left(\frac{m_1(m_3 - m_2) - m_2(m_3 + m_2)}{(m_1 + m_2)(m_2 + m_3)} \right) ; \quad 0 \leq \delta_1 \leq \pi \quad (7)$$

$$\delta_2 = -\arccos \left(\frac{m_2(m_3-m_1)-m_1(m_3+m_1)}{(m_1+m_2)(m_3+m_1)} \right); \quad \pi \leq \delta_2 \leq 2\pi \quad (8)$$

The Schrödinger equation can be written as

$$H\Psi = E\Psi \quad (9)$$

$$\Psi = \sum_{k=-L}^{+L} F_k^L(y, \alpha, \beta) D_{Mk}^L(\phi, \theta, \psi) \quad (10)$$

$$D_{Mk}^L(\phi, \theta, \psi) = e^{iM\phi} d_{Mk}^L(\theta) e^{ik\psi} \quad (11)$$

$$F_k^L(y, \alpha, \beta) = h_\lambda(y) e^{i\mu\beta} f_{k,\mu}^{L,\lambda}(\alpha) \quad (12)$$

where $h(y)$ is solution of

$$\left\{ \frac{\partial^2}{\partial y^2} + \frac{5}{y} \frac{\partial}{\partial y} + \frac{m}{\hbar^2} [E - V(y)] - \frac{4(\lambda+2)\lambda}{y^2} \right\} h_\lambda(y) = 0. \quad (13)$$

With

$$h_\lambda(y) = y^{-5/2} f_\lambda(y) \quad (14)$$

we have

$$f_\lambda'(y) + \left\{ \frac{m}{\hbar^2} [E - V(y)] + \frac{-\frac{15}{4} - 4(\lambda+2)\lambda}{y^2} \right\} f_\lambda(y) = 0 \quad (15)$$

This last equation can be solved numerically, giving the three-quark bound state energy. The bound-state mass is given by

$$M(q_a q_b q_c) = m(q_a) + m(q_b) + m(q_c) + E(q_a q_b q_c) \quad (16)$$

In order to obtain equation (15) we must know $V(y)$. For this we have to perform an average in α, β of the potential $V(y, \alpha, \beta)$. Following Gerck and d'Oliveira⁶, we have that, given a potential

$$V(r) = r^{2\nu} \quad (17)$$

we obtain

$$V(y) = By^2 \quad (18)$$

where B can be written as

$$B = B_1 + B_2 + B_3 , \quad (19)$$

with

$$B_1 = \left[\frac{(m_1+m_2)m}{4m_1m_2} \right]^\nu \frac{1}{2\pi} \int_0^{\pi/2} \int_0^{4\pi} (1 - \sin \alpha \sin \beta)^\nu \sin \alpha \cos \alpha d\alpha d\beta \quad (20)$$

$$B_2 = \left[\frac{(m_2+m_3)m}{4m_2m_3} \right]^\nu \frac{1}{2\pi} \int_0^{\pi/2} \int_0^{4\pi} (1 - \sin \alpha \sin(\beta - \delta_1))^\nu \sin \alpha \cos \alpha d\alpha d\beta \quad (21)$$

$$B_3 = \left[\frac{(m_3+m_1)m}{4m_3m_1} \right]^\nu \frac{1}{2\pi} \int_0^{\pi/2} \int_0^{4\pi} (1 - \sin \alpha \sin(\beta - \delta_2))^\nu \sin \alpha \cos \alpha d\alpha d\beta \quad (22)$$

3. THREE-QUARK BOUND STATE EQUATION

The three-quark bound state equations can be obtained as the quark-antiquark equations. The graphical representation of the three-quark Bethe-Salpeter equation is shown in fig. 2. Assuming two-body interactions only ($G_3=0$), we obtain the three-quark Bethe-Salpeter equation (Flamm and Schröberl)⁷

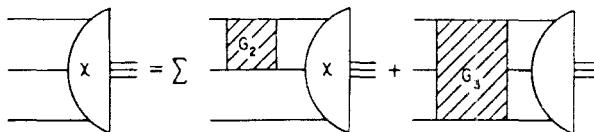


Fig.2 - Graphical representation of the Bethe-Salpeter equation. x represents the $B - S$ amplitude; G_2 is the two-particle and G_3 the three-particle irreducible $B - S$ kernel.

$$(E - H_1 - H_2 - H_3)\psi(\vec{r}, \vec{s}) = -\Lambda^{+++} \sum_{j < k} U(\vec{x}_{jk})\psi(\vec{r}, \vec{s}) \quad (23)$$

where H_1 , H_2 and H_3 are the quark Hamiltonians in the baryon center-of-mass system, Λ^{+++} is the positive energy-projection operator, \vec{r} and \vec{s} are the relative coordinates, $\vec{x}_{jk} = \vec{x}_j - \vec{x}_k$, and $U(x_{jk})$ is the generalized potential that contains a spin-dependent part V_S and a spin-independent part V_{SI} . Omitting Λ^{+++} in the last equation we obtain the generalized Breit equation

$$H_B \Psi(\vec{r}, \vec{s}) = E \Psi(\vec{r}, \vec{s}) \quad (24)$$

with

$$H_B = \sum_{i=1}^3 \{ \beta_i m_{q_i} + \vec{\alpha}_i \cdot \vec{p}_i \} + \sum_{j < k} \{ V_{SI}(r_{jk}) + V_S(x_{jk}) \} \quad (25)$$

where p_i are the momenta of the quarks.

Performing a generalized Foldy-Wouthuysen transformation⁸, we obtain the Breit-Fermi Hamiltonian for three quarks ($\hbar=c=1$)

$$H = \sum_i (m_i + \frac{p_i^2}{2m_i}) + V_{SI}(r) + H_{SS} + \dots \quad (26)$$

where m_i are the constituent quark masses and

$$V_{SI}(r) = -\frac{4\alpha_s}{3r}$$

$$H_{SS} = \sum_{i < k} \frac{2\vec{s}_i \cdot \vec{s}_k}{3m_i m_k} \nabla_{r_{ik}}^2 V(r_{ik}) ; (i, k = 1, 2, 3) \quad (28)$$

In obtaining the Breit-Fermi Hamiltonian we have assumed that the QED equations can be applied to QCD. For this we have to replace a , the QED coupling constant, by α_s , the QED effective coupling constant. Besides, we have to take the quark colors into account. The lowest order diagram for the quark-quark interaction due to gluon exchange is shown in fig.3.

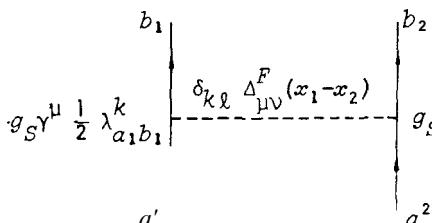


Fig.3 - Lowest order diagram for the quark-quark interaction due to gluon exchange.

Using the Feynman rules, the interaction potential is

$$V = \frac{\alpha_s}{r} \sum_{k=1}^8 \frac{\lambda^k}{2} \cdot \frac{\lambda^k}{2} \quad (29)$$

The sum over the A matrices can be replaced by the scalar product of the F spins of the two quarks. Hence, we obtain

$$V = \frac{\alpha_s}{r} \sum_{k=1}^8 F_k^C(1) F_k^C(2) \quad (30)$$

Since we can construct the quadratic Casimir operator of SU(3) as

$$C^2 = \sum_{k=1}^8 (F_k^C)^2 \quad (31)$$

and the spin of the two-quark system F_k^C is the sum of the F spins of the two quarks

$$F_k^C = F_k^C(1) + F_k^C(2) \quad (32)$$

and assuming that individual quarks belongs to the triplet representation and have $C^2 = 4/3$, we obtain

$$V = \frac{1}{2} \frac{\alpha_s}{r} (C^2 - \frac{8}{3}) \quad (33)$$

Consequently, the bound-state energy is proportional to C^2

For the $\bar{q}\bar{q}$ system, we have

$$3 \otimes \bar{3} = 1 \oplus 8 \quad (34)$$

and for the qq system

$$3 \otimes 3 = 6 \oplus \bar{3} \quad (35)$$

Looking at table 1, we see that

$$E(\bar{q}\bar{q})_1 < E(q\bar{q})_3 < E(q\bar{q})_8 < E(q\bar{q})_6 \quad (36)$$

We conclude that the $\bar{q}\bar{q}$ color singlet is lower in mass than the other

Table 1 - Eigenvalues of the quadratic Casimir operator for the lowest-dimensional representations of SU(3). (*m*=mixed; *s*=symmetric).

Dimension of representation	1	3	$\bar{3}$	6	$\bar{6}$	8	10	10	15	15	15	24	24	27
Quark indices	p	0	1	1	2	2	3	3	3	4	4	4	4	4
	q	0	0	1	0	2	1	0	3	1	2	0	4	1
C^2		0	$\frac{4}{3}$		$\frac{10}{3}$		3	6	$\frac{16}{3}$		$\frac{28}{3}$		$\frac{25}{3}$	8

quark states. This way quark confinement can be made plausible. This argument can also be applied to *n*-quark states, assuming only two-body interactions.

For three-quark states, we have

$$V = \sum_{\substack{a=1 \\ b>a}}^3 \frac{\alpha_s}{r_{ab}} \frac{1}{2} (C^2 - 4) \quad (37)$$

where

$$r_{ab} = |\vec{x}_a - \vec{x}_b|$$

and for the *qqq* system

$$3 \otimes 3 \otimes 3 = 1 \oplus 8 \oplus 8 \oplus 10$$

As can be seen from table 1, we have

$$E(qqq)_1 < E(qqq)_8 < E(qqq)_{10}$$

The color singlet state is again the most favored.

Using eq. (33), we obtain that

$$V(q\bar{q})_1 = - 4/3 \frac{\alpha_s}{r} \quad (41)$$

and

$$V(q\bar{q})_3 = - 2/3 \frac{\alpha_s}{r} \quad (42)$$

then

$$V(q\bar{q}) = 1/2 V(q\bar{q})_1 \quad (43)$$

Although this relation has been obtained for the Coulomb potential, it is believed to be valid to all potentials.

4. PHENOMENOLOGICAL POTENTIAL

The potential

$$V(r) = kr^{1/2} - \frac{4}{3} \frac{\alpha_s}{r} + C \quad (44)$$

has been applied to mesons^{5,9} and with $\alpha_s = 0$ to the bbb and ccc bound states. In this potential the first term is the confining potential, the second term is the potential due to one-gluon exchange, and the parameter C , which depends on each quark-antiquark pair, reflects the fact that we can only calculate energy differences.

Calculating the average potential $V(y)$ and using the parameters for mesons⁵ with the relation (43) we obtain from equation (15) the baryon masses shown in tables 2, 3, 4 and 5.

5. HYPERFINE SPLITTINGS

With the Hamiltonian eq.(26) we obtain, using perturbation theory, the hyperfine splittings between the $J = 3/2$ and $J = 1/2$ states in the 1S level of the baryon spectrum.

The baryon wave functions have been constructed to be completely antisymmetric.

Then, for the $J = 3/2$ baryon, we have

Table 2 - Spectrum of baryon masses (radial excitations). Masses in GeV.

		$K = 0.855 \text{ GeV}^{3/2}$			$\alpha_s = 0.0$	
q_a	q_b	q_c	B_1	$1S$	$2S$	$3S$
uud			2.441	1.344	1.975	2.484
uus			2.434	1.447	2.066	2.564
uss			2.451	1.583	2.193	2.685
sss			2.441	1.701	2.298	2.780
uuc			2.505	2.572	3.167	3.646
usc			2.493	2.683	3.262	3.729
ssc			2.484	2.812	3.379	3.835
ucc			2.490	3.640	4.168	4.593
scc			2.524	3.839	4.366	4.790
ccc			2.441	4.783	5.263	5.649
uub			2.563	5.834	6.419	6.890
usb			2.555	5.927	6.495	6.953
ssb			2.541	6.033	6.586	7.031
ucb			2.656	6.922	7.441	7.859
scb			2.628	7.063	7.572	7.982
ccb			2.484	8.002	8.457	8.823
ubb			2.945	10.152	10.654	11.059
sbb			2.830	10.239	10.722	11.111
cbb			2.524	11.173	11.596	11.937
bbb			2.441	14.300	14.685	14.995

Table 3 - Spectrum of baryon masses (orbital excitations). Masses in GeV.

$q_a\ q_b\ q_c$	1P	1D	2P	1F	2D	1G
uud	1.750	2.096	2.290	2.401	2.573	2.676
uus	1.845	2.184	2.374	2.483	2.651	2.752
uss	1.975	2.310	2.498	2.605	2.772	2.871
sss	2.085	2.412	2.596	2.701	2.864	2.961
uuc	2.954	3.280	3.463	3.568	3.730	3.827
usc	3.055	3.373	3.551	3.653	3.811	3.905
ssc	3.176	3.487	3.661	3.761	3.915	4.007
ucc	3.979	4.269	4.431	4.524	4.668	4.754
scc	4.177	4.466	4.628	4.721	4.865	4.951
ccc	5.092	5.355	5.502	5.586	5.717	5.795
uub	6.210	6.531	6.710	6.813	6.973	7.068
usb	6.292	6.604	6.778	6.878	7.033	7.126
ssb	6.388	6.692	6.862	6.959	7.110	7.200
ucb	7.256	7.540	7.700	7.791	7.933	8.017
scb	7.390	7.669	7.826	7.915	8.054	8.137
ccb	8.294	8.544	8.683	8.763	8.888	8.961
ubb	10.475	10.750	10.905	10.993	11.130	11.212
sbb	10.549	10.814	10.963	11.048	11.180	11.259
cbb	11.445	11.677	11.807	11.881	11.997	12.066
bbb	14.548	14.759	14.877	14.945	15.050	15.113

Table 4 - Spectrum of baryon masses (radial excitations). Masses in GeV.

$q_a \ q_b \ q_c$	B_2	$1S$	$2S$	$3S$	$4S$	$5S$
	$K = 0.767 \text{ GeV}^{3/2}$			$\alpha_s = 0.187$		
uud	5.090	1.339	1.944	2.426	2.834	3.192
uus	5.121	1.448	2.043	2.516	2.916	3.267
uss	5.052	1.587	2.176	2.644	3.039	3.385
sss	5.090	1.707	2.285	2.744	3.131	3.471
uuc	4.872	2.574	3.150	3.607	3.993	4.331
usc	4.923	2.689	3.253	3.700	4.077	4.407
ssc	4.958	2.821	3.375	3.813	4.183	4.506
ucc	4.996	3.646	4.171	4.584	4.932	5.235
scc	4.856	3.851	4.375	4.787	5.134	5.437
ccc	5.090	4.800	5.290	5.672	5.992	6.271
uub	4.692	5.838	6.407	6.857	7.237	7.570
usb	4.730	5.944	6.499	6.939	7.309	7.634
ssb	4.767	6.061	6.604	7.034	7.395	7.712
ucb	4.601	6.935	7.456	7.864	8.208	8.507
scb	4.587	7.092	7.604	8.006	8.343	8.637
ccb	4.958	8.034	8.508	8.875	9.182	9.449
ubb	4.054	10.172	10.685	11.085	11.421	11.713
sbb	4.221	10.281	10.779	11.167	11.491	11.774
cbb	4.856	11.220	11.676	12.027	12.318	12.570
bbb	5.090	14.362	14.800	15.131	15.404	15.639

Table 5 - Spectrum of baryon masses (orbital excitations). Masses in GeV.

$q_a \ q_b \ q_c$	1P	1D	2P	1F	2D	1G
uud	1.740	2.074	2.251	2.364	2.522	2.624
uus	1.843	2.171	2.344	2.456	2.611	2.711
uss	1.978	2.303	2.474	2.585	2.738	2.838
sss	2.092	2.411	2.578	2.688	2.837	2.935
uuc	2.958	3.276	3.442	3.552	3.700	3.798
use	3.066	3.377	3.540	3.647	3.792	3.888
ese	3.192	3.498	3.656	3.763	3.904	3.999
uee	4.001	4.291	4.439	4.541	4.673	4.764
scc	4.205	4.495	4.642	4.745	4.876	4.967
ccc	5.136	5.407	5.541	5.639	5.758	5.845
uub	6.217	6.531	6.695	6.803	6.949	7.046
usb	6.315	6.622	6.782	6.888	7.030	7.125
sbb	6.426	6.726	6.881	6.986	7.124	7.217
ucb	7.288	7.577	7.722	7.824	7.954	8.044
scb	7.440	7.724	7.866	7.967	8.094	8.183
ccb	8.362	8.625	8.752	8.849	8.962	9.046
ubb	10.524	10.808	10.948	11.051	11.176	11.267
sbb	10.624	10.900	11.035	11.136	11.256	11.345
cbb	11.542	11.796	11.913	12.010	12.114	12.197
bbb	14.680	14.923	15.029	15.126	15.221	15.303

$$\Psi = \tau \chi_s \phi_s \psi \quad (45)$$

where

τ is the color wave function

χ_s is the symmetric spin wave function

ϕ_s is the symmetric flavor wave function

ψ is the ground state space wave function

with

$$\chi(3/2, 3/2) = \uparrow\uparrow\uparrow$$

$$\chi(3/2, 1/2) = 1/\sqrt{3} (\uparrow\uparrow\downarrow+\uparrow\downarrow\uparrow+\downarrow\uparrow\uparrow)$$

$$\chi(3/2, -1/2) = 1/\sqrt{3} (\downarrow\uparrow\uparrow+\uparrow\downarrow\downarrow+\uparrow\downarrow\downarrow)$$

$$\chi(3/2, -3/2) = \downarrow\downarrow\downarrow$$

For the baryons with $J = 1/2$

$$\Psi = \frac{\tau}{\sqrt{2}} (\chi_p \phi_p + \chi_\lambda \phi_\lambda) \psi \quad (46)$$

where p and h are the mixed representations

$$\chi_p(\frac{1}{2}, \frac{1}{2}) = 1/\sqrt{2} (\uparrow\downarrow-\downarrow\uparrow) \uparrow$$

$$\chi_p(\frac{1}{2}, -\frac{1}{2}) = 1/\sqrt{2} (\uparrow\downarrow-\downarrow\uparrow) \downarrow$$

$$\chi_\lambda(\frac{1}{2}, \frac{1}{2}) = 1/\sqrt{6} (\uparrow\uparrow\downarrow+\uparrow\downarrow\uparrow-2\uparrow\uparrow\downarrow)$$

$$\chi_\lambda(\frac{1}{2}, -\frac{1}{2}) = 1/\sqrt{6} (\uparrow\downarrow\downarrow+\downarrow\uparrow\downarrow-2\downarrow\uparrow\uparrow)$$

Applying H_{SS} to the wave functions (45), (46), we have obtained that the spin-spin correction is given by

$$\begin{aligned} <\Psi|H_{SS}|\Psi>_{3/2} - <\Psi|H_{SS}|\Psi>_{1/2} &= \frac{1}{3m_1 m_2} <\Psi|\nabla_{r_{12}}^2 V(r_{12})|\Psi> \\ &+ \frac{1}{3m_1 m_3} <\Psi|\nabla_{r_{13}}^2 V(r_{13})|\Psi> + \frac{1}{3m_2 m_3} <\Psi|\nabla_{r_{23}}^2 V(r_{23})|\Psi> \end{aligned} \quad (47)$$

For the potential (44) with $\alpha_s = 0$, we have

$$\begin{aligned} \langle \Psi | H_{SS} | \Psi \rangle_{3/2} - \langle \Psi | H_{SS} | \Psi \rangle_{1/2} &= \frac{K}{8m_1 m_2} \langle \psi | \frac{B_1}{y^{3/2}} | \psi \rangle + \frac{K}{8m_1 m_3} \langle \psi | \frac{B_2}{y^{3/2}} | \psi \rangle \\ &\quad + \frac{K}{8m_2 m_3} \langle \psi | \frac{B_3}{y^{3/2}} | \psi \rangle \end{aligned} \quad (48)$$

where B_1 , B_2 and B_3 represent the average in α , β and are shown in table 6.

Using eq.(48) we have obtained the results shown in tables 7 and 8.

Assuming that the potential is a mixture of scalar and vector potentials, as in the case of the mesons⁵, of the form

$$V(r) = V_v(r) + V_s(r)$$

with

$$V_v = (1-f)V_{\text{conf.}}(r) + V_{\text{coul.}}(r)$$

$$V_s = fV_{\text{conf.}}(r)$$

$$0 \leq f \leq 1$$

we have obtained the results of the tables 9 and 10. The scalar term does not contribute to the spin-spin correction. Then the results obtained for many values of f can be compared to the experimental data to give the correct Lorentz nature of the confining potential.

Tables 11 and 12 show our results and those from Bjorken¹⁰, and Samuel and Moriarty¹¹. Those authors use differerit procedures to calculate the baryon masses.

We have considered $\alpha_s = 0$ in the results shown in tables 7,8, 9 and 10. In this case we have calculated the hyperfine splittihs, taking into account the confining potential. This is due to the fact that the hyperfine splittings involve a 6-function. This is equivalent to having the wave function at the origin. This calculation turns out to be difficult if we use the Zickendraht variables. Besides, the method developed by Zickendraht showed to be in excellent agreement with the

Table 6 - B_1 , B_2 and B_3 values which represent α and β averaged in the Laplacian potential.

q_a	q_b	q_c	B_1	B_2	B_3
<i>uuu</i>			2.439	2.439	2.439
<i>bbu</i>			3.760	0.934	0.934
<i>ccs</i>			3.122	1.883	1.883
<i>ccu</i>			3.285	1.956	1.956
<i>ssc</i>			1.883	2.588	2.588
<i>ssu</i>			2.588	2.327	2.327
<i>uub</i>			1.568	2.478	2.478
<i>uuc</i>			1.819	2.530	2.530
<i>uus</i>			2.327	2.530	2.530
<i>csu</i>			2.767	2.375	1.850
<i>bbc</i>			3.122	1.883	1.883
<i>bbs</i>			3.656	1.103	1.103
<i>ccb</i>			1.883	2.588	2.588
<i>ssb</i>			1.618	2.530	2.530
<i>bcs</i>			3.285	1.671	1.451
<i>bcu</i>			3.656	1.519	1.348
<i>bsu</i>			2.767	2.278	1.568

Table 7 - Masses of the ground states of baryons using spin-spin corrections. Masses in GeV.

$q_a q_b q_c$	Symbol	$K = 0.855 \text{ GeV}^{3/2}$		$\alpha_s = 0.0$		
		$M(1S)(J^P=3/2^+)$		$M(1S)(J^P=1/2^+)$		
		Theor.	Experimental ¹⁴	Theor.	Experimental ¹⁴	
uuu	Δ^{++}	1.344	1.232			
uud	Δ^+	1.344	1.232	p	0.866	0.938216 ± 0.000036
udd	Δ^0	1.344	1.232	n	0.866	0.9395731 ± 0.0000027
ddd	Δ^-	1.344	1.232			
uus	Σ^{*+}	1.447	1.38229 ± 0.00039	Σ^+	1.022	1.189371 ± 0.000060
uds	Σ^{*0}	1.447	1.3820 ± 0.0025	Σ^0	1.022	1.19246 ± 0.00008
dds	Σ^{*-}	1.447	1.38744 ± 0.00058	Σ^-	1.022	1.197388 ± 0.000079
uds				Λ^0	1.022	1.115566 ± 0.000056
uss	Ξ^{*0}	1.583	1.53178 ± 0.00034	Ξ^0	1.215	1.31483 ± 0.00082
dss	Ξ^{*-}	1.583	1.53513 ± 0.00084	Ξ^-	1.215	1.32134 ± 0.00014
sss	Ω^-	1.701	1.67237 ± 0.00034			
uuc	Σ_c^{*++}	2.572		Σ_c^{++}	2.310	
udc	Σ_c^{*+}	2.572		Σ_c^+	2.310	
ddc	Σ_c^{*0}	2.572		Σ_c^0	2.310	
udc				Λ_c^+	2.310	2.2820 ± 0.0031
usc	Ξ_c^{*+}	2.683		Ξ_c^+, Ξ_c^{A+}	2.454	
dsc	Ξ_c^{*0}	2.683		Ξ_c^0, Ξ_c^{A0}	2.454	
ssc	Ω_c^{*0}	2.812		Ω_c^0	2.614	
ucc	Ξ_{cc}^{*++}	3.640		Ξ_{cc}^{++}	3.497	
dcc	Ξ_{cc}^{*+}	3.640		Ξ_{cc}^+	3.497	

Table 8 - Masses of the ground states of baryons using spin-spin corrections. Masses in GeV.

$K = 0.855 \text{ GeV}^{3/2}$				$\alpha_s = 0.0$		
$M(1S) (J^P = 3/2^+)$				$M(1S) (J^P = 1/2^+)$		
$q_a q_b q_c$	Symbol	Theor.	Experimental	Symbol	Theor.	Experimental
scc	R_{cc}^{*+}	3.839		Ω_{cc}^+	3.724	
ccc	Ω_{ccc}^{++}	4.783				
uub	Σ_h^{++}	5.834		Σ_b^+	5.645	
udb	Σ_b^{*0}	5.834		Σ_b^0	5.645	
ddb	Σ_b^{*-}	5.834		Σ_b^-	5.645	
udb				Λ_b^0	5.645	
usb	Ξ_b^{*0}	5.927		$\Xi_b^0, \Xi_b^{A^0}$	5.767	
dsb	Ξ_b^{*-}	5.927		$\Xi_b^-, \Xi_b^{A^-}$	5.767	
ssb		6.033		Ω_b^-	5.895	
ucb	Ξ_{cb}^+	6.922		$\Xi_{cb}^+, \Xi_{cb}^{A^+}$	6.839	
dcb	Ξ_{cb}^0	6.922		$\Xi_{cb}^0, \Xi_{cb}^{A_0}$	6.839	
scb	Ω_{cb}^{*0}	7.063		$\Omega_{cb}^0, \Omega_{cb}^{A^0}$	6.992	
ccb	Ω_{ccb}^{*+}	8.002		Ω_{ccb}^+	7.959	
ubb	Ξ_{bb}^{*0}	10.152		Ξ_{bb}^0	10.110	
dbb	Ξ_{bb}^-	10.152		Ξ_{bb}^-	10.110	
sbb	Ω_{bb}^{*-}	10.239		Ω_{bb}^-	10.201	
cbb	Ω_{cbb}^{*0}	11.173		Ω_{cbb}^0	11.148	
bbb	Ω_{bbb}^-	14.300				

Table 9 - Masses of baryons with $J^P=1/2^+$ using the hyperfine splittings obtained for several values of f ($K=0.855 \text{ GeV}^{3/2}$). Masses in GeV.

$q_a q_b q_c$	Particle	$f=0.1$	$f=0.2$	$f=0.3$	$f=0.4$	$f=0.5$	$f=0.6$	$f=0.7$	$f=0.8$	$f=0.9$	Experimental ¹⁴
uud	P	0.914	0.962	1.010	1.057	1.105	1.153	1.201	1.248	1.296	0.938216 ± 0.000036
uus	Σ^+	1.065	1.107	1.150	1.192	1.235	1.277	1.320	1.362	1.405	1.189371 ± 0.000060
uss	Ξ^0	1.252	1.289	1.326	1.362	1.399	1.436	1.472	1.509	1.546	1.31483 ± 0.00082
uuc	Σ_c^{++}	2.337	2.363	2.389	2.415	2.441	2.467	2.494	2.520	2.546	
usc	$\Xi_c^+, \Xi_c^{A^+}$	2.477	2.500	2.523	2.546	2.568	2.591	2.614	2.637	2.660	
ssc	Ω_{cc}^0	2.634	2.654	2.673	2.693	2.713	2.733	2.753	2.773	2.792	
ucc	Ξ_{cc}^{++}	3.512	3.526	3.540	3.554	3.569	3.583	3.597	3.611	3.625	
scc	Ω_{cc}^+	3.736	3.747	3.759	3.770	3.782	3.793	3.805	3.816	3.827	
uub	Σ_b^+	5.664	5.683	5.702	5.721	5.740	5.759	5.778	5.796	5.815	

Table 10 - Masses of baryons with $J^P=1/2^+$ using the hyperfine splittings obtained for several values of f ($K=0.855 \text{ GeV}^{3/2}$). Masses in GeV.

$q_a q_b q_c$	Particle	$f=0.1$	$f=0.2$	$f=0.3$	$f=0.4$	$f=0.5$	$f=0.6$	$f=0.7$	$f=0.8$	$f=0.9$	Experimental
usb	$\Xi_b^0, \Xi_b^{A^0}$	5.783	5.799	5.815	5.831	5.847	5.863	5.879	5.895	5.911	
scb	Ω_b^-	5.909	5.923	5.936	5.950	5.964	5.978	5.991	6.005	6.019	
ucb	$\Xi_{cb}^+, \Xi_{cb}^{A^+}$	6.847	6.856	6.864	6.872	6.881	6.889	6.897	6.905	6.914	
scb	$\Omega_{cb}^0, \Omega_{cb}^{A^0}$	6.999	7.006	7.014	7.021	7.028	7.035	7.042	7.049	7.056	
ccb	Ω_{ccb}^+	7.964	7.968	7.972	7.976	7.981	7.985	7.989	7.993	7.998	
ubb	Ξ_b^0	10.114	10.118	10.123	10.127	10.131	10.135	10.139	10.144	10.148	
sbb	Ω_{bb}^-	10.204	10.208	10.212	10.216	10.220	10.223	10.227	10.231	10.235	
cbb	Ω_{ccb}^0	11.151	11.154	11.156	11.159	11.161	11.164	11.166	11.168	11.171	

Table 11 - Masses of baryons with $J^P=3/2^+$. Masses in MeV. The columns correspond to: (1) the potential parameters: $K=0.855 \text{ GeV}^{3/2}$, $\alpha_s = 0$; (2) the potential parameters: $K=0.767 \text{ GeV}^{3/2}$, $\alpha_s = 0.187$; (3) the values from Samuel and Moriarty; (4) the values from Bjorken

	(1)	(2)	(3)	(4)
uuu	1.344	1.339	1.262 ± 45	
uus	1.447	1.448	1.441 ± 50	
uss	1.583	1.587	1.575 ± 45	
sss	1.701	1.707	1.694 ± 35	
uuc	2.572	2.574	2.594 ± 45	2.463 ± 30
usc	2.683	2.689		2.608 ± 30
ssc	2.812	2.821	2.768 ± 35	2.755 ± 30
ucc	3.640	3.646		3.695 ± 60
scc	3.839	3.851		3.840 ± 60
ccc	4.783	4.800		4.925 ± 90
uub	5.834	5.838		5.740 ± 60
usb	5.927	5.944		5.890 ± 60
ssb	6.033	6.061		6.035 ± 60
usb	6.922	6.935		5.970 ± 70
scb	7.063	7.092		7.120 ± 70
ccb	8.002	8.034		8.200 ± 90
ubb	10.152	10.172		10.250 ± 120
sbb	10.239	10.281		10.395 ± 120
cbb	11.173	11.220		11.480 ± 120
bbb	14.300	14.362		14.760 ± 180

Table 12 - Masses of baryons with $J^P=1/2^+$. Masses in MeV. The columns correspond to: (1) the potential parameters: $K=0.855 \text{ GeV}^{3/2}$ and $\alpha_s = 0$ for masses and spin-spin correction; (2) $K=0.767 \text{ GeV}^{3/2}$ and $\alpha_s = 0.187$ for masses. $K=0.767 \text{ GeV}^{3/2}$ and $\alpha_s = 0$ for spin-spin corrections; (3) values from Samuel and Moriarty; (4) values from Bjorken.

	(1)	(2)	(3)	(4)
<i>uud</i>	866	910	1.069 ± 45	
<i>uus</i>	1.022	1.064	1.249 ± 40	
<i>uds</i>	1.022	1.064	1.164 ± 60	
<i>uss</i>	1.215	1.255	1.338 ± 55	
<i>uuc</i>	2.310	2.336	2.431 ± 20	2.403
<i>udc</i>	2.310	2.336	2.282 ± 60	2.243
<i>usc</i>	2.454	2.480		2.468(2.558)
<i>ssc</i>	2.614	2.638	2.727 ± 25	2.715
<i>ucc</i>	3.497	3.511		3.635
<i>scc</i>	3.724	3.742		3.800

energy-eigenvalues, but the same could not be said about the wave functions. In the calculations of the hyperfine splittings using only a confining potential, the accuracy of the calculations is equivalent to that of energy levels. In this way we avoided, to consider the value of the wave function at the origin. But in the analysis of the hyperfine splittings we have considered $0 \leq f \leq 1$. In the case where $f=1$ the confining potential does not contribute to the hyperfine splittings. Such confining potential would be a Lorentz scalar. In that case the contributions to the hyperfine splittings can only come from the Coulombic part of the potential. We are studying this question and one possible solution can be found using the method proposed by Hiller et al.¹². This method shows that the value of the wave function at the origin, for two or more bodies, can be related to the derivative of the potential. We hope to present this calculation as soon as possible.

6. CONCLUSION

When our results are compared with those obtained by Guimarães et al.¹³, using the K harmonics method, for the square root potential, we note a difference. This is not surprising because for the harmonic-oscillator potential our results, which agree with the analytic values, don't agree with those authors results.

For the $J = 3/2$ baryons the comparison of our results and the experimental data show a little discrepancy. From table 11 we see that in a general way our results agree with those obtained by other authors^{10,11}. The discrepancy is greater for the uuu systems and for the heavier baryons.

In table 7 the comparison of our results and the experimental data shows that the masses of $J = 1/2$ baryons are below the experimental ones. This fact leads us to the conclusion that the confining potential is not a pure vector but a mixture of vector and scalar, which can be confirmed by table 12 from the comparison with other authors.

In table 9 the experimental values show that f must be below 0.5. For mesons, comparison of the calculated and experimental values showed that f must be between 0.5 and 0.6. If the baryon masses calculated for these f values are compared with those obtained by other authors we find a good agreement.

In the analysis of the $J = 1/2$ baryon results we must recall that we have not calculated the spin-spin correction for the complete potential. The inclusion of the Coulombic term would affect our results probably leading to a better agreement with the f value obtained for mesons .

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Resumo

Considerando que a interação entre três quarks pode ser descrita em termos das interações entre pares e que a interação quark-quark está relacionada à interação quark-antiquark ($V(q\bar{q}) = 1/2 V(\bar{q}\bar{q})$), calculamos as massas dos bárions como estados ligados de três quarks. Calculamos também as correções relativísticas provenientes das interação spin-spin. Comparamos nossos resultados com os dados experimentais disponíveis.