Revista Brasileira de Física, Vol. 14, n? 2, 1984

Gravitational Instability in Newtonian Cosmology with Varying G

J. P. BAPTISTA, A.B. BATISTA and J.C. FABRIS

Departamento de Física e Química, Universidade Federal do Espírito Santo, 29000, Vitória, ES, Brasil

Recebido em 26 de julho de 1984

Abstract Use is made of McVittie's model for Newtonian Cosmologies with time-varying G as unperturbed Universe, in the study of linear density perturbations. A general equation for the density fluctuation is obtained. In the case with null pressure and $G \sim t^{-1}$ (Dirac cosmological model) there results that the fluctuation density grows faster than the corresponding case for G constant. It is, however, insufficient to explain the formation of Galaxies. The situation where $p\neq 0$ and $k=0$, inthe case of ideal monoatomic gas, leads only to oscillating solutions. Finally, a model that "approximates" a particular solution of Brans-Dicke theory leads to a perturbation that differs slightly from classical or relativistic ones, although insufficient for the applications above mentioned.

1. INTRODUCTION

The possibility that the gravitationel constant G be actually a function of time was first pointed out by Dirac[1] in 1937. This assumption was made as an attempt to explain why tiny numbers should appear in fundamental laws of physics, such as the ratio of the gravitational to the eletric forces between electron and proton, which is of theorder of 10^{-40}. At the first sight there are no simple explanation for that. However, the expresslon

$$(\hbar^2 H_0/Gc)^{1/3} \sim m_\pi$$

which relates the fundamental quantities $\hbar$, G, c and m_π (pion mass) to the Hubble constant, seems to indicate that sush tiny numbers are not determined solely by considerations of microphysics, but also by causes of cosmological nature. Others "coincidences" like, for instance (see Weinberg, ref. 2, page 620),

$$(e^2/m_e c^3\ H_0^{-1}) \sim 10^{-40} \tag{2}$$

seem to indicate that it becomes more difficult to assume the trivial point of view that eqs. (1) and *(2)* are meaningless numerical coincidences. Indeed all these questions, at the first sight, seem to look like pythagorian numerology. However, the existence of several expressions like eqs. (1) and (2) lead us to regard these numerical relations as having a real, though mysterious, significance. This point of view has stimulated the search for explanations.

Dirac proposed that in eq. (1) R, c and m_A are constant and G varies like

$$G \propto \dot{R}/R \propto t^{-1} \tag{3}$$

This assumption leads directly to the Dirac's cosmological model. However, this model is an incomplete one, because Diracdid not specified the correspondlng theory of gravitation having a varying "constant" of gravitation G. Nevertheless, the model makes a number of definite predictions, as we shall see in what follows.

On the other hand, the first theory of gravitation having a time dependent gravitational constant was proposed by Jordan[3]. However, this theory was subjected to strong criticism[4] mainly by the fact that its energy-rnomentum tensor has not a null-divergence. Among the attempts inspired by Dirac's theory, the most interesting and complete scalar--tensor theory of gravitation is that proposed by Brans and Dicke[5]. Under a relativistic approach, this theory incorporates, beside the gravitational potential $g_{\alpha\beta}$, a scalar field ϕ that replaces G^{-1} and is generated, in principle, by the distant matter of the universe. This field obeys the equation

$$\Box\ \phi = 8\pi/(3 + 2\omega)T^{\mu}{}_{\mu}$$

where $T_{\mu\nu}$ is the energy-momentum tensor of matter and ω is a dimensionless coupling parameter. As expected, the theory of Brans and Dicke contains the Einstein's theory of general relativity, which is recovered by putting $\omega \to \infty$ in Brans-Dicke theory. So, the isotropic and homogeneous Brans-Dicke cosmological model falls into the Friedmann solution in that limit.

In the last years, several workers have concentrated attention on theories with varying G, because some of its implications are, in principle, observable. For instance, the varying G affects the cosmological expansiori, with consequence in the process of cosmic nucleo synthesis, resulting in an helium production whose abundance differs from the corresponding result obtained in general relativity[6]. Another consequence is the reformulation of the age of the stars and globular clusters, as well as of the thermal history of the Earth[7].

This paper has two aims: 1) To show, uslng a newtonian model, the influence of the varying gravitational constant on the evolution of the density perturbations; we note that, in the present case, the density contrast obeys a forced oscillation equation with damping; in particular, we apply this result in the Dirac's model; 2) To calculate the density contrast in a special frame which constitutes the "newtonian approximate" of the Brans-Dicke theory. We must remark that a cosmological newtonian model with varying G does not constitute an approximation of the Brans-Dicke theory. If we make $\omega \to \infty$ and $\phi = cte,$ we obtain the general relativity theory, and the late reduces to a newtonian theory with constant G. On the other hand if in Brans-Dicke theory, we consider a distribution of matter such that $p \ll \rho c^2$ without imposing $\omega \to \infty$, the gravitational interaction can be described in a newtonian frame, but in this frame we have not a corresponding equation for ϕ.

In what follows we study the evolution of density perturbation using as non-perturbed model the solutions given by McVittie (see ref. 10) which describes a newtonian cosmology modified by introducing the time variation of the gravitational constant G.

2. PERTURBED EQUATIONS

The basic equations used here are Euler, continuity and Poisson equations, with the late one modified by the substituition of G by G = $= G_0 F(t/T)$, with G_0 and T constants, and F a dimensionless function of the time[10]. We set (ρ, p, v, g, F) as a non-perturbed McVittie's solution, and treat $(\rho_1, p_1, \vec{v}_1, \vec{g}_1, F_1)$ as small perturbations. Introducing the combinations

$$\rho + \rho_1,\ p + p_1,\ \vec{v} + \vec{v}_1,\ \vec{g} + \vec{g}_1,\ F + F_1 \tag{4}$$

in the basic equations and neglecting terms of second order in the perturbations, we can write the perturbed equations

$$\begin{aligned}
&\partial \rho_1/\partial t + 3(\dot{R}/R)\rho_1 + (\dot{R}/R)\vec{x}.\vec{\nabla}\rho_1 + \rho\vec{\nabla}.\vec{v}_1 = 0\\
&\partial \vec{v}_1/\partial t + (\dot{R}/R)\vec{v}_1 + (\vec{v}.\vec{\nabla})\vec{v}_1 = -(v_s^2/\rho)\vec{\nabla}\rho_1 + \vec{g}_1\\
&\vec{\nabla}.\vec{g}_1 = -4\pi G_0 F\rho_1 - 4\pi G_0 \rho F_1\ ;\ \vec{\nabla}\times\vec{g}_1 = 0;\ \dot{f} \equiv df/dt
\end{aligned} \tag{5}$$

where v_s is the sound velocity. Supposing that the perturbations will propagate as plane waves, we put

$$\phi(x,t) = \phi(t)\exp(-i\vec{q}.\vec{x}/R) \tag{6}$$

where $\vec{q}$ is the wave vector. The function $R(t)$ in this expression represents the influence of the expansion of the newtonian cosmological fluid in the wave length of the perturbations. Splitting the velocity $\vec{v}_1$ in components parallel and perpendicular to the vector $\vec{q}$

$$\vec{v}_1 = \vec{v}_{1\parallel} + \vec{v}_{1\perp} \tag{7}$$

and following Weinberg[2] we put

$$\vec{v}_{1\parallel} = i\vec{q}\varepsilon(t) \; ; \; \varepsilon(t) = -\vec{q}.\vec{v}_1/q^2 \tag{8}$$

From eqs. (6) and (7) we obtain

$$\vec{g}_1 = 4\pi G_0 iR(\rho_1 F + \rho F_1)\vec{q}/q^2 \tag{9}$$

Taking into account eqs. (7), (8) and (9), the system of eqs. (5) is rewritten as

$$\dot{\vec{v}}_{1\perp} + (\dot{R}/R)\vec{v}_{1\perp} = 0 \tag{10}$$

$$\dot{\varepsilon} + (\dot{R}/R)\varepsilon = -v_s^2(\rho_1/R)/\rho + 4\pi G_0 R(\rho_1 F + \rho F_1)/q^2 \tag{11}$$

There are two normal modes of oscillation for the perturbations. The first one, eq. (10), gives the rotational mode where $\vec{v}_{1\perp}$ falls as $R^{-1}(t)$. This time function is given by the non-perturbed McVittie's solution. The second, eq. (11), gives the compressional or longitudinal mode. If we define the density contrast $\delta(t)$ as $\delta(t)=\rho_1/\rho$ we get $\dot{\delta} = \varepsilon q^2/R$. Putting these expressions in eq. (11), we obtain

$$\ddot{\delta} + (2\dot{R}/R)\dot{\delta} + (v_s^2 q^2/R^2 - 4\pi G_0\rho F)\delta = 4\pi G_0\rho F_1 \tag{12}$$

In this equatfon for the density contrast we recover the classical newtonian result by putting $F_1 = 0$. Obviously, the second member of eq. (12) is a consequence of the assumed hypothesis that G is a function of time.

Eq. (12) represents a pattern of a very simple motion which allows us to see immediately how the variability of G affects the evolution of the perturbation. In this case a density perturbation generated in an expanding cosmological fluid evolves like a forced oscillatory motion with damping. We think that the corresponding equation for the evolution of the density contrast given by a relativistic' theory will not have a forced term: the forces will be absorbed by geometrization. Its effect will be given by the metric tensor of space-time.

The expanding newtonian cosmological model is gravitationally unstable, with a growth of the density perturbation, if we have

$$k < k_J = (4\pi G_0 \rho F/v_s^2)^{1/2} \qquad (13)$$

where $k = q/R$ is the wave vector of the perturbation and k_J stands for Jean's vector. The time evolution of the density contrast $\delta(t)$ will be the consequence of three simultaneous actions: i) the collapsing action of the gravitational field of the perturbation; ii) the damping action produced by the expansion of the background; iii) the forced action, produced by the time variation of G.

However, when we solve eq. (12) for several cosmological models obtaining an expression for the growth of perturbation, we see that the influence of the forced term on this growth is not so important. The numerical contribution of this second member is small, as we shall see.

3. DENSITY PERTURBATIONS IN DIRAC'S COSMOLOGY

According the McVittie's notation for the Dirac's cosmological model, we have $F(t/T) = T/t$. We assume that the perturbation of F in this model behaves like $F_1 = T^2/t^2$. In McVittie's solution for the non--perturbed equation, there is a constant K implying three kinds of solutions corresponding to two relativistic models endowed with a non-null curvature ($K<0$ and $K>0$), and the flat model, ($K=0$). We use here this notation and we will find the solutions for the eq. (12) for $K>0$, $K<0$, $K=0$, in the particular cases where the material distribution is such that $p=0$ and $p\neq 0$. It is well known that for models such that $p=0$, the perturbations are always unstable. For models such that $p\neq 0$, instabilities will be produced by perturbation such that $k<k_J$. In both cases eq. (12) has solutions for which $\lim_{t\to\infty} \delta(t)\to\infty$. In the study of the perturbation for all cases, the solutions of eq. (12) will be written

under the notation δ_+, δ_-, δ_p. The notation δ_+ and δ_- stands for growing and decaying solutions of the homogeneous equation, and δ_p stands for the particular solution of the non-homogeneous eq. (12). As it is known, the general solution comes out from the linear combination of these solutions.

Case I: $p=0$

1.1 - $K > 0$

The parametric solutions for the unperturbed model are[11]

$$\tau \equiv t/T = A^{3/2} (B - \theta + \operatorname{sen}\theta)^{-1} \qquad 0 < \theta \leq B \qquad (14)$$
$$R = (G_0 M T^2)^{1/3} A^{1/2} (1 - \cos\theta)(B - \theta + \operatorname{sen}\theta)^{-1}$$

where R is the scale factor of the expansion, τ is the time and A, B, M, G_0, T are positive constants. Eq, (12) gives for this case

$$(1 - \cos\theta) d^2\delta/d\theta^2 + \sin\theta d\delta/d\theta - 3\delta = 3A^{-3/2}(B - \theta + \sin\theta) \qquad (15)$$

The corresponding solutions are

$$\delta_+ \propto (5 + \cos\theta)/(1 - \cos\theta) - 3\theta\sin\theta/(1 - \cos\theta)^2 \qquad (16)$$

$$\delta_- \propto \sin\theta/(1 - \cos\theta)^2 \qquad (17)$$

$$\delta_p \propto (2\sin\theta - \theta\cos\theta - 3\theta + B)/(1 - \cos\theta) - \sin\theta(6 + \theta^2 + B\theta)/(1 - \cos\theta)^2 \qquad (18)$$

1.2 - $K < 0$

In this case the solutions for the unperturbed model are

$$\tau = A^{3/2}(B_0 + \Omega - \sinh\Omega)^{-1} \qquad (19)$$
$$R = (G_0 M T^2)^{1/3} A^{1/2} (\cosh\Omega - 1)/(B_0 + \Omega - \sinh\Omega)$$

Now eq. (12) has the form

$$(\cosh\Omega - 1) d^2\delta/d\Omega^2 + \sinh\Omega d\delta/d\Omega - 3\delta = 3A^{-3/2}(B_0 + \Omega - \sinh\Omega)$$

with solutions

$$\delta_+ \propto (5 + \cosh\Omega)/(\cosh\Omega - 1) - 3\Omega\sinh\Omega/(\cosh\Omega - 1) \quad (20)$$

$$\delta_- \propto \sinh\Omega/(\cosh\Omega - 1)^2 \quad (21)$$

and

$$\delta_p \propto (2\sinh\Omega - \Omega\cosh\Omega - 3\Omega + B_0)/(\cosh\Omega - 1) + \sinh\Omega(6 + \Omega^2 + B_0\Omega)/(\cosh\Omega - 1)^2 \quad (22)$$

1.3 - $K = 0$

We take a McVittie's solution that gives $R \to 0$ when $\tau \to 0$, and $R \to \infty$ when $\tau \to \infty$. With this choice, we have

$$R = (9G_0MT^2/2)^{1/3}\ \tau^{1/3} \quad (23)$$

The differential equation for δ is, in this case

$$\ddot{\delta} + (2/3)\dot{\delta}/\tau - (2/3)\delta/\tau^2 = (2/3)/\tau^3 \quad (24)$$

and the corresponding solutions are

$$\delta_+ \propto \tau \quad (25)$$

$$\delta_- \propto \tau^{-2/3} \quad (26)$$

$$\delta_p \propto \tau^{-1} \quad (27)$$

Case II: $p \neq 0$

We examine here only the flat model, $K=0$. Assuming that the cosmological fluid is an expanding perfect gas with equation of state $p \propto \rho^\gamma$, we can write

$$v_s^2 \propto \rho^{\gamma-1} \propto R^{-3(\gamma-1)}$$

or, using eq. (23)

$$v_s^2 \propto \tau^{-(\gamma-1)} \quad (28)$$

where $\gamma = C_p/C_v$, the ratio of specific heats. The equation for the density contrast is

$$\ddot{\delta} + (2/3)\dot{\delta}/\tau + \{\Lambda^2/\tau^{\gamma-1} - (2/3)/\tau^2\}\delta = (2/3)/\tau^3 \quad ; A = \text{constant} \quad (29)$$

This equation has homogeneous solutions of the form

$$\delta_\pm \propto \tau^a J_{\pm\mu}(r\tau^s) \quad (30)$$

where $J_{\pm\mu}$ is the Bessel function of order μ. The solution of the non-

-homogenous equation is

$$\delta_p \propto -\delta_+ \int_{x_0}^{\tau} \{\delta_- h/W(\delta_+,\delta_-)\}dx + \delta_- \int_{x_0}^{\tau} \{\delta_+ h/W(\delta_+,\delta_-)\}dx \qquad (31)$$

where $h=(2/3)/\tau^3$ and W is the wronskian

$$W(\delta_+,\delta_-) = \Lambda\tau^{2a+s-1}(J_\mu J'_{-\mu} - J'_\mu J_{-\mu}) = s\tau^{2a-1}\sin(\mu\pi) \qquad (32)$$

Then,

$$\delta_p \propto -\tau^{-1}J_\mu(\Lambda\tau^s/s) \sum_{k=0}^{\infty} \{(-1)^k/k!\Gamma(k-\mu+1)\}(\Lambda\tau^s/2s)^{2k-\mu}(2ks-2)^{-1}$$

$$+ \tau^{-1}J_{-\mu}(\Lambda\tau^s/s) \sum_{k=0}^{\infty} \{(-1)^k/k!\Gamma(k+\mu+1)\}(\Lambda\tau^s/2s)^{2k+\mu}3(6ks-1)^{-1} \qquad (31')$$

Eqs. (30) and (31') are solutions of eq. (29) if

$$\Lambda = \pm rs$$

$$s = (7/3 - \gamma)/2 \qquad (33)$$

and

$$a = 1/6 \ ; \ \mu = 5/6s$$

For a given value of γ, eqs. (30), (31') and (32) give us the complete solution to the density contrast δ in this case.

Some comments about the solution proposed, concerning the Dirac's cosmology, are necessary. We easily see that in the case of $p=0$ and $K\neq 0$ the homogeneous solution given by eqs. (16), (17), (20) and (21) are the classical and well known solution (see for instance, ref. 12). The new contribution is given by eqs. (18) and (22). However, for $\theta \to 0$ and $R \to 0$, $\delta_p(\theta)$ and $\delta_p(R)$ fall like τ^{-1}, i.e., tend to a vanishing contribution.

The description of the evolution of perturbations in an expanding background with $p\neq 0$ is a little more complicated. In spite of the fact that the calculation for the case $K\neq 0$ can be easily made, we have restricted our study to the flat model, because in this case the solutions can be more easily compared to the corresponding solutions given in the case $p=0$ in the asymptotic limit. For large values of τ the influencs coming from the pressure becomes less important; then, the complete solution of eq. (29) (for $K = 0$ and $p \neq 0$) must be asymptotically compatible with the solution given by eqs. (25), (26) and (27) for the case $K=0$ and $p=0$. But we see, by eq. (33) that this happens only if $s < 0$, which implies $\gamma > 7/3$. However, this restriction on γ is not a realistic one; it is generally accepted that, during the time while the perturbation evolves, the cosmological fluid is an expanding perfect gas. On the other hand we can readly verify that for values of γ such that $\gamma < 7/3$ the Bessel function oscillates, in contraction with the behaviour of the solution given by eqs. (25), (26) and (27). For instance, eqs. (30) and (31) give, for $\gamma = 8/3$ and large τ

$$\delta_+ \propto \tau \; ; \quad \delta_- \propto \tau^{-2/3} \quad ; \quad \delta_p \propto \tau^{-1} \tag{34}$$

So, the compatibility between this solution and the asymptotic behaviour given in the case $K=0$ and $p=0$ is obtained only be means of a non-realistic assumption.

4. AN "APPROXIMATE MODEL" TO THE BRANS-DICKE THEORY

For $K=0$, McVittie proposed a famíly of solutions (ref. 10, eq. 3.18) given by

$$\begin{aligned} G &= G_0 \tau^{-(\nu+3)} \\ R &= \{-9G_0 M T^2/(\nu+1)(\nu+4)\}^{1/3} \tau^{-(\nu+1)/3} \\ \rho &= \{(\nu+1)(\nu+4)/(12\pi G_0 T^2)\} \tau^{\nu+1} \end{aligned} \tag{35}$$

The classical newtonian model and the Dirac's model are recovered by choosing $\nu = -3$ and $\nu = -2$, respectively.

We now write the general eq. (12) for density perturbation taking into account *eq.* (35) for the background. Again, we consider the cosmological fluid as an expanding perfect gas with a fixed $\gamma = C_p/C_v$. From eqs. (35) we can write

$$v_s^2 \propto \rho^{\gamma-1} \propto \tau^{(\nu+1)(\gamma-1)} \quad ; \; R^2 \propto \tau^{-2(\nu+1)/3}$$

Then we have

$$(v_s^2 q^2 T^2)/R^2 = \Lambda \, \tau^{(\nu+1)(\gamma-1/3)}$$

Putting these expressions into eq. (12), we have

$$\ddot{\delta} + (2/3)(\nu+1)\dot{\delta}/\tau + \{\Lambda^2/\tau^{-(\nu+1)(\gamma-1/3)} + (\nu+1)(\nu+4)/\tau\}\delta =$$

$$= -(1/3)(\nu+1)(\nu+3)(\nu+4)/\tau^3 \qquad (36)$$

where $\dot{\delta} \equiv d\delta/d\tau$ and $\tau = t/T$. This equation gives the evolution of the contrast in density for all McVittie's background models, i . , for $-1 > \nu \geqq -3$.

The complete solution of eq. (31) is given by the eqs. (30) and (31'), with

$$a = (5 + 2\nu)/6 \quad ; \; s = \{(\nu+1)\gamma + (5-\nu)/3\}/2$$
$$\mu^2 s^2 = (2/3)(\nu+1)a - (a-1)a - (\nu+1)(\nu+4)/3 \qquad (37)$$

The solution for the Dirac's model showed above, as well as the classical results given by Weinberg (ref. *2*, eqs. 15.3.39 and 15.9.43) are obtained by putting into the expression (37), $\nu = -2$ and $\nu = -3$ respectively and replacing the resulting values of a, s and μ into eqs. (30) and (31).

We show now the calculation for the evolution of the contrast δ by considering a newtonian model which we call "newtonian approximate" to an isotropic and homogeneous model of the Brans-Dicke theory. We define a "newtonian approximate" as the newtonian cosmological model with a varying constant G with a time behaviour of the functions G, R and ρ being the same as the corresponding functions given by one of the cosmological solutions from Brans-Dicke[5].

We choose in Brans-Dicke theory the cosmological solutions

$$\phi \propto \tau^{2/(4+3\omega)} \quad ; \; R \propto \tau^{(2\omega+2)/(3\omega+4)} \qquad (38)$$

to be put in correspondence with a "newtonian approximate" model. We remark that this solution goes smoothly to the Friedmann flat model, i.e., ϕ = constant, $R \propto t^{2/3}$.

Taking $\omega=6$ in eq. (38), the "newtonian approxirnate" model will comes out from eqs. (35) with $\nu = -2.91$. Taking into account these values, eq. (37) gives, for a perfect and monoatomic gas with $\gamma = 5/3$, the values $a = -0.13$ and $s = -0.38$. Replacing these values into eq. (30) and (31') we obtain, finaliy, the complete solution of eq. (36), which is asymptotically consistent (for $\tau \gg \Lambda^{-s}$) with the null pressure solutions ($\Lambda=0$)

$$\delta_+ \propto \tau^{0.72} \; ; \; \delta_- \propto \tau^{-0.98} \; ; \delta_p \propto \tau^{-1} \qquad (39)$$

These results are slightly different from the classical ones given by Lifshit's calculations in general relativity, where the density perturbations grow like $\delta_+ \propto \tau^{2/3}$.

5. CONCLUSION

We have considered small perturbations of the solution of the newtonian cosmology with varying G, and we have derlved the differential equations which describes the evolution for the contrast in density for any member of the family of solutions proposed by McVittie. We obtain, in particular, the Dirac's model by putting, for $K=0$, v=-2. The calculation of the contrast for this model is, for a null-pressure matter

$$\delta_+ \propto \tau \; ; \; \delta_- \propto \tau^{-2/3} \; ; \delta_p \propto \tau^{-1} \qquad (40)$$

For $p\neq0$, the complete solution of eq. (36) which is asymptotically consistent with eq. (40) is obtained for $\gamma > 7/3$. For values of γ smaller than 7/3, 6 oscillates. This fact means that if we accept the only mathematically consistent solutions, we must adopt a gas model not compatible with the hypothesis of a perfect and monoatomic gas for the cosmological background.

On the other side, Dirac's cosmology predicts values for observable quantities which do not agree with the available data. For instance: i) deceleratlon factor, $q_0 = 2$; ii) age of the Universeas 1/3 of the value accepted today; iii) abundance of He less than 0.20; iv) temperature on the Earth's surface at 4×10^9 years ago around 500^0 K, which is not in agreement with the geophysical results that show the existence of liquid water at that epoch[7]. From all our results shown above, we can conclude that Dirac's cosmology is not a good candidateto be considered as a "newtonian approximate" model to the Brans-Dicke cosmology.

The "newtonian approximate" is the model obtained by putting, for $K=0$, v = -2.91 in McVittie's solution. The expressions for the general solution in this case are given by eqs. (30), (31') and (37), with $\gamma=5/3$. This solution is asymptotically consistent with the one given by eq. (35) and, in this case, the growing perturbation δ_+ grows faster than the growing perturbation given by general relativity. Although it is not enough to explain the galaxies formation. Nevertheless, this model predicts "good" values for the quantities above mentioned: $q_0 \sim 0.57$, Helium abundance, age of the Universe, description ofthermal history of the Earth, all in agreement with the most recents available values (see refs. 2 and 7).

On the other hand, it is expected that the Brans - Dicke relativistic cosmological models be gravitationally unstable, giving growing perturbations in agreement with eq. (39). We are still studying this problem. We remark that N. Bandyopadhyay[13] has published a paper where he proposes a calculation for density perturbations in Brans-Dicke theory by means of the variation of the Raychaudhuri's equations, obtaining an equation for δ that is not compatible (see equation 3.9 of ref. 13) with the general relativity in the limit $\omega \to \infty$.

REFERENCES

1. P.A.M. Dirac, Nature *139*, 323 (1937).
2. S. Weinberg, *Gravitation and Cosmology* (1972).
3. P. Jordan, Nature 164, 137 (1949).
4. H. Bondi, *CosmoZogy*, University Press, Cambridge (1961).
5. C. Brans-R.Dicke, Phys. Rev. 124, n? 3, 925 (1961).
6. P. J. E. Peeble, *Cosmology: The Physics of Large Scale Structure*, Princeton University Press (1980).
7. V.M. Canuto & I. Goldman, Nature 304, 311 (1983).
8. W.B. Bonnor, Mon.Not.R.Astr.Soc. 116, 351 (1956).
9. E. Lifshlts, Zh. Eksp. Teor. Phys. 16, 587 (1946); Thēorie du Champ, L. Landau and E. Lifshitz, Ed. Mir. Moscow.
10. G.C. McVittie, Mon.Not.R.Astr.Soc., 183, 749 (1978).
11. J.P. Baptista, Rev. Bras. Fis. 8, 396 (1978).
12. G.B. Field, Stars and Stellar Systems, Ed, A.R. Sandage & M.Sandage, University of Chicago; IX (1967).
13. N. Bandyopachyay, J.Phys. A: Math. Gen. 10, n? 2, 189 (1977).

Calculamos a evolução das perturbações na densidade que se propagam em modelos newtonlanos a G variável, gravitacionalmente instáveis. Encontramos uma equação geral para o contraste $\delta(t)$, válida para qualquer modelo do tipo mencionado. A aplicação para a evolução das perturbações em cosmologia de Dirac, onde G é suposto variar como t^{-1}, revela que as perturbações crescentes evoluem sensivelmente mais rapidamente que aquelas já conhecidas em teoria newtoniana clássica. Por outro lado, as soluções para $\delta(t)$ em cosmologia de Dirac se apresentam assintoticamente inconsistentes. Definimos em seguida um modelo newtoniano a G variável "aproximativo" ã teoria relativística de Brans-Dicke, o qual admite perturbações crescentes ligeiramente diferentes das perturbações clássicas, newtonianas ou relativísticas, porém ainda insuficientemente rãpidas para o fim a que se destinam.