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Use of the Heat-Kernel Expansion with the Background Field Method to Regularize Supersymmetric (Gauge) Theories

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Abstract We use the proper time variable of the heat-kernel expansion to regularize field theories in the framework of the background field method. The method can be naturally applied to supersymmetric and gauge theories. Explicit computations have been done including superfield perturbation theory.

1. INTRODUCTION

Renormalization is one of the most difficult techniques in quantum field theory. Besides difficulties of principle with some schemes breaking down at very high orders of perturbation theory[1], some symmetries of the theory can be mutilated in the process. The problem of recovering gauge symmetry in the BPHZ scheme[2] is notorious[3]. Dimensional regularization[4] is the best method to deal with gauge theories, since the gauge principle is maintained in arbitrary dimensions. However, concerning supersymmetric theories the situation is not yet in good shape. This comes from the fact that the number of degrees of freedom of fields of different spin does not behave in the same way with respect to the dimension. A way out is the process called dimensional regularization via dimensional reduction, or supersymmetric dimensional regularization (SDR)[5]. In this process, the space-time is D-dimensional, but the fields and the algebra are kept 4-dimensional. However the process is ambiguous at higher orders of perturbation theory[6]. In this work we propose a regularization procedure based on a cut in the integration over the proper-time variable of the heat kernel expansion. This is made as follows — one uses the background field method[7], splitting the field into a classical and a quantum part. The quantum piece is integrated, and we get an effective action in terms of the classical fields, from which the renormalization constant can be read. In order to perform the integration over the quantum fields, the corresponding Green's function are expanded in a series whose termsare successively less divergent at short distances, and the coefficients are functions of the background fields, being called Seeley coefficients[8]. Introducing at this point a cut in the integration over the proper-time, all products of different Green's functions are regularized in an explicitely supersymmetric way, since the above procedure can be equally made using superfields.

Some of these results were also partially shown in a previous communicatíon[9].

2. THE HEAT KERNEL AND BACKGROUND FIELD METHOD

The background field method consists in splitting the field into a classical and a quantum piece. The specific way one does itis rather arbitrary, and one can use this arbitrariness in order to simplify the computations. We shall study 3 cases, namely the ϕ^4 theory, the (supersymmetric) Wess-Zumino model, and the supersymmetric $N = 1$ Yang-Mills theory. The ϕ^4 model is described by the lagrangean:

$$L = \frac{1}{2} (\partial_\mu \phi)^2 + \frac{g}{4!} \phi^4 \tag{2.1}$$

The background quantum splitting is simply

$$\phi \to \phi + \xi$$

and we have

$$L = L(\phi) + L'(\phi)\xi + \frac{1}{2} (\partial_\mu \xi)^2 + \frac{g}{4} \xi^2\phi^2 + \frac{g}{6} \xi^3\phi + \frac{g}{4!} \xi^4 \tag{2.2}$$

the quadratic operator

$$\Box_B = - \Box + \frac{g}{2} \phi^2 \tag{2.3}$$

defines a heat-kernel $G(x,y;\tau)$, such that

$$\frac{\partial}{\partial\tau} G(x,y;\tau) - \Box_B G(x,y;\tau) = 0 \tag{2.4}$$

with the boundary condition

$$G(x,y;0) = \delta(x-y) \tag{2.5}$$

The Green's function is given by

$$G(x,y) = \int_0^\infty d\tau\, G(x,y;\tau) \tag{2.6}$$

The heat-kernel (or equivalently the **Green's** function) can be expanded in a series[8].

$$G(x,y;\tau) = \sum_{n=0}^{\infty} \frac{\tau^n\, a_n(x,y)\, e^{-(x-y)^2/4\tau}}{(4\pi\tau)^2} \tag{2.7}$$

Plugging (2.7) back into (2.4) and (2.5) we can compute the a_n**'s. In** fact what we actually need are the lowest three Seeley coefficients at coinciding points. From (2.5)

$$a_0(x,y) = 1 \tag{2.8}$$

and from (2.4), to lowest order **in** τ:

$$-\frac{g}{2}\phi^2 \frac{e^{-x^2/4\tau}}{(4\pi\tau)^2} = \frac{e^{-x^2/4\tau}}{(4\pi)^2}\left\{-\frac{a_1}{\tau^2} + \frac{x^2}{4\tau} a_1\right\} \tag{2.9}$$

from which

$$a_1(x,x) = -\frac{g}{2}\phi^2 \tag{2.10}$$

To second order **in** τ we have

$$a_2(x,x) = \frac{g^2}{8}\phi^4(x) - \frac{g}{12}\Box\phi^2 \tag{2.11}$$

So much for the ϕ^4 theory. We turn now to the computation of the Seeley coefficients for the Wess-Zumino model. The theory is defined by **the** lagrangean dens**ity**[10]**.**

$$L = \int d^4\theta\ \bar{\phi}\phi - \frac{\lambda}{3!}\int d^2\theta\ \phi^3 - \frac{\lambda}{3!}\int d^2\bar{\theta}\ \bar{\phi}^3 \tag{2.12}$$

where $\phi(\bar{\phi})$ is a chiral (antichiral) superfield:

$$\bar{D}_{\dot{\alpha}}\phi = 0 = D_{\alpha}\phi \tag{2.13}$$

and D is the usual covariant derivative (see ref. 9 for our conventions)

The background quantum splitting is also very simple

$$\phi \to \phi + \xi$$
$$\bar{\phi} \to \bar{\phi} + \bar{\xi}$$

and we have the lagrangean,:

$$L = \int d^4\theta\ \bar{\xi}\xi - \int d^2\theta\ \left(\frac{\lambda}{2}\ \phi\xi^2 + \frac{\lambda}{3!}\ \xi^3\right) - \int d^2\bar{\theta}\ \left(\frac{\lambda}{2}\ \bar{\phi}\bar{\xi}^2 + \frac{\lambda}{3!}\ \bar{\xi}^3\right) \tag{2.14}$$

As usual we can turn the two dimensional Grassmann integrals into four dimensional by means of the identities.

$$\int d^2\theta\ \phi\xi^2 = \int d^2\theta\ d^2\bar{\theta}\ \xi\phi\ \frac{D^2}{\Box}\ \xi \tag{2.15a}$$

$$\int d^2\bar{\theta}\ \bar{\phi}\bar{\xi}^2 = \int d^2\theta\ d^2\bar{\theta}\ \bar{\xi}\bar{\phi}\ \frac{\bar{D}^2}{\Box}\ \bar{\xi} \tag{2.15b}$$

The part of the lagrangean, which is quadratic in the quantum fields is now:

$$L = \int d^4\theta\ \widehat{\underbrace{\xi\ \bar{\xi}}}\ \ O\begin{pmatrix}\xi\\ \bar{\xi}\end{pmatrix} \tag{2.16a}$$

$$O = \begin{bmatrix} -\lambda\dfrac{D^2}{\Box}\phi & 1 \\ & \\ 1 & -\lambda\dfrac{\bar{D}^2}{\Box}\bar{\phi} \end{bmatrix} \tag{2.16b}$$

The heat kernel method has already been employed in supersymmetric theories[10]. It is possible to find the heat-kernel expansion for $(O\Box)^{-1}$.

This is done in a very simple way, just considering the heat-kernel corresponding to

$$\begin{bmatrix} 0 & \Box^{-1} \\ \Box^{-1} & 0 \end{bmatrix} \delta(x-y) = \int_0^\infty \frac{d\tau}{(4\pi\tau)^2} e^{-(x-y)^2/4\tau} \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \tag{2.17}$$

and treating the background field as a perturbation. The first Seeley coefficient *(ai)* is given by the expansion of

$$\begin{bmatrix} 0 & \Box^{-1} \\ \Box^{-1} & 0 \end{bmatrix} \begin{bmatrix} -\lambda D^2\phi & 0 \\ 0 & -\lambda\bar{D}^2\bar{\phi} \end{bmatrix} \begin{bmatrix} 0 & \Box^{-1} \\ \Box^{-1} & 0 \end{bmatrix}$$

so that

$$\int \frac{d\tau}{(4\pi\tau)^2} e^{-(x-y)^2/4\tau} a_1(x,y) = -\lambda \int \frac{d\tau_1 \, d\tau_2}{((4\pi)^2\tau_1\tau_2)^2}$$

$$\times \; e^{-(x-z)^2/4\tau_1} \begin{bmatrix} -\lambda\bar{D}^2\bar{\phi} & 0 \\ 0 & -\lambda D^2\phi \end{bmatrix} e^{-(z-y)^2/4\tau_2} \tag{2.18}$$

from which we get

$$a_1(x,x) = -\lambda \begin{bmatrix} \bar{D}^2\bar{\phi} & 0 \\ 0 & D^2\phi \end{bmatrix} \tag{2.19}$$

The a_2 coefficient can be computed also easily:

$$a_2(x,x) = \frac{\lambda^2}{2} \begin{bmatrix} 0 & \bar{D}^2 D^2 \bar{\phi}\phi \\ D^2\bar{D}^2\phi\bar{\phi} & 0 \end{bmatrix} \tag{2.20}$$

Finally we turn to the case of supsrsymmetric gauge theories. The lagrangean density of a N=1 Yang-Mills theory is given by (11), (12):

$$L = \frac{\mathrm{tr}}{2g^2} \int d^2\theta \;\; w^2 + \frac{\mathrm{tr}}{2g^2} \int d^2\bar{\theta} \;\; \bar{w}^2 \quad + \text{gauge fixing} + \text{ghosts} \tag{2.21}$$

where

$$w_\alpha = \bar{D}^2 \; (e^{-V} D_\alpha \, e^V)$$

and V is the gauge field, a general real superfield, in the adjoint representation of the gauge group G.

Gauge transformations act as

$$e^V \to e^{i\bar{\Lambda}} e^V e^{-i\Lambda} \tag{2.22}$$

where $\Lambda(\bar{\Lambda})$ is a chiral (antichiral) gauge parameter. A suitable gauge fixing lagrangean is[9]:

$$L_{\mathrm{Gf.}} = -\frac{1}{g^2}\int d^4\theta\, D^2V\,\bar{D}^2V \tag{2.23}$$

The gauge transforrnation (2.22) is highly nonlinear, but can be re-expressed as

$$\delta V = L_{\frac{1}{2}V}\left[-i(\bar{\Lambda}+\Lambda) + \coth L_{\frac{1}{2}V}\, i(\bar{\Lambda}-\Lambda)\right] \tag{2.24}$$

where

$$L_x Y = [X,Y]$$

It is now possible to write the Faddev-Popov action

$$L_{\mathrm{FP}} = \mathrm{tr}\int d^4\theta\,(c'+\bar{c}')\; L_{\frac{1}{2}V}\left[c+\bar{c}+\coth L_{\frac{1}{2}V}(c-\bar{c})\right] \tag{2.25}$$

The background field rnethod is not less cornplicated, although the results are quite sirnple, once we know the way to proceed. It turns out[12] that the best procedure is to consider the quantum field as above (V) in the chfral representation, and the background field in a vector representation, defined by the fields Ω and $\bar{\Omega}$. The splitting is

$$e^V \to e^{\Omega}\, e^V\, e^{\bar{\Omega}} \tag{2.26}$$

This has the advantage that we can use the following very simple rules, in order to realize the method[12].

a) Covariant derivatives are now background covariant:

$$D_\alpha \to \mathcal{D}_\alpha = e^{-\Omega}\, D_\alpha\, e^{\Omega} \tag{2.27a}$$

$$\bar{D}_{\dot{\alpha}} \to \bar{\mathcal{D}}_{\dot{\alpha}} = e^{\bar{\Omega}}\, \bar{D}_{\dot{\alpha}}\, e^{-\bar{\Omega}} \tag{2.27b}$$

b) The Laplacian operator in the quadratic part of the gauge action, after expanding in powers of V, turns out to be

$$\Box_B = \mathcal{D}^\alpha \mathcal{D}_\alpha - i\omega^\alpha \mathcal{D}_\alpha - \bar{\omega}^{\dot{\alpha}} \bar{\mathcal{D}}_{\dot{\alpha}} \tag{2.28}$$

where $\omega(\bar{\omega})$ are the background field strenghs.

c) The ghost (and eventually matter) fields, which previously obeyed a chirality or antichirality conditions obey a background covariant chirality or antichirality condition:

$$\begin{aligned} D_\alpha \bar{c} = 0 &\rightarrow \mathcal{D}_\alpha \bar{c} = 0 \\ \bar{D}_{\dot{\alpha}} c = 0 &\rightarrow \bar{\mathcal{D}}_{\dot{\alpha}} c = 0 \end{aligned} \tag{2.29a}$$

As a result of c), when the ghosts are introduced one must divide out the (spurious), contribution from the determinant of their square operator, since it is no longer constant, but depends on the background field. This introduces another ghost, the Nielsen-Kallosh ghost [13], which however, has only one loop contribution:

$$L = \mathrm{tr} \int d^4\theta\, \bar{b} b \tag{2.30}$$

now we can compute all the necessary Seeley coefficients. In order to do that, we must look at the propagator of both the gauge potential V, whose inverse propagator is:

$$(\quad a V \quad)^{-1} = (\mathcal{D}^a \mathcal{D}_a - i\omega^\alpha \mathcal{D}_\alpha - i\bar{\omega}^{\dot{\alpha}} \bar{\mathcal{D}}_{\dot{\alpha}})\, \delta(a-a') \tag{2.31}$$

and the ghosts, which are chiral (antichiral), and have as propagators.

$$\langle \bar{c}(z) . c(z') \rangle^{-1} = (\mathcal{D}^a \mathcal{D}_a - i\omega^\alpha \mathcal{D}_\alpha - \frac{i}{2} \mathcal{D}^\alpha \omega_\alpha)\, \delta(z-z') \tag{2.32}$$

The heat-kernel corresponding to the operator

$$\Box' - i\omega^\alpha \mathcal{D}_\alpha - i\bar{\omega}^{\dot{\alpha}} \bar{\mathcal{D}}_{\dot{\alpha}}$$

where $\Box' = \mathcal{D}^a \mathcal{D}_a$, can be computed using the identity (the chiral case (2.32) can be obtained trivially, substituting $\bar{\omega}^{\dot{\alpha}} \bar{\mathcal{D}}_{\dot{\alpha}}$ by $\frac{1}{2} \mathcal{D}^\alpha \omega_\alpha$).

$$G(z,z') = \int_0^\infty d\tau \; \left[\exp\{-\tau\,(\Box' - i\omega^\alpha \mathcal{D}_\alpha - i\bar{\omega}^{\dot\alpha} \bar{\mathcal{D}}_{\dot\alpha}\,)\}\right] \delta(z-z') =$$

$$= \int d\tau_1\, e^{-\tau_1 \Box} \delta(z-z') + \int d\tau_1 d\tau_2 \int d^4z \;\; e^{-\tau_1 \Box} \;\; \delta(z-z'')\,(-\, i\omega^\alpha \bar{\mathcal{D}}_\alpha -$$

$$i\omega^\alpha \mathcal{D}_\alpha)\; e^{-\tau_2 \Box'} \;\; \delta(z''-z') + \frac{1}{2}\int d\tau_1 d\tau_2 d\tau_3 \int d^4z''\, d^4z'''\, e^{-\tau_1 \Box} \;\; \delta(z-z'')$$

$$(-i\bar{\omega}^\alpha \bar{\mathcal{D}}_\alpha - i\omega^\alpha \mathcal{D}_\alpha)\;\;(z'')\;\; e^{-\tau_2 \Box'} \;\; \delta(z''-z''')\;\;(-i\bar{\omega}^{\dot\alpha}\bar{\mathcal{D}}_{\dot\alpha} - i\omega^\alpha \mathcal{D}_\alpha)\;\;(z''')$$

$$e^{-\tau_3 \Box'} \delta(z'''-z') \; + \text{ higher orders.} \tag{2.33}$$

For our purposes, the background fields in $\Box'$ do not contribute,since the corresponding a_2 coefficient is:

$$a_2(x,x) \approx F^2_{ab} = -\,4\mathcal{D}^2\omega^2 \tag{2.34}$$

which is less divergent, because of the derivatives (as a rule, we must have a maximal number of derivatives on the $\delta(z-z')$ function to have a non zero result, and derivatives on the exponential functions, in order to have contributions to the infinite parts).

We have for the Seeley coefficients a_2:

$$a_2(z,z) = \omega^\alpha \mathcal{D}_\alpha\, \omega^\beta \mathcal{D}_\beta \quad + \quad (\text{terms with}) \tag{2.35}$$

3. REGULARIZATION AND PRODUCT OF THE GREENS – FUNCTIONS AT THE SAME POINT. COMPUTATION OF COUNTERTERMS

We shall now rearrange the heat kernel expansion in such away that all the divergent structures become transparent. We write[14]:

$$G(x,y) = G_0(x-y)\,a_0(x,y) + G_1(x-y)\,a_1(x,y) + G_2(x-y)\,a_2(x,y) + \tilde{G}(x,y) \tag{3.1}$$

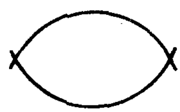
$$G_0^2(x)\; G_1(x) = \frac{1}{2(4\pi)^2}\left[\ln e^{\gamma}\Lambda\; 2\varepsilon\right]^2 \delta(x) \tag{3.8}$$

and

$$G_0^3(x) = C_1\;\delta(x) - \frac{1}{(4\pi)^2}\ln(e^{\gamma+1}\Lambda\; 2\varepsilon)\;\partial^2\delta(x) \tag{3.9}$$

where C_1 is linearly divergent, and contribute to mass renormalization

Now we are able to compute the divergences is perturbation theory. In the ϕ^4 theory, we have for the one loop contribution the diagram (1) (Fig.1).



(fig.1)

which contributes as

$$\int d^4x\;\; d^4y\; \frac{g}{4}\,\phi^4(x)\;\; G_0^4(x-y)\;\frac{g}{4}\,\phi^2(y) = -\int \frac{d^4x}{16(4\pi)^2}\;\; g^2\phi^4(x)\;\;\ln(e^{\gamma+1}\Lambda\; 2\varepsilon) \tag{3.10}$$

implying that we must take a conterterm

$$\delta L = -\frac{1}{4!}\;\frac{3g^2}{32\pi^2}\ln(e^{\gamma+1}\Lambda\; 2\varepsilon)\;\;\phi^4 \tag{3.11}$$

or equivalently

$$Z_g = 1 - \frac{3g^2}{32\pi}\ln(e^{\gamma+1}\Lambda\; 2\varepsilon) \tag{3.12}$$

giving the β-function at lowest order

$$\beta^{(1)}(g) = \frac{3g^2}{16\pi^2} \tag{3.13}$$

For the two loop computation we compute the three diagrams in Fig.2

(a) (b) (c)

(fio. 2)

The first contains the lowest order counterterm and its contribution is

$$(a) = \int d^4x \, \frac{3g^2}{4.32\pi^2} \phi^2(x) \; \ln(e^{\gamma+1} \; \Lambda \; 2\varepsilon) \, a_1(x,x) \, G_1(x,x)$$

$$= \int d^4x \, \frac{3g^3}{4.64\pi^2 (4\pi)^2} \; \phi^4 \; (\ln e^{\gamma+1} \; \Lambda \; 2\varepsilon)^2 \tag{3.14}$$

The second contribution is

$$(b) = -\int d^4x \, \frac{g}{8} \left[G_1(x,x) \, a_1(x,x)\right]^2 = -\frac{g^3}{32(4\pi)^4} \int d^4x \; \ln(e^{\gamma+1} \Lambda \; 2\varepsilon)^2 \, \phi^4 \tag{3.15}$$

Finally (c) has two contributions, one exactly as above for the ϕ^4 term:

$$\frac{3g^2}{12} \int d^4x \; d^4y \; \phi(x) \, \phi(y) \, a_1(x,y) \, G_0^2(x-y) \, G_1(x-y) \quad =$$

$$= -\frac{g^3}{16(4\pi)^4} \; (\ln \, e^{\gamma} \; \Lambda \; 2\varepsilon)^2 \int d^4x \phi^4(x) \tag{3.16}$$

and one contribution to the 2-point functíon

$$\int d^4x d^4y \; \frac{g^2}{12} \; \phi(x) \, G_0^3(x-y) \, \phi(y) \; = \; \frac{g^2}{12(4\pi)^4} \ln(e^{\gamma+\prime} \; \Lambda \; 2\varepsilon) \int d^4x \; (\partial_\mu \; \phi(x))^2 \tag{3.17}$$

We have as a result

$$= 1 - \frac{3}{2(4\pi)^2} \; \ln \, e^{\gamma+1} \; \Lambda \; 2\varepsilon \; + \frac{gg^2}{4(4\pi)^4} \; (\ln \, e^{\gamma+1} \; \Lambda \; 2\varepsilon)$$

$$+ \frac{3g^3}{(4\pi)^4} \; \ln \, e^{\gamma+1} \; \Lambda \; 2\varepsilon \tag{3.18}$$

and

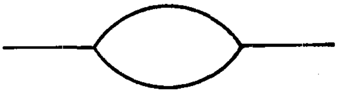
$$Z_\phi = 1 - \frac{g^2}{12(4\pi)^4} \; \ln \, e^{\gamma+1} \; \Lambda \; 2\varepsilon \tag{3.19}$$

This gives the β-function

$$\beta(g) = \frac{3g^2}{(4\pi)^2} - \frac{17}{3}\frac{g^2}{(4\pi)^4} \tag{3.20}$$

which is a well-know result.

For the Wess-Zumino action we have at lowest order thediagram of Fig. 3



(fig.3)

contributing as:

$$2\int d^4x d^4y d^4\theta_x d^4\theta_y \left(\frac{\lambda}{2}\right)^2 \phi(x,\theta_x)\, G_0^2(x-y)\, \bar{\phi}(y,\theta_y)\, \delta^4(\theta_x-\theta_y)\, \bar{D}^2 D^2 \delta^4(\theta_x-\theta_y)$$

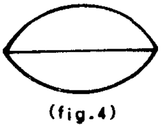
$$= -\frac{\lambda^2}{2(4\pi)^2}\, \ell n\, e^{\gamma+1}\, \Lambda \;\; 2\varepsilon \int d^4x\, d^4\theta\, \bar{\phi}\phi(x,\theta) \tag{3.21}$$

giving for the β-function the value

$$\beta^{(1)}(\lambda) = \frac{3\lambda^3}{2(4\pi)^2} \tag{3.22}$$

where we have used the Feynman rules for (anti) chiral fieds, which demands factors of D^2 $(\bar{D}^2)$ in all lines, except one[11].

The two loop computation is very simple, since it involves only one diagram (Fig. 4).



(fig.4)

As a result of the Feynman rules we have.

$$\frac{3\lambda^2}{3!}\int d^4x \;\; d^4y\; d^4\theta_x\; d^4\theta_y\;\; G_0^2(x-y)\, \Box\, G_2(x-y)\, \delta^4(\theta_x-0_y)$$

$$D^2\bar{D}^2\delta^4(\theta_x-\theta_y)\;\; D^2\bar{D}^2\delta^4(\theta_x-\theta_y)\, \lambda^2\; \bar{\phi}\phi(x) \tag{3.23}$$

We have as a result:

where

$$G_n(x) = \int_0^\infty \frac{d\tau}{(4\pi\tau)^2} \tau^n \quad e^{-x^2/4\tau}$$

$G_0(x)$ is regularized by a cut-off in the τ integration:

$$G_{0\varepsilon}(x) = \int_\varepsilon^\infty \frac{d\tau}{(4\pi\tau)^2} \quad e^{-x^2/4\tau} \tag{3.2}$$

To regularize the remaining functions, we force them to obey the following relation, valid in the non regularized theory

$$G_{m+n+1}(x-y) = \int d^4z \; G_m(x-z) \quad G_n(z-y) \tag{3.3}$$

As a result we have

$$G_n(x) = \int_{(n+1)\varepsilon}^\infty \frac{d\tau}{(4\pi\tau)^2} \quad e^{-x^2/4\tau} \left(1 - \frac{(n+1)\varepsilon}{\tau}\right)^n \tau^n \tag{3.4}$$

In order to find the divergences characteristic of perturbation theory, we Fourier transform the product of Green's functions at the same point, and consider the first few terms in a Taylor series around zero momentum, the remaing terms being convergent (e.g. by power counting). The first product is given by $G_0^2(x)$. In the above procedure it is only necessary to consider the first term (zero-momentum)

$$\int d^4x \; G_0^2(x) = \int_\varepsilon^\infty \frac{d\tau_1 d\tau_2}{(4\pi)^4 \tau_1\tau_2} \frac{(4\pi)^2}{\left(\frac{1}{\tau_1} + \frac{1}{\tau_2}\right)^2} \; e^{-\Lambda(\tau_1+\tau_2)} \tag{3.5}$$

where a small mass Λ was introduced to avoid infrared divergences. The above integral can be performed, and we have, for small ε:

$$\int d^4x \; G_0^2(x) = - \frac{1}{(4\pi)^2} \; \ell n \; (e^{\gamma+1} \Lambda \; 2\varepsilon) \tag{3.6}$$

so that

$$G_0^2(x) = - \frac{1}{(4\pi)^2} \ell n \; (e^{\gamma+1} \Lambda \; 2\varepsilon) \, \delta(x) \tag{3.7}$$

Other products can be computed by the same procedure. We need the following

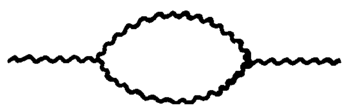
$$\frac{\lambda^2}{4\,(4\pi)^4}\ (\ln e^{\gamma}\,\Lambda\ \ 2\varepsilon)^2 \int d^4x\ d^4\theta\ \bar{\Phi}\phi \qquad (3.24)$$

and the β-function can be computed[15]:

$$\beta = \frac{3\,\lambda^3}{2\,(4\pi)^4} - \frac{3\lambda^5}{2\,(4\pi)^4} \qquad (3.25)$$

Now we can turn to the Yang-Mills theory. The one loop computation will have the following contributions:

i) The gauge field loop, fig. 5, which corresponds to the trace of the a_2 coefficient

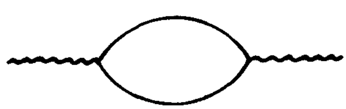


(fig.5)

ii) The ghost loop, fig. 6, corresponding to the trace of the a_2 coefficient in the chiral case. It comes with a factor of 3 (2 Fadeev Popov ghosts, and one Nielsen Kallosh ghost).

(fig.6)

iii) The matter fields, fig.7, which only appear when we have extended supersymnetry. It comes with multiplicity 1 for N = 2, and 3 with N = 4. Each contribution is numerically equal to the ghost contribution, but withopposite sign, to that they cancel for N = 4.



(fig.7)

It is not difficult to see, in the present approach, that i) does not contribute. This happens because in the a_2 coefficient there are two derivatives, namely $D_\alpha\, D_\beta$ (or $\bar{D}_{\dot\alpha}\, \bar{D}_{\dot\beta}$) which are not enough to delete the four dimensional deltas in the Grassmann variables, so that the term vanishes at coinciding points. The only contribution comes from the chiral integrals:

$$\int d^4x\, d^2\theta \int \frac{d\tau}{\tau}\, G^{ch}((x,\theta_x),(x',\theta_{x'});\tau) =$$

$$= \int d^4x\, d^2\theta\, C_2(G) \int_{3\varepsilon}^{\infty} \frac{d\tau}{\tau}\, \frac{e^{-(x-x')^2/4\tau}}{(4\pi)^2} \left(1 - \frac{3\varepsilon}{\tau}\right)^2 g^2\omega^{\alpha\beta} D_\alpha D_\beta \delta^2(\theta-\theta')\Big|_{\theta=\theta'}$$

$$= \frac{C_2(G)}{(4\pi)^2}\, g^2\, \ln(\Lambda\varepsilon) \int d^4x\, d^2\theta\, \omega^2 \qquad (3.26)$$

Taking into account the corract multiplicity and the matter fields, we have

$$\beta^{(1)}(g) = -\frac{(4-N)\,C_2(G)\,g^3}{16\,\pi^2} \qquad (3.27)$$

Now we can turn to the superconformal anomalies. It was shown[16] that the axial current J^5_μ, the supersymnetry current S_μ, and the energy momentum tensor $\theta_{\mu\nu}$ belong to a supermultiplet, and that their transformations under supersymmetry are related by:

$$\delta S^\mu = 2\theta^{\mu\nu}\gamma_\nu\varepsilon + i\gamma^\mu\gamma_5\varepsilon\partial_\rho J^{5\rho} - i\gamma_\rho\gamma_5\varepsilon\partial^\rho J^{5\mu} - \frac{i}{2}\gamma^\lambda\varepsilon\,\varepsilon_{\lambda\mu\nu\rho}\partial^\nu J^{5\rho} \qquad (3.28)$$

Also the axial anomaly, the y-trace anomaly of the supersymmetry current, and the trace of the energy momentum tensor are in a supermultiplet. As a matter of fact, contractlng *(3.28)* with γ_μ we get

$$\delta\gamma_\mu S^\mu = 2\theta_\mu{}^\mu\varepsilon + 3i\gamma_5\varepsilon\partial_\mu J^{\mu 5} \qquad (3.29)$$

In our superfield language, *(3.28)* means that those objects are part of a supercurrent $J_{\alpha\dot\alpha}$, and (3.29) are connected with the supertrace J. The classical conservation is given by

$$\bar{D}^{\dot\alpha} J_{\alpha\dot\alpha} = 0 \qquad (3.30)$$

which quantum mechanically has an anomaly, and (3.30) is transformed into

$$\bar{D}^{\dot\alpha} J_{\alpha\dot\alpha} = c\, D_\alpha J \qquad (3.31)$$

Recently it has been proued using components that (3.31) holds, sothat the supersymnetric structure holds[17]. Using our method, we can easily derive (3.31).

In the Wess-Zumino model the superconformal Noether current is

$$J_{\alpha\dot{\alpha}} = \frac{1}{3} D_a\phi\bar{D}_{\dot{a}}\bar{\phi} + \frac{i}{3}\phi\overleftrightarrow{\partial}_{\alpha\dot{\alpha}}\bar{\phi} \tag{3.32}$$

Classicaly we have

$$D^{\alpha}J_{\alpha\dot{\alpha}} = -\frac{1}{3}D^2\phi\bar{D}_{\dot{\alpha}}\bar{\phi} + \frac{1}{6}\bar{D}_{\dot{\alpha}}D^2\phi\bar{\phi} = 0 \tag{3.33}$$

where the equations of motion where used in the last step. Quantum mechanically we introduce the heat kernel expansion, and we have.

$$\begin{aligned} D^{\alpha}J_{\alpha\dot{\alpha}} &= -\frac{1}{3}D^2\phi\bar{D}_{\dot{\alpha}}\bar{\phi} + \frac{1}{6}\bar{D}_{\dot{\alpha}}D^2\phi\bar{\phi} \\ &= \left[-\frac{1}{3}\Box\bar{D}_{\dot{\alpha}}G(z,z') + \frac{1}{6}\bar{D}_{\dot{\alpha}}\Box G(z,z')\right]_{z=z'} \end{aligned} \tag{3.34}$$

The laplacian operator gives a non zero value when applied to the G_{ℓ} function, selecting out the *a,* coefficient and the result is non zero due to the mismatch (-1/3, 1/6) in (3.34), and we have:

$$D^{\alpha}J_{\alpha\dot{\alpha}} = -\frac{1}{6}\frac{1}{(4\pi)^2}\lambda^2\bar{D}_{\dot{\alpha}}(\bar{\phi}D^2\phi) = -\frac{1}{9}\frac{\beta(\lambda)}{\lambda}\bar{D}_{\dot{\alpha}}(\phi D^2\phi) \tag{3.35}$$

We can use the same procedure for the Yang-Mills theory. The current is (in the following we consider only $N=1$):

$$J_{\alpha\dot{\alpha}} = \omega_{\alpha}\bar{\omega}_{\dot{\alpha}} \tag{3.36}$$

and we shall look at its divergence

$$\bar{D}^{\dot{\alpha}}(\omega^{a}_{\alpha}\bar{\omega}^{a}_{\dot{\alpha}}) = \bar{D}^{\dot{\alpha}}(\bar{D}^2D_{\alpha}V^{a}D^2\bar{D}_{\dot{\alpha}}V^{a}) \tag{3.37}$$

Again we have laplacian operator applied to a Green's function.

$$\bar{D}^{\dot{\alpha}}J_{\alpha\dot{\alpha}} = C_2(G)\left[\Box D_{\alpha}\bar{D}^2G(z,z')\right]_{z=z'} \tag{3.38}$$

selecting out the a, coefficient. As before, a maximum number of derivatives must be applied on the Grassmann δ functions, and we are left with:

$$\bar{D}^{\dot{\alpha}} J_{\alpha\dot{\alpha}} = C_2(G) \frac{g}{(4\pi)^2} D_\alpha(\omega^2) = -\frac{1}{3} \frac{\beta(g)}{g} D_\alpha(\omega^2) \qquad (3.39)$$

showing that the anomalies belong to a supermultiplet.

4. CONCLUSION

We present a regularization scheme which can be used for supersymmetric theories in a superfield formulation. The present scherne works at two loop order, and it is possible to obtain the usual values for the β-function and the anomalies. The latter were obtained only at one loop, but the two loop results are in progress. A convergenceproof for the renormalized theory working to all orders is not available, although this is a very desirable result.

The present scheme has an advantage over supersymrnetric dimensional regularization (SDR), since always work on the physical 4 dimensional space.

Finally, it seems that (3.39) is in contradiction with the Adler-Bardeen theorem. Recently this issue was discussed at lengh[18]. In order to study the problem with the present method, a thorough 2-loop computation is needed for the Yang-Mills field, which is presently under progress.

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Usamos a variável de tempo próprio definida na expansão do "heat-kernel" para regularizar teorias de campos no contexto do método de campo de fundo. O método pode ser naturalmente aplicado a teorias supersimétricas e teorias de calibre. Cálculos explícitos foram feitos, incluindo teoria de perturbação com supercampos.