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# A New Cosmological Coincidence and the Value of the Cosmological Constant

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**Abstract** An alternative way is used to calculate the cosmological constant through a cosmological coincidence exibited recently by Landsberg. In the relationship between physical constants it is reasonable to replace the Hubble parameter by the cosmological constant multiplied by the velocity of light.

## 1. INTRODUCTION

Landsberg[1] has recently called attention to the newcosmological coincidence to wit:

$$M_u/M \sim M/m \sim \sqrt{N} \quad ,$$
$$M \sim (hc^4/G^2H_0)^{1/3} \quad , \tag{1}$$

where $M_u$ is the mass of the known universe, $M$, the mass of a black-hole with evaporation time of the order of the Hubble time, $m$ the mass of an elementary particle and N, the number of baryons of the observable universe. In the present paper we follow closely the reasoning of Zel'dovich[2] and of ourselves[3] through a special argument involving some constants of physics plus relation (1), avoiding the Hubble parameter, H, which is replaced by the cosmological constant, $\Lambda^{1/2}$, multiplied by the velocity of light, c.

## 2. NO FUNCTION G(t). A COLUMBUS EGG ARGUMENT

It was through the use of the present time valueofthe Hubble parameter, $H_0$, in his numerological relations, that Dirac[4] raised the hypothesis that Newton's G could be a function of the cosmic time. Geological and astrophysical data contradicted Dirac's function $G(t)$. Besides, recent astronomical observations[5] are highly unfavourable to it also. It seems to us that if we aim at establishing a connection among the constants of the micro and the macro worlds, the cosmological constant plus the velocity of light should be used, instead of the Hubble parameter, which is a cosmological time function derived from the scaling function $R(t)$ of the Robertson-Walker metric. Thus, it is likely that laws of nature of the type $G(t)$ cannot be inferred from numerology.

In 1980, I showed[3] through a Columbus-egg argument that the relation which suggested $G(t)$,

$$GM_u/c^2 r_0 \simeq 1 \quad (r_0,\ \text{Hubble length}) \ , \qquad (2)$$

which can be inferred from the Whitrow-Randall[6] relation

$$(4\pi/3)\ G\ \rho_0\ H_0^{-2} \simeq 1 \ , \qquad (3)$$

($\rho_0$ and $H_0$, being the present values of the cosmic density and Hubble parameter) is in fact not a law of nature but an approximation, at the present stage of the universe, of the exact equation:

$$(4\pi/3)\ G\ \sigma\ q^{-1} H^{-2} = 1 \ , \qquad (4)$$

q being the deacceleration parameter and

$$\sigma = \rho + 3p - 2\Lambda/\kappa \ . \qquad (5)$$

$\rho$, $p$ and $\kappa$ are respectively the cosmic density, pressure and Einstein's gravitational constant.

Equation (4) exhibits the deacceleration and Hubble parameters at any age of the universe and follows from the relativistic differential equations for uniform universe models, to wit:

$$(4\pi/3)\ G\ \sigma (R/R'') = -1 \ , \qquad (6)$$

where R is the scaling function and $R''$ its second order time derivative.

Assuming that for the present epoch

$$\sigma_0\ q_0^{-1} \simeq \rho_0 \ , \qquad (7)$$

then relation (3) follows, which is in effect compatible with observational values of $H_0$ and $\rho_0$. Therefore, relation (2) is not a law of nature, and hence cannot be invoked as an argument for a time-varying G.

Zel'dovich[2], on different grounds, has inferred also that the Dirac relations are approximately valid only for the present stage of cosmic evolution. Assuming the present argument, I propose the forthcoming reasoning to calculate the cosmological constant.

### 3. A STATISTICAL RELATION

Relation (1) may have a statistical meaning, if we bear in mind that the mass M of the black hole considered evaporates according to Hawking's theory during a long time interval of the order of the universe's age. Thus, the quantity $\sqrt{N}$ is the fluctuation in the number N of baryons in the observable universe. It is therefore convenient that $m$ in (1) should be viewed as the mass $m_p$ of the proton. In order to avoid the Hubble parameter, let us assume that the curvature is closed and obeys the differential equations of the uniform model of Lemaître, i.e., with cosmological constant different from zero. In this case, N is the total number of baryons in the universe and $M_u$ the total mass.

### 4. CALCULATION OF THE COSMOLOGICAL CONSTANT

Putting into Lemaître's equation the conditions,

$$\tilde{R}' \simeq 0, \ \tilde{R}'' \simeq 0, \ \tilde{p} \simeq 0, \tag{8}$$

for an epoch $\tilde{t}$, i.e. in the plateau region of the function $R(t)$, it follows:

$$\kappa\tilde{\rho} \simeq 2\Lambda \ , \ \Lambda \simeq \tilde{R}^{-2} \ . \tag{9}$$

Since for a closed model,

$$\rho = M_u (2\pi^2 \ R^3)^{-1} \ , \tag{10}$$

and recalling that $\kappa = 8\pi G/c^2$, it is easy to see that it follows:

$$(2/\pi) \ G \ M_u \ \Lambda^{1/2} \ c^{-2} \simeq 1 \ . \tag{11}$$

Let us postulate an exact relation of the type (11):

$$(4\pi/3) \ (GM_u \ \Lambda^{1/2}) c^{-2} = 1 \ . \tag{12}$$

Of course we can put:

$$M_u = Nm_p \quad . \tag{13}$$

Relation (11) is also equivalent to a Heisenberg-type relation:

$$(4\pi/3)\ GNm_p^{\ 2}\ \Lambda^{+1/2}\ (\lambda_p/c) = h \tag{14}$$

where $\lambda_p$ is the proton Compton length.

This suggests the assumption of the following hypothesis:

$$\beta(Gm_p^{\ 2}/\lambda\ )\ (\Lambda^{-1/2}/c) = h \tag{15}$$

β being a numerical factor. This form of Heisenberg's relation as pointed out by Sivaran[8] was considered by Weinberg in 1972[9]. I used it before in 1963[10] in order to calculate the cosmological constant in a de Sitter universe. Without its Heisenberg fcrm it can be traced back as far as 1931 in a famous Eddington's paper[11].

Relations (1) could be reduced to a single one, if $m$ is identified with $m_p$, the mass of the proton. Replacing $H_0$ by $c\Lambda^{1/2}$ in (1) and postulating the exact relation,

$$\left[(h\ c^3\ \Lambda^{-1/2})/G^2\right]^{1/3} = m_p\sqrt{N} \tag{16}$$

it comes that and combining it with (12),

$$\Lambda^{-1/2} = (4\pi/3)^3\ h^2\,(m_p^3\ G)^{-1} \quad . \tag{17}$$

From (15) and (17) the numerical values are obtained:

$$\Lambda^{-1/2} = 9.2 \times 10^{27}\ \text{cm}\ ,$$
$$\beta = 1.55 \times 10^{-2} \quad . \tag{18}$$

On the other hand,

$$N = (3/4\pi)\ (c^2\ \Lambda^{-1/2})/Gm_p\ , \tag{19}$$

gives,

$$N = 1.77\ \times 10^{79}\ . \tag{20}$$

This new calculation of the cosmological constant via the modification of (1) furnishes a result of the same order of magnitude as that given by Eddington[11]

$$\Lambda^{-1/2} = 1.01 \times 10^{27} \text{ cm} , \tag{21}$$

by Zel'dovich[2],

$$\Lambda^{-1/2} \simeq 4.6 \times 10^{27} \text{ cm} , \tag{22}$$

and by myself in 1980[3],

$$\Lambda^{-1/2} = 1.4 \times 10^{27} \text{ cm} . \tag{23}$$

Landsberg relation[1,12]

$$m(b) \sim \left(\frac{h^3 H_0}{G}\right)^{1/5} \left(\frac{c^5}{h H_0^2 G}\right)^{b/15} \tag{24}$$

(h being a numerical parameter and $m(b)$ a mass which could be $m_\pi$, $M_u$, or Planck's mass, according to a suitable value of $b$) given in a context of constant G, could be reformulated as:

$$m(b) \sim \left(\frac{h^3 c \Lambda^{1/2}}{G}\right)^{1/5} \left(\frac{c^3}{hG\Lambda}\right)^{b/12} , \tag{25}$$

by putting $c\Lambda^{1/2}$ in place of $H_0$. Assuming $H_0 = 3.2 \times 10^{-18}$ s$^{-1}$ (100 km. s$^{-1}$. Mp$^{-1}$) and considering that $\Lambda^{1/2}c = 3.3 \times 10^{-18}$ s$^{-1}$, relation (25) would give values of the same order of magnitude.

Of course Landsberg's relation in the present context has to do with closed cosmological models of the Lemaître type with a plateau for the scaling function.

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**Resumo**

Apresenta-se um caminho alternativo para se calcular a constante cosmológica através uma nova coincidência cosmológica exibida recentemente por Landsberg. Nas relações entre as constantes fundamentais da física é razoável a substituição do parâmetro de Hubble pela constante cosmológica multiplicada pela velocidade da luz.