

## Center-of-Mass Corrections for the Harmonic S+V Potential Model

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**Abstract** Center-of-Mass corrections to the mass spectrum and static properties of low-lying S-wave baryons are discussed in the context of a relativistic, independent quark model, based on a Dirac equation, with an equally mixed scalar and vector confining potential of harmonic type. A more satisfactory fitting of the parameters involved is obtained, as compared with previous treatments in which CM corrections were neglected.

### 1. INTRODUCTION

As a dynamical theory of strong interactions, Quantum Chromodynamics has gained wide acceptance. It provides a field of theoretical framework for a systematic analysis of hadronic phenomena based on which a considerable progress in their understanding has been achieved. The quantitative success of QCD in the realm of very high momentum transfers is quite impressive. A reason for this stems from the fact that the effective coupling constant ( $\alpha_s$ ) in that region is small and perturbation theory can be applied. The same cannot be said about the hadron structure problem at ordinary mass scales (circa 1 GeV), where a large amount of data exists. For this reason, it is still of interest to study simple and workable models which incorporate some features inspired by QCD, such as asymptotic freedom and confinement.

Among these, in the non-relativistic domain, special mention must be made to the pioneering work of De Rujula, Georgi and Glashow<sup>1</sup> and to the extensive analysis of Isgur and Karl<sup>2</sup>.

In the relativistic domain, the original bag model<sup>3</sup> gives us a possible description of hadrons as composite systems of relativistic spin 1/2 Dirac quarks, enclosed in the interior of a cavity with sharp boundary.

As an alternative to the models of the bag type, we have the relativistic potential models<sup>4</sup>, in which the idea of a sharp boundary

is abandoned in favor of a Lorentz scalar potential of the confining type.

This is tantamount to confinement because it corresponds to attributing an effective mass to the Dirac constituent quark which grows indefinitely for increasing distances.

The present work is based on a relativistic potential model in which the constituent quarks obey a Dirac equation in a central, confining potential, which is a mixture, in equal parts, of a Lorentz scalar ( $S$ ) and a Lorentz vector ( $V$ ).

The Dirac equation with an equally mixed  $S+V$  confining potential has several interesting properties: (i) it provides a relativistically consistent confinement, in view of the absence of Klein paradox for quarks or anti-quarks<sup>5</sup>; (ii) it admits exact solutions whenever the corresponding Schrödinger equation is exactly solvable for the same potential<sup>6</sup>; (iii) the one-body spin-orbit interaction vanishes<sup>7</sup>; (iv) it realizes an exact  $SU(2)$  spin symmetry, independent of the radial dependence of the potential<sup>7</sup>; (v) it admits an exact Melosh transformation<sup>8</sup>; (vi) for high energies, it admits linear Regge trajectories for a confining linear potential<sup>5</sup>.

An independent quark model based on the equally mixed  $S+V$  Dirac equation has been applied, with a reasonable degree of success, to the study of the properties of the low-lying  $S$ -wave baryons, particularly the mass spectrum<sup>9,10</sup> and static baryonic properties<sup>9-11</sup>. The description of baryons as an independent quark model requires, however, the introduction of center-of-mass (CM) corrections in order to remove, from the total single-quark energy, the spurious energy of the CM motion. These CM corrections are of importance not only for the calculation of the mass spectrum<sup>12</sup> but also for the static properties such as the axial charge  $g_A$ , the rms radius and magnetic moment of the nucleon<sup>13</sup>.

In this work, the CM corrections are worked out following the treatment of Wong<sup>12</sup>, who makes use of an appropriate projection procedure, known in nuclear physics as the generator coordinate method. The radial dependence of the  $S+V$  potential will be taken to be harmonic. This choice of the confining potential has the additional advantage that the calculations may be performed analytically.

This paper is organized as follows: in section 2, the S+V relativistic model is briefly discussed, together with the expressions of the static baryonic properties derived from it. Section 3 is devoted to the calculation of the mass spectrum for the S-wave ground state baryons. This is done by calculating the one-gluon-exchange (OGE) energy contributions, to first order in  $\alpha_c$ , following the lines of Amaral and Zagury<sup>10</sup>. In order to make the paper reasonably self-contained, the gluon exchange corrections are presented with some detail for the case of a harmonic potential, a case amenable to an exact, analytical treatment, as mentioned before.

Section 4 describes the calculation of the CM corrections. The parameter fitting of the mass spectrum and of the main baryonic static properties is discussed in section 5. Finally, section 6 will be devoted to the main conclusions.

## 2. THE HARMONIC S+V MODEL

The present model, as applied to ordinary hadrons is a relativistic, independent quark model whose main properties are the following<sup>9</sup>: (i) baryons are color singlets; (ii) the SU(3) flavour group is only broken by taking the mass of the strange quark to be different from that of the non-strange quarks; (iii) quarks are confined, in a first approximation, by a relativistic S+V confining potential which, in the center-of-mass system of the hadron, takes the form of a central potential

$$U(r) = \frac{1}{2} (1+\beta) V(r) ; \quad (2.1)$$

(iv) the mass spectrum obtained is corrected by one-gluon exchange (OGE) among different quarks inside the hadrons, calculated to first order in  $\alpha_c = g^2/4\pi$  where  $g$  is the quark-gluon coupling constant. For mathematical simplicity, the confining potential is taken here to be harmonic

$$V(r) = V_0 + \frac{1}{2} Kr^2 \quad (2.2)$$

Then, each quark in the hadron obeys the Dirac equation

$$[\vec{\alpha} \cdot \vec{p} + \beta m + \frac{1}{2} (1+\beta) V(r)] \psi(\vec{r}) = E \psi(\vec{r}) . \quad (2.3)$$

The S-wave solutions to this equation have the form

$$\psi(\vec{r}) = \begin{pmatrix} \psi_A \\ \psi_B \end{pmatrix} = N \begin{pmatrix} \phi(r) \chi \\ \frac{\vec{\sigma} \cdot \vec{p}}{E+m} \phi(r) \chi \end{pmatrix} , \quad (2.4)$$

where  $\chi$  is a Pauli spinor and  $\phi(r)$  is a normalized eigenfunction of the radial equation

$$\vec{p}^2 \phi_i(r) = (E+m_i) [E - m_i - V(r)] \phi_i(r) \quad (2.5)$$

where  $i$  is the quark-flavor index.

Putting

$$R_i = \left( \frac{2}{x_i K} \right)^{1/4} \quad \text{and} \quad x_i = E + i \quad (2.6)$$

the functions  $\phi(r)$  are given by

$$\phi(r) = \left( \frac{1}{\pi R^2} \right)^{3/4} \exp\left(-\frac{1}{2} \frac{r^2}{R^2}\right) . \quad (2.7)$$

Defining

$$\int \psi_A^\dagger \psi_A d^3\vec{r} = N_A , \quad \int \psi_B^\dagger \psi_B d^3\vec{r} = N_B ,$$

one has, for normalized  $\psi$ 's

$$\int \psi^\dagger \psi d^3\vec{r} = N_A + N_B = 1 .$$

So, it is easily seen that

$$N_A = N^2 , \quad N_B = \frac{3}{2} \frac{N^2}{x^2 R^2} . \quad (2.8)$$

Hence, the normalization constant in eq.(2.4) is given by

$$N^2 = \left[ 1 + \frac{3}{2} \frac{1}{(xR)^2} \right]^{-1} \quad (2.9)$$

For the S-wave quark energy we have

$$E = m + V_0 + \frac{3}{xR^2} . \quad (2.10)$$

The static baryonic properties in the harmonic case have simple forms:

(i) The axial charge  $g_A$  of the nucleon is

$$g_A = \frac{5}{9} (4N_0^2 - 1) , \quad (2.11)$$

(ii) for the rms radius of the proton we have

$$\langle r^2 \rangle_p = \frac{3}{2} N_0^2 \left( R_0^2 + \frac{5}{2x_0^2} \right) \quad (2.12)$$

(iii) and for the proton magnetic moment, in units of nuclear magnetons (n.m.)

$$\mu_p = 2M_p \frac{N_0^2}{x_0} \text{ n.m.} . \quad (2.13)$$

The subscript zero on the r.h.s. of eqs.(2.11)-(2.13) indicates that it refers to non-strange quarks.

For the hyperonic  $\beta$ -decays, the following formula for the axial charge holds

$$g_A = g_A^{\text{SU}(6)} \cdot 2\sqrt{2} \cdot N_0 N_s \left( \frac{R_0 R_s}{R_0^2 + R_s^2} \right)^{3/2} \cdot \left[ 1 - \left( \frac{R_s}{R_0} \right)^4 \frac{1}{x_0^2 (R_0^2 + R_s^2)} \right] , \quad (2.14)$$

where  $g_A^{\text{SU}(6)}$  stands for the well-known matrix element associated with the corresponding decay.

One sees that the model has five free parameters: the ground-state potential  $V_0$ , Hook's spring constant  $K$ , the mass of the non-strange quark  $m_0 (=m_u = m_d)$ , the strange quark mass  $m_s$  and the quark-gluon coupling constant  $a_c$ .

### 3. OGE CORRECTIONS TO THE MASS SPECTRUM

We now discuss the OGE corrections to the mass spectrum of the ground-state hadrons, in the harmonic S+V model.

Following closely ref.(10), we take into account the quarkself-energy terms, which were neglected in previous treatments<sup>3</sup>. This has

the effect of reducing considerably the *gluon-electric contribution* giving rise to a better fitting of the baryonic properties.

The OGE corrections can be written as a sum of a *gluon-electric energy* and a *gluon-magnetic energy*

$$\begin{aligned}\Delta E_E &= \frac{1}{8\pi} \sum_a \sum_{i,j} \int \langle h | \frac{\rho_i^a(\vec{r}) \rho_j^a(\vec{r}')}{|\vec{r}-\vec{r}'|} | h \rangle d^3\vec{r} d^3\vec{r}' , \\ \Delta E_M &= -\frac{1}{8\pi} \sum_a \sum_{i,j} \int \langle h | \frac{\vec{j}_i^a(\vec{r}) \vec{j}_j^a(\vec{r}')}{|\vec{r}-\vec{r}'|} | h \rangle d^3\vec{r} d^3\vec{r}' ,\end{aligned}\quad (3.1)$$

where  $a$  denotes the color index and  $i, j$  are the quark flavor indices.

The densities  $\rho$  and currents  $\vec{j}$  are given by<sup>9</sup>

$$\begin{aligned}\rho_i^a(\vec{r}) &= g \lambda_i^a \frac{N_i^2}{x_i^2} [\vec{x}_i^2 \cdot \vec{\phi}_i^2(r) + \phi_i^{\dagger i}(r)] , \\ \vec{j}_i^a(\vec{r}) &= 2g \lambda_i^a \frac{N_i^2}{x_i^2} \phi_i(r) \phi_i^{\dagger}(r) (\hat{r} \times \vec{\sigma}_i) ,\end{aligned}\quad (3.2)$$

where  $g$  is the quark-gluon coupling constant and  $\lambda_i^a$  are the Gell-Mann matrices of the SU(3) color group.

Summing over the color indices, the integral expressions in eqs. (3.1) can be written as:

(i) *gluon-electric* part

$$\Delta E_E = -\frac{128\pi^2}{3} \alpha_G \langle h | n \sum_{i>j} J_{ij}^E - \sum_i J_{ii}^E | h \rangle \quad (3.3)$$

where  $n$  is equal to 1 for baryons and 2 for mesons, and

$$J_{ij}^E = \frac{N_i^2 N_j^2}{x_i x_j} \int_0^\infty \frac{1}{r^2} F_i(r) F_j(r) dr , \quad (3.4)$$

with

$$F_i(r) = \frac{1}{x_i} \phi_i(r) \phi_i^{\dagger}(r) r^2 + \int_0^r (2E_i - V(r')) \phi_i^2(r') r'^2 dr'$$

(ii) *gluon-magnetic* part

$$\Delta E_M = \frac{256\pi^2}{9} \alpha_c \langle h | n \sum_{i>j} (\vec{\sigma}_i \cdot \vec{\sigma}_j) J_{ij}^M - \sum_i J_{ii}^M | h \rangle \quad (3.5)$$

where

$$J_{ij}^M = \frac{N_i^2 N_j^2}{x_i^2 x_j^2} \int_0^\infty \phi_i^2(r) \phi_j^2(r) r^2 dr \quad (3.6)$$

Performing the integrations in eqs. (3.4) and (3.6), we get the following expressions

$$\begin{aligned} J_{ij}^E = & \frac{1}{8\pi^3} \left( \frac{\pi}{R_i^2 + R_j^2} \right)^{1/2} \left[ 1 + \frac{1}{x_i^2 (R_i^2 + R_j^2)} \left( 1 + \frac{3}{2} \frac{R_i^2}{R_j^2} \right) + \right. \\ & \left. + \frac{1}{x_j^2 (R_i^2 + R_j^2)} \left( 1 + \frac{3}{2} \frac{R_j^2}{R_i^2} \right) + \frac{15}{4} \frac{1}{x_i^2 x_j^2} \frac{1}{(R_i^2 + R_j^2)^2} \left( 1 + \frac{2}{5} \frac{R_i^4 + R_j^4}{R_i^2 R_j^2} \right) \right] \cdot N_i^2 N_j^2 \end{aligned} \quad (3.7)$$

and

$$J_{ij}^M = \frac{1}{4\pi^3} \left[ \frac{\pi}{(R_i^2 + R_j^2)^3} \right]^{1/2} \cdot \frac{1}{x_i x_j} \cdot N_i^2 N_j^2 \quad (3.8)$$

Defining

$$I_{ij}^E = -\frac{128}{3} \pi^2 J_{ij}^E, \quad I_{ij}^M = \frac{256}{9} \pi^2 J_{ij}^M, \quad (3.9)$$

we can write

$$\begin{aligned} \Delta E_M = & \alpha_c (a_{00} I_{00}^M + a_{0s} I_{0s}^M + a_{ss} I_{ss}^M) \quad , \\ \Delta E_E = & \alpha_c (b_{00} I_{00}^E + b_{0s} I_{0s}^E + b_{ss} I_{ss}^E) \quad . \end{aligned} \quad (3.10)$$

In eqs. (3.10), the subscript zero indicates a quark  $u$  or  $d$  and the subscript  $s$ , a strange quark. The coefficients  $a$ 's and  $b$ 's are easily evaluated from eq.(3.3) and eq.(3.5) for the different hadron states  $|h\rangle$ . The results are contained in table 1, which also includes the ground-state meson coefficients, as a complement to the results of ref.(10).

Table I - Coefficients appearing in the calculation of the magnetic and electric OGE energies, eqs. (3.10), for baryons (ref.10) and mesons.

Baryons	$a_{00}$	$a_{0s}$	$a_{ss}$	$b_{00}$	$b_{0s}$	$b_{ss}$
$N$	-12	0	0	0	0	0
$\Lambda$	-9	0	-3	-1	2	-1
$\Sigma$	-5	-4	-3	-1	2	-1
$\Xi$	-3	-4	-5	-1	2	-1
$\Delta$	-6	0	0	0	0	0
$\Sigma^*$	-5	2	-3	-1	2	-1
$\Xi^*$	-3	2	-5	-1	2	-1
$\Omega$	0	0	-6	0	0	0

  

Mesons	$a_{00}$	$a_{0s}$	$a_{ss}$	$b_{00}$	$b_{0s}$	$b_{ss}$
$\rho$	-4	0	0	0	0	0
$\omega$	-4	0	0	0	0	0
$\phi$	0	0	-4	0	0	0
$K$	-3	-6	-3	-1	2	-1
$K^*$	-3	2	-3	-1	2	-1
$\pi$	-12	0	0	0	0	0

With the OGE corrections, the hadron masses will be given by

$$E = E_0 + \Delta E_M + \Delta E_E = E_0 + E_1 \quad (3.11)$$

where  $E_0$  is the sum of the single-particle energies and  $E_1$  is the total OGE correction.



#### 4. THE CM CORRECTIONS

Donoghue and Johnson<sup>12</sup> pointed out, in their analysis of the CM corrections to the bag model, that in an independent quark system the independent particle state is not directly identified with the hadron to which it corresponds, but rather with a wave-packet of the hadron momentum states in which the CM is neither at rest nor in uniform motion but fluctuates about a mean position. Although the average value of the total momentum  $\vec{P}$  satisfies  $\langle \vec{P} \rangle = 0$ , one has  $\langle \vec{P}^2 \rangle \neq 0$ . For hadrons of sufficiently high masses  $m_h$  this gives rise to an energy  $\langle \vec{P}^2 \rangle / 2m_h$ , which, being spurious, must be subtracted from the hadron energy as calculated in eq. (3.11).

This approach, known as the wave-packet method, has been discussed by several workers<sup>12,13</sup>. Here we follow the formulation of the wave-packet method as discussed by Wong<sup>12</sup>, which corresponds to a particular instance of the generator coordinate method well known in nuclear physics<sup>14</sup>.

Accordingly, a single-particle hadron state with center at  $\vec{x}$ ,  $|h(\vec{x})\rangle$ , is expressed as

$$|h(\vec{x})\rangle = \int d^3\vec{P} e^{i\vec{P}\cdot\vec{x}} \frac{\Phi(\vec{P})}{W_h(\vec{P})} |h(\vec{P})\rangle, \quad (4.1)$$

where  $|h(\vec{P})\rangle$  are momentum eigenstates of the hadron  $h$ , normalized as usually

$$\langle h(\vec{P}) | h(\vec{P}') \rangle = (2\pi)^3 \delta(\vec{P}-\vec{P}') W_h(\vec{P}), \quad (4.2)$$

where

$$W_h(\vec{P}) = \begin{cases} 2w_{\vec{P}}, & \text{for mesons.} \\ E_{\vec{P}}/m_B, & \text{for baryons.} \end{cases}$$

The function  $\Phi(\vec{P})$ , the momentum-profile of the wave-packet, is easily seen to be given by

$$\Phi(\vec{P}) = \left[ \frac{1}{(2\pi)^3} W_h(\vec{P}) \tilde{I}(\vec{P}) \right]^{1/2}, \quad (4.3)$$

where

$$\tilde{I}(\vec{P}) = \frac{1}{(2\pi)^3} \int d^3\vec{r} e^{-i\vec{r} \cdot \vec{P}} \langle h(0) | h(\vec{r}) \rangle \quad (4.4)$$

is the Fourier transform of the Hill-Wheeler overlap function  $\langle h(0) | h(\vec{r}) \rangle$ . From eqs. (4.4) and (4.1), we get for the expectation value of a function  $F(\vec{P})$  the expression

$$\langle h(0) | F(\vec{P}) | h(0) \rangle = \int d^3\vec{P} \cdot \tilde{I}(\vec{P}) F(\vec{P}) \quad (4.5)$$

Applying this formalism to the harmonic  $\mathcal{L}+V$  model, we have, for a general configuration with  $n$  constituents

$$\tilde{I}_n(\vec{P}) = \frac{1}{(2\pi)^3} \int d^3\vec{r} \cdot e^{-i\vec{r} \cdot \vec{P}} \prod_{i=1}^n I_{q_i}(r) \quad , \quad (4.6)$$

with

$$I_q(r) \equiv \langle h(0) | h(\vec{r}) \rangle = \int d^3\vec{a} \psi^+(\vec{a}) \psi(\vec{r}+\vec{a}) \quad , \quad (4.7)$$

where  $\psi(\vec{a})$  is given by eq. (2.4), in the present model.

For configurations containing strange and non-strange quarks, we have, for the Hill-Wheeler overlap function

$$I_n = \left( 1 - c_0 \frac{r^2}{R_0^2} \right)^{n_0} \cdot \exp \left( -n_0 \frac{1}{4} \frac{r^2}{R_0^2} \right) + \left( 1 - c_s \frac{r^2}{R_s^2} \right)^{n_s} \cdot \exp \left( -n_s \frac{1}{4} \frac{r^2}{R_s^2} \right) \quad (4.8)$$

where

$$c_0 = \frac{v_0^2}{4+6v_0^2} \quad , \quad v_0 = \frac{1}{x_0 R_0} \quad ; \quad c_s = \frac{v_s^2}{4+6v_s^2} \quad , \quad v_s = \frac{1}{x_s R_s} \quad . \quad (4.9)$$

Here,  $n_0$  denotes the number of non-strange quarks in the hadron and  $n_s$  thenumberof strangequarks, with  $n_0 + n_s = 3$  for baryons. *Mutatis mutandis*, eq. (4.8) is valid also for mesons ( $n_0 + n_s = 2$ ).

For the CM corrections, we calculated eq.(4.5) with  $F(\vec{P}) = \vec{P}^2$  and obtained

$$\langle \vec{p}^2 \rangle_n = \frac{n_0}{R_0^2} \left( \frac{5}{2} - N_0^2 \right) + \frac{n_s}{R_s^2} \left( \frac{5}{2} - N_s^2 \right) . \quad (4.10)$$

In computing the CM corrections, we followed the procedure advocated by Bartelski et al.<sup>13</sup>, who have shown that the assumption  $\langle f(\vec{v}^2) \rangle = f(\langle \vec{v}^2 \rangle)$ , where  $\vec{v}^2 = \vec{p}^2/E^2$ , is a better approximation than that corresponding to expanding in  $\langle \vec{p}^2 \rangle/M^2$ . Hence, we took

$$E = \langle (M^2 + \vec{p}^2)^{1/2} \rangle \approx (M^2 + \langle \vec{p}^2 \rangle)^{1/2} ,$$

instead of  $E \approx M + \frac{\langle \vec{p}^2 \rangle}{2M}$ , so that for the CM-corrected masses, one has

$$M = [(E_0 + E_1)^2 - \langle \vec{p}^2 \rangle]^{1/2} , \quad (4.11)$$

with  $\langle \vec{p}^2 \rangle$  given by eq. (4.10).

Finally, our analysis of the CM corrections to the static properties of the ground-state baryons is based on the following expressions<sup>13</sup>

$$g_A = g_A^{\text{static}} \left( 1 + \frac{1}{3} \frac{\langle \vec{p}^2 \rangle}{M^2} \right) , \quad (4.12-a)$$

$$\langle r^2 \rangle = \left[ \langle r^2 \rangle^{\text{st.}} - Q \frac{9}{4 \langle \vec{p}^2 \rangle} \right] \left( 1 + \frac{1}{3} \frac{\langle \vec{p}^2 \rangle}{M^2} \right) , \quad (4.12-b)$$

$$\mu = \mu^{\text{st.}} \left( 1 + \frac{1}{2} \frac{\langle \vec{p}^2 \rangle}{M^2} \right) + Q \frac{\langle \vec{p}^2 \rangle}{6M^2} , \quad (4.12-c)$$

where  $Q$  is the baryon charge and  $M$  the mass. The expressions for  $g_A^{\text{st.}}$ ,  $\langle r^2 \rangle^{\text{st.}}$ , and  $\mu^{\text{st.}}$  are given in eqs. (2.14), (2.12) and (2.13), respectively.

## 5. NUMERICAL RESULTS

We shall now discuss the fitting of the five free parameters of the model. The fitting was made by taking as input the mass of the nucleon,  $A$  and  $R$ , the axial charge of the neutron  $g_A(n)$  and the rms radius of the proton  $\langle r_p^2 \rangle$ . For this, use was made of eqs. (4.11), (4.12-a) and (4.12-b). The results obtained for the baryon masses are

displayed in table 2, where the corresponding results of Amaral and Zagury<sup>10</sup>, without CM corrections, are also indicated for comparison.

Table 2 - Masses obtained for the low-lying S-wave baryons. (in MeV).

Baryons	(Ref.10)	Fitted masses	Experiment (ref.15)
N	INPUT	INPUT	938
$\Lambda$	INPUT	1114.6	1116
$\Sigma$	1150	1141.8	1193
$\Xi$	1304	1292.5	1318
$\Delta$	INPUT	INPUT	1232
$\Sigma^*$	1393	1387.2	1385
$\Xi^*$	1547	1533.7	1530
$\Omega$	1693	INPUT	1672

Although the agreement between our results and the experimental values is very good for the cases of the  $\Lambda$ ,  $\Sigma^*$  and  $\Sigma^*$ , our fitting is a little worse than that of ref.(10) for the  $\Sigma$  and  $\Xi$  particles. If we define a mean-square deviation by

$$\langle \delta M \rangle = \left[ \frac{\sum (M_B - M_{\text{exp.}})^2}{N} \right]^{1/2}, \quad (5.1)$$

where  $N$  is the number of fitted masses, we get for our fitting  $\langle \delta M \rangle = 25.7$  MeV, to be compared with the value 23.8 MeV corresponding to the fitting of ref.(10), without CM corrections.

However, the numerical values of the parameters found in our fitting are much more pleasant: in ref. (10), they were:  $m_{\bar{u}} = 38$  MeV,  $m_s = 211$  MeV,  $V_0 = 219$  MeV,  $K = 8.10^6$  MeV<sup>3</sup> and  $\chi_C = 1.586$ . In the present fitting, we obtained:  $m_{\bar{u}} = 27.8$  MeV,  $m_s = 229.1$  MeV,  $V_0 = 110.3$  MeV,  $K = 21.4.10^6$  MeV<sup>3</sup> and  $\alpha = 0.82$

These results show that by including the CM correction, we obtained a decrease in the mass of the  $u$  quark. Besides, a more satisfactory value for the coupling constant  $a_c$  was obtained.

By using the same parameters as above, we have also calculated the CM-corrected mass spectrum of the S-wave mesons. With exception of the pion, we have found sensible results, which are listed in table 3.

Table 3 - Masses of the low-lying S-wave mesons, obtained with the same parameters of the baryonic fitting. (in MeV).

Mesons	Fitted masses	Experiment (ref.15)
$\rho$	763.1	770
$\omega$	763.1	783
$\phi$	1065.1	1020
$K$	560.8	497
$K^*$	923.4	892
$\pi$	267.9	135

In tables 4 and 5, we list the results obtained for the magnetic moments of the octet baryons and for the axial vector constants  $g_A$ , occurring in the  $\beta$  decays of the baryonic  $\frac{1}{2}^+$  octet, as calculated by eqs. (4.12-c) and (4.12-a), respectively.

We note that in our fitting  $\langle r^2 \rangle_p$  is an input quantity whereas the proton magnetic moment  $\mu_p$  is predicted to be equal to 3.05 n.m., which is 9% higher than the empirical value. On the other hand, in the fitting without CM corrections<sup>10</sup>, the proton magnetic moment was taken as input and a value  $\langle r^2 \rangle_p^{1/2} = 1.196$  fm was predicted for the rms radius of the proton, while the experimental value is 0.877 fm. Besides, our magnetic moment results show a mean-square deviation of the experimental

Table 4 - Magnetic Moments of the  $\frac{1}{2}^+$  octet baryons in nuclear magnetons (n.m.).

Baryons	(Ref. 10)	Fitted $\mu$	Experiment (ref. 15)
$p$	INPUT	3.05	2.7928
$n$	-1.86	-1.98	-1.9130
$\Lambda$	-0.66	-0.69	$-0.613 \pm 0.004$
$\Sigma^+$	2.70	2.78	$2.379 \pm 0.20$
$\Sigma^0$	0.84	0.85	$0.46 \pm 0.28$
$\Sigma^-$	-1.02	-1.08	$-1.10 \pm 0.05$
$\Xi^0$	-1.50	-1.50	$-1.250 \pm 0.014$
$\Xi^-$	-0.57	-0.64	(ref. 16) $-0.69 \pm 0.04$
$(\Sigma^0, \Lambda)$		-1.62	(ref. 17) $-1.82^{+0.18}_{-0.25}$

values equal to 0.23 n.m., which is similar of ref. (10), where  $\langle \delta\mu \rangle = 0.22$  n.m. .

We finally point out that our fitting differs considerably from that of Barik, Dash and Das<sup>11</sup>, although both papers refer to the harmonic  $S+V$  model and indeed make use of the same type of CM corrections. One reason for this is that those authors did not include the baryonic mass spectrum in their analysis. Further, they considered their model simply as a model of the baryonic quark-core and not as a model of the baryons properly. As far as a complete model of baryons has to include the contribution of a pionic cloud, both fittings may be considered, in our view, as independent indications of the viability of the harmonic  $S+V$  model and of the relevance of CM corrections.

Table 5 - Axial vector constants  $g_A$  for the  $\beta$ -decay of the  $\frac{1}{2}^+$  octet baryons.

Decay	(Ref.10)	Fitted $g_A$	Experiment (ref.15)
$n \rightarrow p \ e^- \bar{\nu}$	INPUT	INPUT	$1.254 \pm 0.006$
$\Lambda \rightarrow n \ e^- \bar{\nu}$	0.78	0.801	$0.694 \pm 0.025$
$\Sigma^- \rightarrow n \ e^- \bar{\nu}$	-0.26	-0.265	$\pm 0.372 \pm 0.050$
$\Xi^- \rightarrow \Lambda \ e^- \bar{\nu}$	0.26	0.174	$0.25 \pm 0.05$
$\Xi^- \rightarrow \Sigma^0 \ e^- \bar{\nu}$	1.3	1.304	
$\Xi^0 \rightarrow \Sigma^+ \ e^- \bar{\nu}$		1.304	

## 6. CONCLUSIONS

The harmonic S+V relativistic quark model is characterized, in its original form, by a number of interesting properties and by a great mathematical simplicity. As an independent quark model, however, it requires the remedy of the center-of-mass-corrections. We have shown that, as far as the mass spectrum and the static properties of the ground-state baryons are concerned, this can be done in a satisfactory way by applying the method of refs. (12) and (13).

Our fitting of the baryonic spectrum corresponds to an average accuracy  $\langle \delta M \rangle = 25.7$  MeV, which is only slightly worse than that without CM corrections (23.8 MeV). However, it yields a more satisfactory set of parameters, showing a welcome decrease of the coupling constant  $\alpha_c$  and also of the non-strange quark-mass.

It would be worthwhile to extend the present analysis to include, for instance, the first radially excited positive parity baryons<sup>18</sup>.

An important extension of the model is concerned with the introduction of a pionic cloud<sup>19</sup> surrounding a quark-core, a relevant

feature inspired by QCD. Nevertheless, the contribution of the pion cloud modifies the model substantially and requires a detailed and separate treatment. A preliminary account of this extension, however, seems to indicate promising results<sup>20</sup>.

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#### Resumo

Neste trabalho são discutidas as correções de centro-de-massa para o espectro de massas e propriedades estáticas dos bárions de estados fundamentais em ondas-S, no contexto de um modelo relativístico a quarks independentes, baseado em uma equação de Dirac com um potencial confinante do tipo harmônico e com partes escalar e vetorial igualmente misturadas. Obtém-se um ajuste mais satisfatório dos parâmetros envolvidos do que em tratamentos anteriores nos quais as correções de centro-de-massa não foram aplicadas.