Revista Brasileira de Física, Vol. 15, n? 4, 1985

The Hall Effect in a Unipolar Inductor: A Possible Dynamo or Antidynamo

H. HEINTZMANN

Institut für Theoretische Physik der Universität, 5 Köln 41, FRG

and

M. NOVELLO

Centro Brasileiro de Pesquisas Física, Rua Dr. Xavier Sigaud 150, 22290, Rio de Janeiro, RJ, Brasil

Recebido em 04 de outubro de 1985

Abmact The Hall effect in a magnetized disc is considered. If the disc is used as a unipolar inductor the original magnetic field decreases whereas if a current of appropriate direction is driven through a rotating disc the magnetic field increases whithout limit in our simple analysis for constant current I. The effects should be testable for laboratory sized objects and may be of crucial importance in pulsars. The magnitude of the relative change of the magnetic field is of order βN for pulsars. $\beta = \Omega R/c$ is the equatorial velocity of the pulsar in units of the velocity of light and N is the dimensionless Hall constant $N = e\,B\tau/mc$. Under favourable conditions $\delta B/B = 10^{-5}$ for laboratory sized objects and for pulsars BN = 1.

1. INTRODUCTION

It is well known that the magnetic field in an ordinary disc dynamo is amplified if the inequality

$$M\Omega/R > 2\pi$$

is satisfied, where M is the mutual inductance between the solenoid and the disk, R is its angular velocity, and R is the Ohmic resistance of the circuit. If the inequality is not satisfied, the current will decay as soon as the initially given magnetic field has disappeared.

The disk dynamo depends on the special arrangement of wires and it has remained doubtful, for a long time, whether a dynamo process can occur in a singly connected domain of nearly uniform electrical conductivity, such as that represented by the Earth's core. These doubts have been dispelled by the work of different authors who demonstrated in a mathematically rigorous way that the so-called "homogeneous dynamo" is indeed possible. Since then, a large number of solutions have been obtained, and it is generally believed nowadays that almost all velocity fields are capable of generating magnetic fields in a singly connected body of fluid if the rnagnetic Reynolds number R_m is sufficiently high. R_m is defined in analogy to the ordinary Reynolds number except that the kinematic viscosity is replaced by the magnetic diffusivity $\lambda = (\mu\sigma)^{-1}$, where o is the electric conductivity and μ is the magnetic permeability.

Here we shall pursue the alternative idea that in the case of pulsars, i.e. neutron stars, the Hall-effect may play a crucial part in determining the exterior magnetic field.

2. THE HALL EFFECT IN A MAGNETIZED, NONROTATING DISC

Consider a magnetized disc with inner radius R_i and outer radius R_a. Let σ be the conductivity of the disc, I the total current which is driven through the disc (from R_a to R_i) and $\vec{N} = e\tau\vec{B}/mc$, where τ is related to σ by the well known relation[1] $\sigma = ne^2\tau/m$. Ohm's law[1,2] reads:

$$\vec{j} + \vec{N}\times\vec{j} = \sigma\vec{E} \tag{1a}$$

or in tensor notation

$$\sigma^{-1}_{ab}\, j_b = E_a \quad ;$$

$$\sigma_{ab} = \frac{1}{1+N^2}\,(\delta_{ab} + N_a N_b + e_{abc}\, N_c) \tag{1b}$$

with the inverse

$$j_a = \sigma_{ab}\, E_b \quad . \tag{1c}$$

We use the summation convention and give, for later use, eq (1c) also in vector notation:

$$\vec{j} = \overleftrightarrow{\sigma}\,\vec{E} = \sigma_\perp\,\vec{E} + (\sigma_\parallel-\sigma_\perp)\,(\vec{b}\vec{E})\,\vec{b} + \sigma_\perp\,\vec{E}\times\vec{N} \tag{1d}$$

where b is the unit vector in the direction of B and

$$\sigma_\perp = \frac{\sigma}{1+N^2} \quad ; \quad \sigma_\parallel = \sigma \ .$$

If we apply a potential difference $\Delta\Phi$ between the innei- and outer, rim, the eletric field in the disc will be given by $\vec{E} = -\vec{\nabla}\Phi$, div $E=0$ so that we have

$$\vec{E} = \frac{\Delta\phi}{\rho \log \frac{R_a}{R_i}} \vec{e}_\rho .$$

We use cilindrical coordinates (ρ,ϕ,z) in the disc and $\vec{e}_\rho$, $\vec{e}_\phi$, $\vec{e}_z$ are the unit vectors in (ρ,ϕ,z) direction respectively.

In the following consideration it is convenient to consider a disc of finite height $h \ll R_i$. In order to determined the energy dissipated in the disc we evaluate $W = \int jE.dV$ and obtain the following three equivalent expressions:

$$W = I\Delta\phi = \sigma \frac{2\pi h}{1+N^2} \frac{(\Delta\Phi)^2}{\log \frac{R_a}{R_i}} = \frac{1+N^2}{2\pi\sigma h} (\log \frac{R_a}{R_i}) I^2 \tag{2}$$

whence we deduce by means of $\Delta\Phi = \hat{R}I$ that the resistance $\hat{R}$ is

$$\hat{R} = \frac{1+N^2}{2\pi\sigma h} \log \frac{R_a}{R_i} . \tag{3}$$

Note that $I = 2\pi h\, j_\rho(a)\, R_a = 2\pi h\, j_\rho(i)\, R_i$. Due to the Hall effect the dissipation increases by a factor $1+N^2$ for a fixed current I and decreases by the same factor for a fixed potential difference $\Delta\Phi$. The reason for that due to the magnetic field the current I spirals from R_a to R_i in N turns. The current density j_ϕ, which is just $-N j_\rho$, according to eq. (1b), will however generate an extra magnetic field B_z in the disc, which is of order $\delta B_z = -NI/cR_a$ (see below). Depending on the direction of the current I the existing magnetic field will either increase of decrease. If we keep I fixed (which implies that the Voltage $\Delta\Phi$ increases by $1+N^2$!) we can arrive at very large magnetic fields without using any coils. As a matter of fast, in our simple analysis, which ignores any dependence of τ on B^2 and the mechanical magnetic stresses exerted on the disc[3], infinite magnetic fields are possible for a finite current. The additional magnetic field can be calculated by standard techniques[4] introducing the Green's dyadic in polar coordinates (r,θ,ϕ). Setting $\delta\vec{B} = \text{rot}\,\delta\vec{A}$ and $\text{div}\,\delta\vec{A} = 0$, we obtain

$$\text{rot rot}\,\delta\vec{A} = -\Delta\delta\vec{A} = \frac{4\pi}{c} j_\phi = \frac{4\pi}{c} (-I.N \frac{1}{2\pi\delta^2} \delta(\Theta - \frac{\pi}{2}) \tag{4a}$$

which gives the following for $r \geq R_a$:

$$\delta A_\phi (r,\theta) = - \frac{NI}{cR_n} \sum_{n=1}^{\infty} \alpha_n \left(\frac{r}{R_a}\right)^{-2n-1} P^1_{2n}(\cos\theta) \quad , \tag{4b}$$

where α_n can be inferred e.g. from ref. 4. For the dipole part we find

$$\delta B_r = - \frac{NI}{cR_a} \left(\frac{R_a}{r}\right)^3 \cos\theta \tag{4c}$$

as stated above.

3. THE UNIPOLAR INDUCTOR

We apply now the foregoing consideration to the rotating disc (unipolar inductor). For an observer at rest, Ohm's law now reacls:

$$\vec{J} + \vec{N}\times\vec{J} = \sigma(\vec{E} + \vec{\beta}\times\vec{B}) \tag{5}$$

where $\vec{\beta} = \frac{\rho}{c} \vec{\Omega}\times\vec{e}_\rho$ and terns of order β^2 have been dropped. The electric field at the rim of the disc (index a) is therefore:

$$\vec{E}_a = -\vec{\beta}_a\times\vec{B} + \frac{1}{\sigma}(\vec{J}_a + \vec{N}\times\vec{J}_a) \quad . \tag{6}$$

For good conductivity the second term in eq. (6) can usually be neglected but it is clear that it is this term limits the maximaly extractable current I. If we connect inner and outer rim by nonrotating wires and a resistance $\hat{R}$, a current I will flow and energy will thus be extracted from the rotation of the disc. Ignoring the Hall effect for a moment, the analysis goes as follows. The electric field $\vec{E}$ is given by eq. (6):

$$\vec{E} = - \frac{\rho}{c} (\vec{\Omega}\vec{B})\, \vec{e}_\rho \tag{7a}$$

and the potential in the disc by

$$\Phi = \frac{1}{2c} (\vec{\Omega}\vec{B})\, \rho^2 \quad , \tag{7b}$$

so, the potential difference is

$$\Delta\Phi = \Phi(a) - \Phi(i) = \frac{1}{2c} (\vec{\Omega}\vec{B})\, (R_a^2 - R_i^2) \quad ,$$

and a current I is set up

$$I = \frac{\Delta\Phi}{\hat{R}} = \frac{1}{2c\hat{R}} (\vec{\Omega}\vec{B}) (R_a^2 - R_i^2) \quad . \tag{8}$$

The dissipated energy is given by $W = I\ \Delta\Phi$. This dissipated energy derives from the rotational energy of the disc and therefore angular momentum must be dissipated at a rate $\dot{J}$ such that $W = \vec{\Omega}\dot{\vec{J}}$. Externally, this is achieved in the fixed wires and resistance and inside the disc, angular *momentum* is taken away by means of the electromagnetic torque $\vec{T}$,

$$\vec{T} = \dot{\vec{J}} = \frac{1}{c} \int \rho\ \vec{e}_p \times (\vec{j}\times\vec{B})\ dV \quad . \tag{9}$$

Multiplying eq. (9) by we have

$$\vec{\Omega}\vec{T} = \frac{1}{c}\vec{\Omega} \int \rho\ j_\rho\ \vec{B}\ dV = \frac{1}{2c} (\vec{\Omega}\vec{B}) (R_a^2 - R_i^2)\ I = W \tag{10}$$

as it should.

Let us now take into consideration the Hall effect. Under stationary conditions $\frac{\partial}{\partial t} = 0$ and for rotationally symmetric flow $\frac{\partial}{\partial\phi} = 0$. Since rot $\vec{E} = -\frac{1}{c}\dot{B} = 0$ and $\frac{\partial}{\partial\phi} = 0$ it is clear that E_ϕ must vanish as E vanishes at the boundaries R_a and R_i. As $E_\phi = 0$ and neglecting the section term in Ohm's law, eq. (6), we have $j_\phi + N\ j_\rho = 0$. Then, as in the nonrotating disc, an extra current will flow which will give rise to an additional magnetic field δB_z in the disc and the analysis can directly be carried over from the previous.

According to Lenz' rule the induced magnetic field will oppose the original field if we put in a resistance. However, the reverse is obviousiy also possible, that is, if we drive a current through the unipolar inductor so that the disc speeds up, the magnetic field will then increase and the energy must be supplied by the external e.m.f. which drives the current.

4. APPLICATION TO LABORATORY AND ASTROPHYSICS

For a pure copper crystal at $4K$, a value of $\tau \approx 2\ 10^{-9}$ sec is feasible, whereas at room temperature $\tau = 2\ \ 10^{-14}$ sec for ordinary copper. We find, therefore, that $N = 100\ B_4\tau_{-9}$ and $N = 10^{-3}B_4\ \tau_{-14}$ for a magnetic field of 1 Tesla = 10^4 Gauss. Setting a copper disc, with a radius of 10 cm, into rotation at $R = 10^3\ sec^{-1}$ ($\approx$160 Hz) with a mag-

magnetic field of 1 Tesla, the potential difference will be 5 Volts. Putting a resistance of 10^{-4} Ohm as a load, a current of 1 Ampère will/ /flow, which will give rise to an induced magnetic field (eq. (4c))

$$B = 2 \text{ Gauss } (N/200)(I/1 \text{ Ampère}) . \tag{11}$$

Compared to the original 1 Tesla, this field should be measurable inthe laboratory.

In pulsars, which are believed to work essentially like unipolar inductors, the effect may be much larger for low enough temperatures i.e. for high enough conductivity at the surface. If the surface is something like a lattice we expect a temperature dependence of the conductivity due to electron-phonon scattering, which scales with temperature T like $(T/\theta)^{-5}$ where θ is the Debye temperature of the lattice. In one model calculation[6] for the matter just below the pulsar surface ($\rho \approx 10^4$ gcm^{-3} $B \approx 10^{12}$ Gauss) a conductivity a = $10^{29}\, T_6^{\,5}$ sec^{-1} was calculated which, with $n_s \approx 10^{26}$ cm^{-3}, gave (eq. (1)) $\tau = 10^{-14}$ sec and

$$N = 10^5\, B_{12}\, \tau_{-14}\, T_6^{-5} ,$$

For the induced magnetic field, due to the Hall-effect, we obtain for known current I (which can observationally be inferred from slow-down[7])

$$\frac{\delta B}{B} \approx \frac{\Delta F}{F} \beta\, N \approx 1 \tag{14}$$

if we use I = qc $AF = -\frac{(\vec{\Omega}\vec{B})}{2\pi} \Delta F$, where ΔF ir the area of the polar cap.

5. EXTENDED BODIES AND THE BUILD-UP PROBLEM

So far we have considered only a thin disc and for this case the calculations can be done analytically. The most interesting case as far as astrophysical considerations are concerned is the magnetized sphere. What happens if a current is driven through a magnetized sphere? (The resistance of a non magnetized sphere is given in ref. 2). The build-up problem is determined by Maxwell's equations:

$$\text{rot } B = \frac{4\pi}{c}\, j + \frac{1}{c} \dot{E}$$

$$\text{rot } E = -\frac{1}{c}\dot{B}$$

$$j = \overleftrightarrow{\sigma} E \tag{15}$$

with $\overleftrightarrow{\sigma}$ given by eq. (1d). A good approximation is, in our case, to drop the term $\frac{1}{c}\dot{E}$ and we end up with one of the following equivalent sets of equations

$$\text{rot rot } E = -\frac{4\pi}{c^2}\overleftrightarrow{\sigma}\dot{E} \tag{15a}$$

Neglecting the Hall-effect, the solution of the magnetic field decay in a sphere is of course well known[2]. Separating the time dependence $B = B_0 e^{-\gamma t}$, one finds for the eigenvalues of eq. (15) $\gamma_n = \frac{c^2}{4\pi\sigma} k_n^2$ where $j_0(k_n R) = 0$ in the case of a sphere of radius R, and $J_1(k_n R_a) = 0$ for a cylinder (which is infinitely long in the z-direction and for which $R_i = 0$), where j_0 and J_1 are appropriate Bessel functions[2].

The lowest decay or build-up mode is $k_0 = \pi/R$ for the sphere and $k_0 = 1{,}22\ \pi/R_a$ for the infinite cylinder, i.e. the magnetic field in a cylinder decays $(1.22)^2 \approx 1.5$ times slowlier than a sphere of equal radius. For the cylinder it is still possible to find and analytic solution if the Hall-field is taken into account. The complete solution reads:

$$\vec{B} = B_0\, J_0(k_n\rho)\, e^{-\gamma_n t}\, \vec{e}_z$$

$$\vec{j} = -\frac{cB_0}{4\pi}\cdot J_0'(k_n\rho)\, e^{-\gamma_n t}\, \vec{e}_\phi \tag{16}$$

$$\vec{E} = \frac{B_0}{\sigma}\left(-J_0'(k_n\rho)\, e^{-\gamma_n t}\, \vec{e}_\phi + N J_0'(k_n\rho)\, e^{-\gamma_n t}\, \vec{e}_\rho\right)$$

with unchanged eigenvalues as can be seen from eq. (16) if one bears in mind the relation $J_0' = -J_1$. The only effect of the magnetic field is therefore to produce an additional electric field $E_\rho = -N E_\phi$.

In order to stant this result better we consider the dissipated energy by means of the phenomenological equation[2] (for a thin wire)

$$\frac{1}{c^2}\hat{E}\,\dot{I} + \hat{R}\, I + \hat{C}^{-1} Q = U^{\text{ext}} \tag{17}$$

where $\hat{L}$ is the self-inductance, $\hat{R}$ the resistance and $\hat{C}$ the capacitance of the system. U^{ext} is the externally applied e.m.f. In the decay problem the net charge is zero and no external e.m.f. is present. We have therefore

$$\frac{1}{c^2}\hat{L}\,\dot{I} + \hat{R}\,I = 0$$

and[2]

$$\gamma = \frac{1}{\tau} = c^2\hat{R}/\hat{L} \tag{18}$$

New, as both $\hat{R}$ and $\hat{L}$ depend on N^2 in the same way, we understand that the decay time, which is the ratio of the two quantities, is independent on N^2. For a sphere we should use the telegraph equation[8]:

$$\frac{1}{c^2}\hat{L}\,\dot{I} + \hat{R}\,I + \frac{\partial}{\partial r}V = 0 \tag{19}$$

$$\frac{1}{r^2}\frac{\partial}{\partial r}r^2\;I + \hat{C}\,\frac{\partial V}{\partial c} + \hat{R}V = 0 \tag{20}$$

which leads to

$$\frac{1}{c^2}\hat{R}\,\hat{L}\,\dot{I} + \hat{R}^2\;I - \frac{\partial}{\partial r}r^{-2}\frac{\partial}{\partial r}r^2\;I = 0 \tag{21}$$

It appears therefore that in sphere the Hall-effect will not lead to a qualitatively different decay (or build-up) of the magnetic field either. We turn now to a detailed application to pulsars.

6. EVOLUTION OF PULSARMAGNETISM BY VIRTUE OF A FARADAY DYNAMO MECHANISM

We shall first give some background informatlon on pulsars and some motivation why we think the preceding considerations could be of importance for pulsars.

Soon after the identification of pulsars with neutron stars it was pointed out that the observation of the absence of long period pulsars could be understood if one assumed that the magnetic field decayed on a time scale of some 10 years. As shown in section 5 the time-scale τ_d for a magnetized body to lose its magnetic field dueto Ohmic dissipation is

$$\tau_d = \frac{4\sigma R^2}{c^2\pi} \tag{22}$$

and this amounts to $\tau_d \approx 10^6$ years if the average conductivity is $\sigma \approx 10^{23}$ sec^{-1}, a rather large value for nondegenerate matter. However, unlike in ordinary stars the matter of a neutron star is extremely degenerate and due to the Pauli-principie the conductivity is very large. In fact, the protons may in some site of the neutron star actually form a type II superconductor. Consequently, only in the crust of a neutron star can the magnetic field decay and this would not lead to any appreciable reduction in the star's dipole moment.

Unimpressed by such theoretical considerations, observers continued, essentially until today, to discuss their observational results in terms of magnetic field decay[6]. How do pulsars turn-off then if magnetic field decay is not possible? Three further ideas have been suggested. The first is a clever variant of the magnetic field decay hypothesis[7]. It is clear that a magnetic field cannot be anchored stably in a liquid body. So either a crust or a toroidal field which confines the poloidal field must be present. If this toroidal fields is mainly anchored in the crust it may decay by Ohmic dissipation (see sec. 5), and the poloidal field may subsequently crack the crust and reorient itself, thereby, lowering the magnetic energy, and forming a quadrupole field.

The second is based on the fact that externa1 or internal torques may lead to considerable alignment of the pulsars spins axis with the axis of the dipole moment[8]. There is little observational evidence that alignment alone is the mechanism that turns-off a pulsar, although the angle between dipole and spin axis is probably important for pulsar evolution[9]. Therefore some other mechanism must be at work.

In line with earlier work by Sturrock[10], Ruderman and his group[11] have developes the idea that sparking in gaps is responsible for the coherent radio emission of radio pulsars and that this process depends sensibly on the surface temperature and on the rotation period.

The following discussion is also in line with these consideration, and it stresses, as will be seen, the importance of the surface temperature. As it was explained in detail elsewhere[9,12] a valid model for slowing-down or speeding-up a neutron star is provided by the disc dynamo, which is shown in fig.1, if one replaces the wires by those conducting field lines of the magnetic dipole which cannot corotate

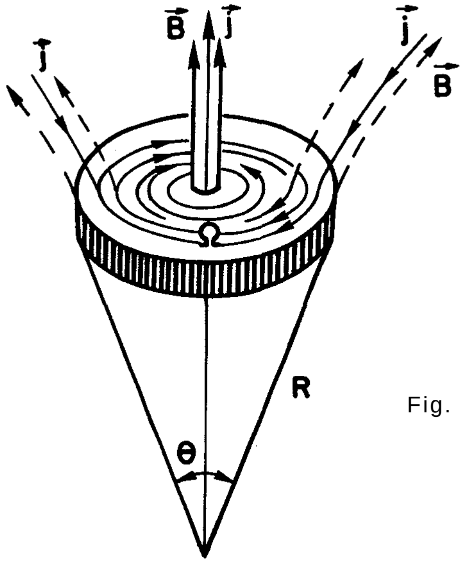

Fig. 1 - Unipolar inductor.

rigidly, i.e., the so called open field lines. In ref. 12 the details of the current flow in the noncorotating magnetosphere are explained and it is demonstrated that the anomalous braking index of the crab--nebula pulsar can bederived in a quite natural manner. This is a major success of the model since the explanation of the anomalous braking index has presented so far a major difficulty for any theory. Basically, the neutron star is slowed-down or accelerated by a magnetic torque provided by a current which flows along the magnetic field lines away from the surface area AF centered on the magnetic poles (polar caps). In the simplest case the return current will flow symmetrically in the axial direction around the forward current and further way from the center of the polar cap.

According to the considerations presented in section 4 no Hall -field can be established for geometrical reasons so the current which flows across the magnetic field lines inside the neutron star must spiral and satisfy Div $\vec{j} = 0$. The total current I can be inferred from the observed slow-down or speed-up and we can write instead of eq. (14)

$$\frac{\delta B_p}{B_p} = \frac{\sigma I \Omega}{e n_e C R^3 \theta^3} \tag{23}$$

where θ is explained in fig. 1. If the conductivity σ is mainly due to electron-phonon scattering σ will denote on temperature like T^{-5} i.e., the conductivity will be dependent extremely temperature. In this case it is convenient to turn around the meaning of eq. 23 and solve it for T instead. For those pulsars[13,14] which are believed to turn-off, we have typically $\dot{\Omega} = -10^{-16}$ sec^{-2} $\theta \approx 10^{-1}$, $B_p \approx \delta B_p \approx 10^{12}$ Gauss, which implies a temperature T $\approx 10^{4,7}$ $^{\circ}$K. For Rontgen-pulsars, we may on the other hand put $\Theta \approx 10^7$ $^{\circ}$K for the surface temperature, $\Theta \approx 10^{-1,5}$ to obtain $\delta B_p \approx 10^{12}$ Gauss that is, in good agreement with the observations if we use $\dot{\Omega} \approx 10^{-12}$ sec^{-2} as inferred from their speed-up[15].

7. SYNOPSIS

The evidence that radio-pulsars are slowed-down and Röntgen-pulsars are accelerated predominantly by magnetic torques is now very strong. Angular momentum is transferred away from the neutron star in the case of radio-pulsars and down to the neutron star in the case of Röntgen-pulsars by means of a magnetic spring. The physical origin of this is an appropriate current flowing along the magnetic field lines. As this current must be closed at the neutrons star surface and no Hall-field can be built-up, a Faraday dynamo mechanism is set-up. It is pointed out that this mechanism could switch-off a radio-pulsar or turn -on a Rontgen-pulsar. Many disconcerting pulsar observations could thus be explained[15] if radio-pulsars can be reactivated in the galactic plane by means of accretion in dense clouds, and if Röntgen-pulsars must first create a sufficiently strong magnetic field to function as a regularly pulsed emitter.

REFERENCES

1. E.M.Lifshitz, L.P.Pitaevckii, *Course of Theoretical Physics*, Vol.10, *Physical Kinetics* §§ 43, 58, 59, Pergarnon Press (1981).
2. L.D.Landau, E.M.Lifshitz, *Elektrodynamik der Kontinua*, Vol.8, §§ 21, 45, 47, 48, Akademie Verlag, Berlin (1967).

3. Ph.M.M. Morse, H. Feshbach, *Methods of Theoretical Physics*, Vol. 2, § 13.2 Mc Graw Hill, N.Y. (1953).

4. G.M.Ewart, R.A.Guyer, G.Greenstein, G.Ap.J. *202*, 238 (1975).

5. H.Heintzmann, E.Schrüfer, Astron. Astrophys.*111*, L4 (1982).

6. J.M.Cordes, D.J.Helfand, Ap.J.*239*, 640 (1980).

7. E.Flowers, M.A.Riderman, Ap.J.*215*, 302 (1977).

8. P.B.Jones, Nature 262, 121 (1967).

9. H.Heintzmann, Nature 292, 811 (1981).

10. P.A.Sturrock, Nature 227, 465 (1970), Ap.J. 164, 529 (1971).

11. M.A.Ruderman, Proc. IAU Symp. nº 95, 87-98 (1980).

12. HHeintzmann, E.Schrüfer, Astron.Astroph. 111, L4 (1982).

13. F.S.Fujimura, C.F.Kennel, Ap.J. 236, 245 (1980).

14. A.G.Lyne, R.T.Ritchings, F.G.Smith, MNRAS 271, 579 (1975).

Resumo

Neste trabalho examina-se o efeito Hall em um disco magnético. Se o disco é usado como indutor unipolar o campo magnético original decresce, enquanto se uma corrente de direção apropriada é inserida no disco em rotação, o campo magnético cresce sem limite — em uma análise simplista, usando-se corrente constante *I*. O efeito pode ser testável em laboratório, e pode ser particularmente importante em pulsares. A magnitude da mudança relativa do campo magnético é de ordem N para pulsares. $\beta = \Omega R/c$ é a velocidade equatorial do pulsar em unidades da velocidade da luz *e* N é a constante adimensional de Hall $N = e\frac{B\tau}{mc}$ Em condições favoráveis $\frac{\delta B}{B} \approx 10^{-5}$ para objetos típicos de laboratõrio e para pulsares BN ≈ 1.