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Specific Heat of the Ising Linear Chain in a Random Field

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Abstract Starting from correlation identities for the Ising model we study the effect of a random ficld on the one dimension version of the model. Explicit results for the magnetization, the two-particle correlation function and the specific heat are obtained for an uncorrelated distributio-of the random fields.

The comparison of the energy gained by forming small randomly oriented domains in the random field with the energy lost at the boundaries of the domains, lead Imry and Ma[1] to conclude that only three-dimensional Ising systerns would remain oriented in the presence of a random field. They showed that the effect of a small random field is to reduce the lower critical dimensionality (d_c) of these systems to d_c=2. Fishman and Aharony[2] argued that the behavior of the Ising ferromagnet in a random field would be the same as in a randomly diluted Ising antiferromgnet in the presence of an external constant field. Since then, a great number of both experimental and theoretical works[3,4] have been published on the subject.

Two experimental techniques have been used in order to investigate such systems: neutron-scattering and cptical birefringence experiments. Early neutron-scattering investigations performed by Yoshizawa et *al*[5] showed that the randoin field destroys the order in both two and the three-dimensional systems. On the other hand optical birefringence measurements[3,6] show that the random field destroys long-range magnetic order at two-dimensions as in the $Rb_2 Co_{0.85} Mg_{0.15} F_4$ case, and increases the sharpness of trançition at three-dlmensions as in the $Fe_{0.6} Zn_{0.4} F_2$ case. Also, according to Belanger et *al*[3], recent

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neutron scattering studies support the conclusions obtained via the optical birefringence measurements cited above.

Theoretical investigations in the random field Ising model (RFIM) have been done both by Aharony[7] and Schneider and Pytte[8] using the mean field approximation on the treatment of different probability distributions for the random fields. Grinstein and Ma[9] have found based on an interface model that the lower critical dimension of such systems is two. Also Fernandez et al[10] studying by numerical methods the solid-on-solid interface of the random field Ising model in two- dimensions predicted that $d_c = 2$. Finally exact results for one-dimensional RFIM have been obtained by Grinstein and Mukamel[11] which show that the structure factor consists of both Lorentzian and Lorentzian-squared for $T>0$ and only Lorentzian at $T=0$.

In this work we use the differential exponential operator technique developed by Kaneyoshi and co-workers[12] applied to a set of exact relations established by Callen[13], in order to obtain the internal energy and the specific heat expressions for the one-dimensional RFIM.

Let us write the hamiltonian as

$$H = -\frac{1}{2}\sum_{ij} J_{ij}\sigma_i\sigma_j - \sum_i h_i\sigma_i \tag{1}$$

where h_i is the random field, $\sigma_i = \pm 1$ are Ising variables and J_{ij} is the exchange interaction between nearest neighbors sites.

The set of correlation identities due to Callen[13] are given by

$$<\{i\}\,\sigma_i> = <\{i\}\tanh\beta E_i> \tag{2}$$

where $<\ldots>$ means thermal average, $\{i\}$ represents any function of Ising variables as long as it is not a function of the site i, and E_i is given by

$$E_i = \sum_j J_{ij}\sigma_j + h_i \tag{3}$$

As usual, $\beta \equiv 1/kT$.

Setting $\{i\} = 1$ and $\{i\} = \sigma_k$ in eq.(2) we get the following identities

$$<\sigma_i> = <\tanh\beta E_i> \tag{4}$$

and

$$< \sigma_k \sigma_i > = < \sigma_k \tanh\beta E_i > \tag{5}$$

which are related to the magnetization and to the two-particle correlation function, respectively.

Introducing into the previous expressions the differential exponential operator

$$e^{\alpha D} f(x) = f(x + \alpha)$$

with $D \equiv \partial/\partial x$, we obtain

$$<\sigma_i> = < \prod_{j=\text{neighbors}} (\cosh \beta JD + \sigma_j \sinh \beta JD)>.\tanh(x + \beta h_i)\Big|_{x=0} \tag{6}$$

and

$$<\sigma_k\sigma_i> = <\sigma_k \prod_{j=\text{neighbors}} (\cosh \beta JD + \sigma_j \sinh \beta JD)> \tanh(x + \beta h_i)\Big|_{x=0} \tag{7}$$

Now let the distribution of random fields be uncorrelated and given by

$$P(h_i) = \frac{1}{2} p[\delta(h_i-h) + \delta(h_i+h)] + q\ \delta(h_i) \tag{8}$$

such that $p+q = 1$. The above distribution (with $h=\infty$) has been treated by Grinstein and Mukamel[11] and in the particular case $p=1$ $(q=0)$ we get the distribution proposed by Aharony[7] which has treated the RFIM in the mean field approximation.

Using eq. (8) into eqs. (7) and (6), and taking the configurational average we get

$$\begin{aligned} <<\sigma_i>>_h = & << \prod (\cosh \beta JD + \sigma_j \sinh \beta JD)>>_h \\ & \times\{ \frac{1}{2} p[\tanh(x+\beta h) + \tanh(x-\beta h)] + q \tanh x\}\Big|_{x=0} \end{aligned} \tag{9}$$

and

$$\begin{aligned} <<\sigma_k\sigma_i>>_h = & << \sigma_k \prod_j (\cosh \beta JD + \sigma_j \sinh \beta JD)>>_h \\ & \times \{ \frac{1}{2} p[\tanh(x+\beta h) + \tanh(x-\beta h)] + q \tanh x\}\Big|_{x=0} \end{aligned} \tag{10}$$

where the symbol $< \dots >_h$ means configurational average. Oneshould note that eqs. (9) and (10) are no longer exact. We have neglected possible field correlations between the terms $<\sigma_i>$ (or $<\sigma_i \sigma_3>$) and $\tanh(x+\beta h_i)$. In other words, we assumed that $<<\sigma_i> \tanh(x+\beta h_i)>_h = <<\sigma_i>><\tanh(x+\beta h_i)>_h$.

For the one-dimensional case the index j runs over the two nearest-neighbors of the site i. We get for this case

$$<<\sigma_i>>_h = (\cosh^2 \beta JD + \frac{1}{2} <<\sigma_{i-1}>>_h \sinh 2\beta JD$$

$$+ \frac{1}{2} <<\sigma_{i+1}>>_h \sinh 2\beta JD + <<\sigma_{i+1}\sigma_{i-1}>>_h \sinh^2\beta JD) \{ \frac{1}{2} p[\tanh(x+\beta h)$$

$$+ \tanh(x-\beta h)] + q \tanh x\}\Big|_{x=0} \tag{11}$$

We notice that the function

$$f(x) = \frac{1}{2} p[\tanh(x+\beta h) + \tanh(x-\beta h)] + q \tanh x$$

is an odd function of the argument x and therefore the action of an even operator on it will give null result. We obtaln for the magnetization

$$m = m \{ \frac{1}{2} p[\tanh\beta (2J+h) + \tanh\beta (2J-h)] + q \tanh 2\beta J\} \tag{12}$$

where we have used

$$<<\sigma_i>>_h = <<\sigma_{i+1}>>_h = <<\sigma_{i-1}>>_h = m$$

From eq.(12) we see that m can only be different from zero at $kT = 0$ ($\beta \to \infty$) and for $h < 2J$.

We can also in the same way obtain for the two-particle correlation function the result

$$<<\sigma_k\sigma_i>>_h = \frac{1}{2} (<<\sigma_k\sigma_{i-1}>>_h + <<\sigma_k\sigma_{i+1}>>_h)$$

$$\times \{ \frac{1}{2} p[\tanh\beta(2J+h) + \tanh\beta(2J-h)] + q \tanh 2\beta J\} \tag{13}$$

In order to pursue further let us define the quantities

$$\alpha = \frac{1}{2} p\left[\tanh\beta(2J+h) + \tanh\beta(2J-h)\right] + q \tanh 2\beta J \tag{14}$$

$$\langle\langle \sigma_k \sigma_i \rangle\rangle_h = g(r) \;, \; \langle\langle \sigma_k \sigma_{i+1} \rangle\rangle_h = g(r+1) \tag{15a}$$

and

$$\frac{g(r+1)}{g(r)} = \gamma \tag{15b}$$

where $\vec{k} - \vec{i} = \vec{r}$ and $g(0) = 1$. The assumption implied by eq. (15b) is consistent with the approximate relations (9) and (10). Taking eqs. (14) and (15) into eq. (13) we obtain

$$1 = (\gamma + \frac{1}{\gamma}) \frac{1}{2} \alpha \tag{16}$$

Solving this equation for γ we obtain

$$\gamma_{1,2} = \frac{1}{\alpha} \pm (\frac{1}{\alpha^2} - 1)^{1/2} \tag{17}$$

The exact limit for $h = 0$ is accomplished by choosing for γ the solution

$$\gamma_2 = \frac{1}{\alpha} - (\frac{1}{\alpha^2} - 1)^{1/2} \equiv \varepsilon$$

We obtain

$$\varepsilon \equiv \langle\langle \sigma_i \sigma_{i+1} \rangle\rangle_h = \frac{1}{\alpha} - (\frac{1}{\alpha^2} - 1)^{1/2} \tag{18}$$

In figure 1, we show the correlation function ε for $p = 1$ $(q = 0)$, as a function of kT/J, for various values of h/J. We notice that the effect of the random field in the one-dimensional Ising model is to destroy partially the short-range order at any finite temperature.

The internal energy U is given by

$$U = \langle\langle H \rangle\rangle_h = \langle\langle -\frac{1}{2} \sum_{ij} J_{ij} \sigma_i \sigma_j \rangle\rangle_h = -NJ\varepsilon \tag{19}$$

As we can see the random field only contributes to the internal energy via the correlation function.

The specific heat obtained from eq. (19) is

$$\frac{C_v}{Nk} = \frac{J}{k} \left\{ \frac{1}{\alpha^2} \left[1 - \frac{1}{(1-\alpha^2)^{1/2}}\right] \right\} \frac{d\alpha}{dT} \tag{20}$$

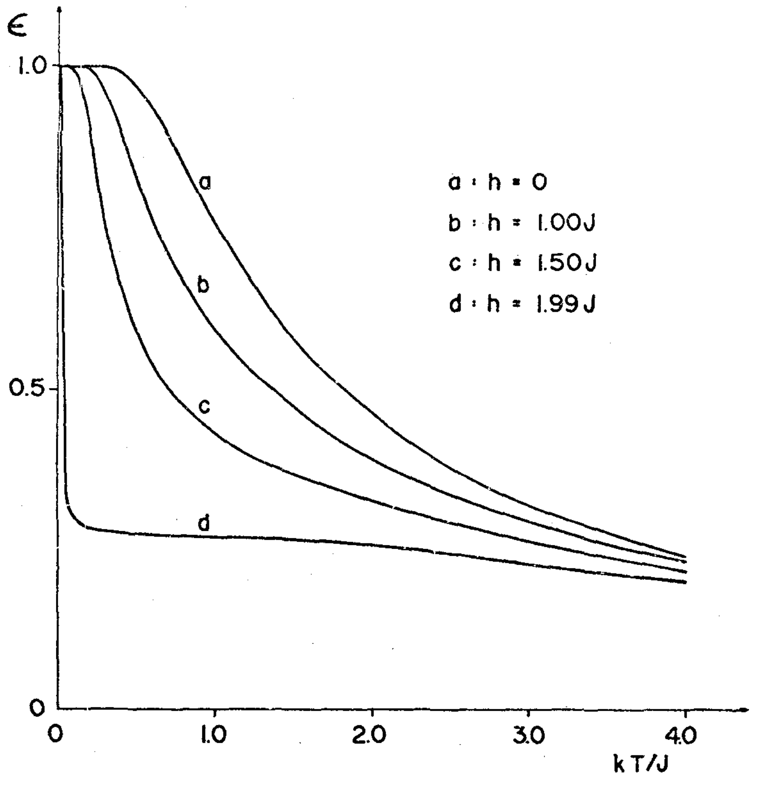

Fig.1 - Nearest-neighbors two-particle correlation function of the one-dimensional Ising model in a random field, as a function of kT/J, for various values of h/J and for $p = 1$.

where

$$\frac{d\alpha}{dT} = -\left(\frac{J}{kT^2}\right)\left\{\frac{1}{2}\,p\left[\left(2+\frac{h}{J}\right)\,\mathrm{sech}^2\,\frac{J}{kT}\left(2+\frac{h}{J}\right)\right.\right.$$

$$\left.\left.+\left(2-\frac{h}{J}\right)\,\mathrm{sech}^2\,\frac{J}{kT}\left(2-\frac{h}{J}\right)\right] + 2q\,\mathrm{sech}^2\,\frac{2J}{kT}\right\} \qquad (21)$$

and a is given by the relation (14).

In figure 2 we plot C_v, for $p = 1$ $(q=0)$, as a function of kT/J, for various values of h/J. For high field the maximum of C_v increases in intensity and occurs in lower temperatures.

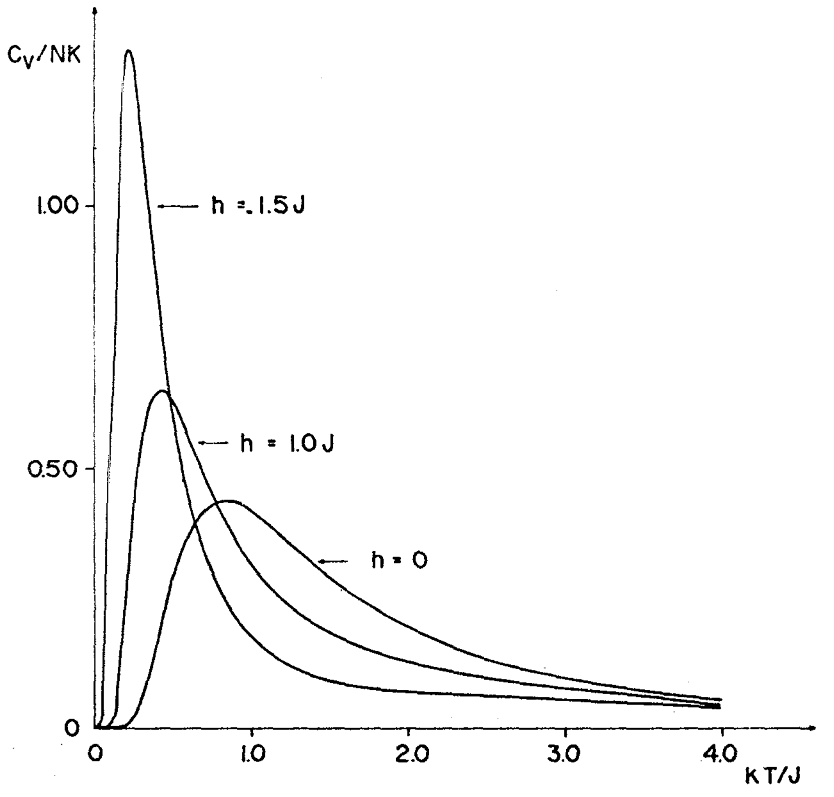

Fig.2 - Specific heat of the one-dimensional Ising model in a random field, as function of kT/J, for various values of h/J and for $p = 1$.

In order to calculate m at T = 0 we use the relation

$$m^2 = \lim_{r\to\infty} \varepsilon^r \tag{22}$$

Looking at eqs. (18) and (14), we note that a = 1, at $T = 0$, for any value of $p \neq 0$, as long as $h < 2J$. As a = 1 implies that $\varepsilon = 1$, we have in this situation $m^2 = 1$. For any finite temperature or for $h > 2J, \alpha < 1$ implies that $\varepsilon < 1$, and consequently that $m=0$. We see that h = 25 is the critical field which frustrates the bonds, destroying the long-range order at T = 0.

As a last remark we recall that recently have been reported optical birefringence measurements on the quasi-one-dimensional spin $\frac{1}{2}$

Ising diluted antiferromagnet crystal Cs $Co_{(1-x)}Mg_xCl_3$[14]. The present results could be tested if such measurements could be performed in the presence of uniform magnetic fields.

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Resumo

O modelo de Ising em um campo aleatório, na sua versão unidimensional, é estudado a partir de um conjunto de identidades para as funções correlação. Obtemos resultados explícitos para a magnetização, a função correlação de duas partículas e o calor especifico, usando uma distribuição não correlacionada de campos aleatórios.