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I.R. and F.I.R. Laser Polarimetry as a Diagnostic Tool in High-β and Tokamak Plasmas

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Abstract The change of the polarization state of an electromagnetic wave (E.M.W.) propagating across a magnetized plasma may be used to determine plasma parameters. In a plasma machine of the Tokamak type, the Faraday rotation of the E.M.W. allows for the determination of the product of the plasma electronic density by the poloidal magnetic field. In this paper we propose a novel optical configuration which permits simultaneous measurements of these two parameters without the use of an auxiliary interferometric set up. By choosing appropriate laser wavelength this method can be used in Tokamaks ($\lambda \gtrsim 1mm$) and also in θ-Pinches plasmas ($\lambda \approx 10\mu m$). We discuss the application of these results to plasma machines now in operation in Brazil, like the Tokamak/USP and θ-Pinch/UNICAMP, using lasers developed at UNICAMP.

1. INTRODUCTION

Electromagnetic waves (E.M.W.) propagating through an anisotropic, birefringent and optically active plasma medium may be used to determined the plasma electron density and the internal magnetic field strength without perturbing the plasma by analysing the change on the polarization state of the propagating electromagnetic waves[1,2,3].

We will briefly review this diagnosis method, following the work in Tokamaks by Ascoli et *al*[1] and De Marco and Segre[2], and extend the work to the linear θ-Pinch systems.

We also propose, an optical apparatus where the simultaneous measurements of the electron density and the poloidal magnetic field are possible both in a Tokamak and in a θ-Pinch. The Infrared (I.R.) and Far Infrared (F.I.R.) Lasers needed for these measurements were built at our Institute and diagnosis parameters are analysed in two machines, the Tokamak and the θ-Pinch in operation at USP and UNICAMP, respectively.

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2. ELECTROMAGNETIC WAVE POLARIZATION STATE EVOLUTION THROUGH AN ANISOTROPIC, BIREFRINGENT AND OPTICALLY ACTIVE MEDIUM

The evolution of the polarization state of an E.M.W. can be caracterized completely by the two parameters represented in fig. 1, Y (the azimuth) and χ (related to eccentricity E) where

$$0 \leqslant \Psi \leqslant \pi \quad ; \quad \chi = \text{arc tan } \varepsilon \ ,$$

with (1)

$$-\frac{\pi}{4} \leqslant \chi \leqslant \frac{\pi}{4} \quad \text{and} \quad \varepsilon = \pm \frac{b}{a} \ .$$

and the + or - is associated with the counter clockwise or clockwise rotation.

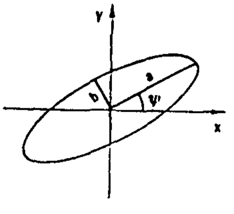

Fig.1 - The polarization ellipse.

In the case where h (E.M.w. wavelength) is much smaller than the characteristic lengths of the medium, each propagation direction can be associated with two E.M.W. with different phase velocities and orthogonal polarization vectors[g], named ordinary and extraordinary waves. These waves possess phase velocities c/μ_1 and c/μ_2, where μ_1 and μ_2 are locally defined refraction indices with μ_1 > p2. These indices depend on z, the direction of propagation of the E.M.W.. For a typical plasma lenght L, these two waves will have different phase veloçities which leads to a phase shift $\Delta\phi$ given by

$$\Delta\phi = \int_0^L \frac{\omega}{c} \Delta\mu_{1,2} \, dz$$

The two waves are then recombined to give a wave whose polarization state is $\Delta\phi$ dependent. In order to represent the evolution of this polarization state, the Poincaré sphere[4,5] (fig. 2) is used. This

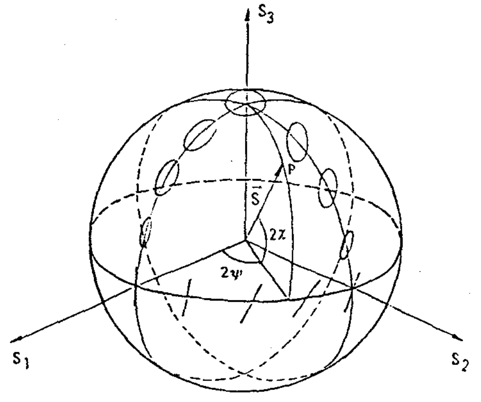

Fig. 2 - Poincarê sphere representation of polarization.

is a sphere of unit radius where each state of polarization is uniquely represented by a point P on the surface whose latitude and longitude are 2χ an 2Ψ (fig. 2). The evolution of the polarization during propagation may be represented on the Poincarē sphere by infinitesirnal rotations about an axis passing through the points representing the characteristic polarizations, the angle of rotation being equal to A$. Thus the evolution of the polarization may be represented by

$$\frac{dS}{dz} = \vec{\Omega}(z) \times \vec{S}(z) \tag{2}$$

where

$$\vec{S} = \begin{cases} S_1 = \cos 2\chi \cos 2\Psi \\ S_2 = \cos 2\chi \text{ sen } 2\Psi \\ S_3 = \text{sen } 2\chi \end{cases} \tag{3}$$

is the polarization vector in terms of Stoke's parameters S_1, S_2 and S_3 which are the cartesian coordinates of the poínt P in figure 2. The vector $\vec{R}$ has rnodulus

$$|\vec{\Omega}| = \frac{d(\Delta\phi)}{dz} \tag{4}$$

and the direction of the fastest wave, $\hat{S}_{\mu 2}$:

$$\vec{\Omega}(z) = \frac{\omega}{c} \Delta\mu_{1,2}\ \hat{S}_{\mu 2} \tag{5}$$

From eqs. 1 and 3 we get:

$$\Psi = 0.5 \text{ arc tan } \left(\frac{S_2}{S_1}\right) \tag{6}$$

and

$$\varepsilon = \tan\chi = \frac{|S_3|}{1 + (1 - S_3^2)^{1/2}} \tag{7}$$

Let $\vec{S}_0 = \vec{S}(0)$ be the initial polarization vector: Solving eq. (2) up to second order in $\vec{\Omega}$, we have[6]:

$$\vec{S}(z) = \vec{S}_0 + \int_0^z \vec{\Omega}(z')\, dz' \times \vec{S}_0 + \int_0^z \vec{\Omega}(z')\, dz' \times \left[\int_0^{z'} \vec{\Omega}(z'')\, dz'' \times \vec{S}_0\right]$$

or

$$\vec{S}(z) = \vec{S}_0 + \vec{W}(z) \times \vec{S}_0 + \vec{W}(z) \times \vec{W}(z') \times \vec{S}_0 \tag{8}$$

where

$$\vec{W}(z) = \int_0^z \vec{\Omega}(z')\, dz' \quad .$$

3. WAVE PROPAGATION IN A NONUNIFORM PLASMA WITH MAGNETIC FIELD

So far, we have assumed that the E.M. wavelength should be much smaller than the characteristic lengths of the plasma, as for example, the scale length of the plasma density gradient. Nevertheless, for many applications, it is also true that the E.M.W. frequency ω is much larger than the plasma frequency ω_p ($\omega^2 >> \omega_p^2$), giving, therefore $\Delta\mu_{12} << 1$ and $\vec{W}(z) << 1$. Furthermore, if for simplicity we ignore, collisions, ion mouvements and thermal effects, we can consider thei-esults of an electromagnetic wave propagating through the electron dominant plasma[7] to find $\vec{\Omega}(z)$[6]:

$$\vec{\Omega}(z) = \frac{\omega_p^2}{2c\omega^3} \begin{cases} \left(\frac{e}{mc}\right)^2 (B_y^2 - B_x^2) \\ -2 \left(\frac{e}{mc}\right)^2 B_x B_y \\ 2\omega \left(\frac{e}{mc}\right) B_z \end{cases} \tag{9}$$

where B_x, B_y and B_z are the components of the magnetic field $\vec{B}$. From eq. 8 we get:[8]

$$\vec{S}(z) = \begin{cases} S_1 = S_{01} - W_3 S_{02} - \frac{W_3^2}{2} S_{01} + W_2 S_{03} \\ S_2 = S_{02} + W_3 S_{01} - W_2 S_{03} - \frac{W_3^2}{2} S_{02} - \frac{W_1^2}{2} S_{02} \\ S_3 = S_{03} + W_1 S_{02} - \frac{W_1^2}{2} S_{03} - \frac{W_2^2}{2} S_{03} \end{cases} \qquad (10)$$

where

$$W_i = \int_0^z \Omega_i(z')\, dz'$$

Equations (10) give us the output values of $\vec{S}$ as a function of the input parameter $\vec{S}_0$ and the plasma parameter of the medium.

4. ELECTRON DENSITY AND MAGNETIC FIELD MEASUREMENTS USING THE POLARIMETRY

4a -

Consider, in the toroidal configuration, the wave propagating through a vertical cord, and the y axis in the toroidal direction[9]. Assume that the plasma electron density and the current density dependent on $r = (x^2+z^2)^{1/2}$ in such a way that the poloidal magnetic field possess a purely azimuthal component. In the cylindrical approximation it can be written as[10]:

$$B_p(r) = B_I\, b(r) \qquad (11)$$

where $B_I = 2I_T/ac$ is the poloidal magnetic field at the outer radius of the plasma, and I_T the poloidal plasma current. Then the following expressions can be written:

$$n(z) = n_0 f(z)$$

$$B = \begin{cases} B_x = B_I\, b(r)\, \frac{z}{r} = B_I\, g(z) \\ B_y = B_T \\ B_z = B_I\, b(r)\, \frac{x}{r} = B_I\, h(z) \end{cases} \qquad (12)$$

where $b(r)$ and $f(z)$ are the profile function of the B field and the density, respectively.

Since for the Tokamak configurations $f(z)$ and $h(z)$ are even functions of (z), and $g(z)$ is an odd function of (z), we can find the expressions for $\vec{W}(z)$ as:

$$W(z) = \begin{cases} W_1 \simeq M \int_{0}^{z_0} dz\, f(z) \\ W_2 = 0 \\ W_3 \simeq P \int_{-z_0}^{z_0} dz\, f(z)\, h(z) \end{cases} \tag{13}$$

or

$$W(z) = \begin{cases} W_1 \simeq \frac{M}{n_0} \int_{-z_0}^{z_0} dz\, n(z) \\ W_2 = 0 \\ W_3 \simeq \frac{P}{n_0 B_I} \int_{-z_0}^{z_0} dz\, n(z)\, B_z(z) \end{cases} \tag{14}$$

with

$$M = \frac{\omega}{2c} \frac{\omega_{p0}^2}{\omega^4} \left(\frac{e}{mc}\right)^2 B_T^2 = 2.46 \times 10^{-21}\, n_0\, \lambda^3\, B_T^2 \tag{15}$$

and

$$P = \frac{\omega}{c} \frac{\omega_{p0}^2}{\omega^3} \frac{e}{mc} B_I = 5.27 \times 10^{-17}\, n_0\, \lambda^2\, B_I \tag{16}$$

where CGS units were used.

We can see from the above eqs. that W_1 behaves as λ^3 whereas W_3 behaves as λ^2, suggesting that the wavelength of the incident beam should be as long as possible, but keeping in mind that $\omega^2 \gg \omega_p^2$. From these considerations, in the case of the Tokamaks ($n \sim 10^{14}$ cm^{-3}) the beam wavelenght should be on F.I.R. region. Furthemore, we should point out that from the measurements of W_1 we can obtain the electron density, $n(z)$, and from W_3 the product $n(z)\ B_z(z)$, which gives the simultaneous measurements of densíty and poloidal magnetic field.

However, so far, as to our knowledge, these measurements have been done using only the independent measurements of W_3 or W_1 and using auxiliary interferometer techniques[11-14]. The simultaneous measurements of W_1 and W_3 has been suggested by Segre[15] using a high resolution ellipsometer and electro-optical system to induce the necessary external birefringency to measure the eccentricity. Meanwhile, this kind of measurement involves high quality optical equipment (polarizers, quarter wave plates, ferrite and detector), which is not easily obtained at the F.I.R. region. We propose here a way to overcome these requirements using a much simpler optical set up and less sophisticated equipment.

4b - Optical Set up and Measurements

The key question for simultaneous measurements of density and magnetic fields is the determination of the variables W_1 and W_3. It has been shown that under certain conditions of the input polarization vector, the plasma behaves either as a birefringent medium, for which the eccentricity changes without changing the azimuth angle (Cotton--Motton Effect), or as an optically active medium, for which the azimuth angle changes without a change in the eccentricity (Faraday effect)[16].

However, if we define the input polarization state in such a way as to define the medium as birefringent and optically active, we can measure W_1 and W_3 simultaneously. The optical set up proposed for this is:

1 - Define the input wave polarization state

$$\vec{S}_0 = (0,\ \text{sen } 2\Psi,\ 0) \tag{17}$$

where $\psi = \omega_r t$, which implies on a rotating polarizer with ω_r as a modulation frequency which in turn can give the resolution of the polarimeter.

2 - From equation (10) and using $B_I << B_T$, the output wave polarization state is

$$\vec{S} \approx (-\,W_3\ \text{sen } 2\Psi,\ (1 - \frac{W_1^2}{2})\ \text{sen } 2Y,\ W_1\ \text{sen } 2Y) \tag{18}$$

At this point we can use a beam splitter in order to get two beams which can be coupled to two polarizers

a) Setting the first polarizer to be paralel to B_T or

$$\vec{P}_1 = (1,0,0) \ , \tag{19}$$

b) and the second at 45^o with respect to B_T or

$$\vec{P}_2 = (0,1,0) \ , \tag{20}$$

the output in this case will be:

a) from the first polarizer:

$$\frac{2I}{I_0} = 1 - W_3 \text{ sen } 2\ \omega_r t \tag{21}$$

which gives us information about the product $n(z)B_z(z)$;

b) from the second polarizer:

$$\frac{2I}{I_0} = 1 + \left(1 - \frac{W_1^2}{2}\right) \text{ sen } 2\ \omega_r t, \tag{22}$$

which gives us information about the density $n(z)$.

This configuration is shown in figure 3 and can be analyzed in a multichannel system.

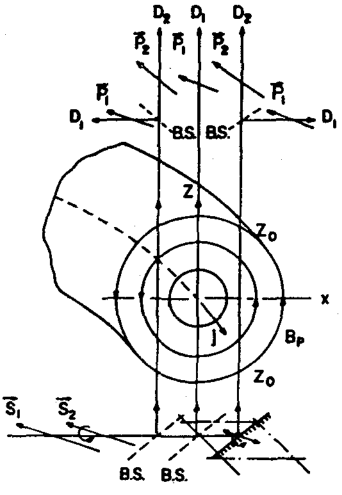

Fig.3 - Multichannel system configuration for simultaneous measurements of electronic density and poloidal magnetic field in Tokamaks by the polarimetric technique.

For the specific application on USP's Tokamak with propagation along the diameter (L = 16cm, $n \sim 10^{12} \rightarrow 10^{13}$ cm^{-3}, $B_T = 5.0$ KG), and using the FIR laser line with $\lambda = 3030.0\mu m$ of a CD_3OH laser optically pumped by a CO_2 laser (both built by UNICAMP's laser group[17]), will result in an equivalent rotating angle of -8.7°.

4c - For the case of UNICAMP's 8-Pinch

In this case we assume that the magnetic field in the z-direction is:

$$\vec{B} = (0,0,B\ ) \tag{23}$$

which from eq. (9) as gives the value of $\vec{\Omega}(z)$:

$$\vec{\Omega}(z) = (0,0,\Omega_3(z)) = \frac{\omega_p^2}{c\omega^2}\frac{e}{mc} B_0 \tag{24}$$

or

$$\Omega_3(z) = 5.27 \times 10^{-17} \lambda^2 n(z) B_0 \ . \tag{25}$$

4d - Opticat Set up and Measurements

If the input light is linearly polarized at 90° with respect to B_z, i.e., parallel to the Y axis, for example let the input polarization state vector be given by $\vec{S}_0 = (1,0,0)$, then the output polarization vector (using $\vec{W}(z) \ll 1$) will be given by

$$\vec{S}(z) = (1,W_3,0) \ . \tag{26}$$

From eq. (6) we have defined the value of W_3 which, in this specific case, is a purely Faraday rotation. This angular rotation of the polarization state is given by:

$$\Psi = (\frac{5.27}{2}) \times 10^{-17} \lambda^2 \int_0^L n B_0 \, dL \tag{27}$$

This result is the same found by Turner et al[18] with neither the infinitesimal rotation approximation nor the Poincaré sphere representation. For the polarimetric measurement, we place at the end of 8-Pinch, a polarizer perpendicular to the input polarizer. This optical set up is shown in fig. 4. Using the usual formula of two crossed polarizers we find:

$$I = I_0 \operatorname{sen}^2 \psi$$

where the output intensity (I) is given in terms of the input intensity (I_0) and of the rotation angle (ψ).

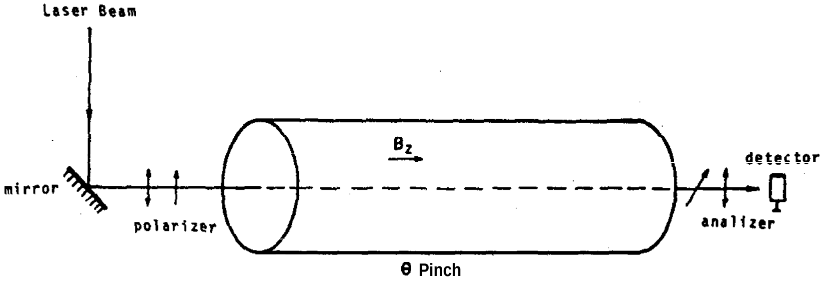

Fig.4 - Optical configuration for polarimetric measurements of electronic density in θ-Pinches.

The output intensity versus electronic density is plotted in figure 5 for the plasma parameters of the θ-Pinch, UNICAMP (B_0 = 8, 10, 12 kG; L = 100 cm) for three wavelenghts: λ = 10.6μm (CO_2 laser line), λ = 75.6μm (CH_3OH laser line) and λ = 118.8μm (CH_3OH laser line).

REFERENCES

1. V.Ascoli-Bartoli, W.Bozzolan, M.Grolli and L.Pieroni, Int. Rep, L. GI/R71/11/1; Lab. Gas Ionizado, Frascati Rome, Italy.
2. F.de Marco and S.E.Segre, Plasma Phys. 14, 245 (1972).
3. A.D.Craig, Plasma Phys. *18*, 777 (1976).
4. G.N.Ramachandran and S.Ramaseschan, Cristal Optics Encyclopedia of Physics vol 25/1, Springer-Verlag Berlin (1961).
5. M.Born and E.Wolf, *Principles of Optics*, Pergamon-Press (1964).
6. S.E.Segre, Plasma Phys. 20, 295 (1978).
7. M.A.Heald and Wharton, *Plasma Diagnostics with Microwaves*, Wiley, N. Y. (1965).
8. S.E.Segre, Int.Rep. 22-19, ENEA-CENTRO, Frascati, Rome, Italy (1982).

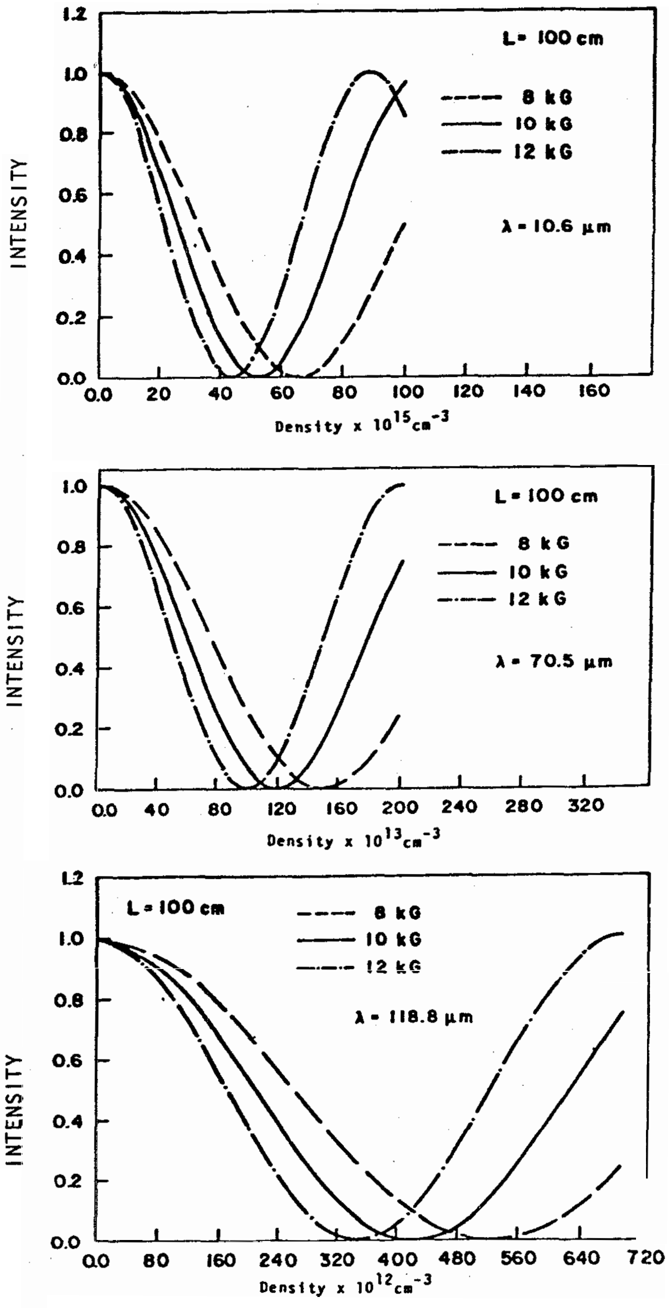

Fig.5 - Outuput intensity versus electronic density for the UNICAMP-6-Pinch parameters.

9. J.H.Vuolo and R.M.O.Galvão had developed a formulation for polarimetric measurements analyzing the case of horizontal propagation in Yaxis (toroidal direction) to determine the electron density using the Faraday rotation effect. Revista Brasileira de Física *22,* 912 (1982).
10. L.A.Artsimovich, Nuclear Fusion 12, 215 (1972).
11. W.Kunz, Equipe TFR, Nuclear Fusion, 18, 1729 (1978).
12. C.H.Ma, P.D.Hutchinson and K.L.Vander Sluiz, Appl.Phys.Lett. *34*, 218 (1979).
13. C.H.Ma, P.D.Hutchinson, P.A.Staats and K.L.Vander Sluiz, Int.Journal of Infrared and Milimeter Waves 3, 2, 263 (1982).
14. M.Grolli and G.Madalluno, Nuclear Fusion 22, 7, 961 (1982).
15. S.R.Segre, S.Opt.Soc.Am., 72, 1, 167 (1982).
16. J.H.Vuolo and R.M.O.Galvão, Int.Rep., IEAv, 013/82 (1982).
17. D.Pereira and A.Scalabrin, to be published in the "International J. of Infrared and Millimeter Waves".
18. R.Turner and T.O.Poehler, The Phys. of Fluids *13*, 4 1072 (1970).

Resumo

A mudança do estado de polarização de uma onda eletromagnética propagando-se através de um plasma magnetizado pode ser utilizada para a determinação de parâmetros desse. Num Tokamak, a rotação de Faraday permite a determinação do produto densidade eletrônica por campo magnético poloidal. Propomos aqui, uma apropriada configuração Õptics, que permite a medida desses parâmetros independente e simultaneamente, sem o uso de um interferômetro auxiliar. Para nosso caso, medidas podem ser feitas tanto em Tokamaks como em 8-Pinches, utilizando-se Lasers no IVL ($\lambda > 1$mm) no IV ($h \sim 10.0 \mu m$) respectivamente. Analiza-se a viabilidade dessas medidas, utilizando-se parãmetros típicos das máquinas em operação no Brasil (Tokamak-USP e θ-Pinch-UNICAMP), e lasers construídos no Grupo de Lasers e Aplicações da UNICAMP.