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Magnetic Scattering in Born Approximation

J.B. BASTOS FILHO *

Institut für Theoretische Physik der ETH, Zürich, Switzerland

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Abstract The magnetic electron scattering by magnons is studied in the Born approximation. For ferro and ferrimagnetic targets the difference between the spin-flip amplitudes is proportional to the absolute square of the exchange integral.

1. INTRODLICTION

In the preceding article[1] we have shown that the general Hermitian Hamiltonian consisting of a spin independent potential plus a spin-orbit coupling does not lead to polarization effects in Born approximation (B.A.). For magnetic scattering, however, the final polarization effects in B.A. are different from zero.

The aim of this note is the study of the spin polarization effects in B.A. when the electron beam is scattered by magnons.

In scattering theory the problem of the difference between polarization $\vec{P}$ and asymmetry $\vec{A}$ is very important, because it is related to the time reversal concept[2], and the quantity ($\vec{P}-\vec{A}$) is related to the difference between spin-flip amplitudes. Special attention has been given to this difference in the field of elementary particles[3-4], but little ãttention has been given to this problem in solid state physics. Vredevoe et al[5] and Saldaña et al[6] have studied the magnetic scattering by magnons in B.A., but they have not considered the difference between the spin-fllp amplltudes.

In this note we show that for ferromagnetic target materials this diff'erence is proportional to $|J_{\vec{K}}|^2$, where $J_{\vec{K}}$ is the exchange integral in Fourier space, with the index $\vec{K}$ denoting the momentum transferred by the scattered electrons.

For ferrimagnetic targets the above effect is proportional to

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* Present: address: Departamento de Física da Universidade Federal de Alagoas, 57.000, Maceió, Alagoas, Brasil

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$|J_{\vec{K}}|^2$, but it is smaller than the effect Found for ferromagnetic materials.

For antiferromagnetic targets the polarization effects are found to be equal to zero.

2. FERROMAGNETIC TARGET

The interaction between an incident beam electron at position $\vec{r}$ and with spin $\vec{S}$, and the atoms of the magnetic target located at positions $\vec{r}_j$ and with spins $\vec{S}_j$, is given by the Hamiltonian

$$H = \varepsilon \sum_j V(\vec{r}-\vec{r}_j) + \sum_j J(\vec{r}-\vec{r}_j)\vec{S}.\vec{S}_j \qquad (1)$$

where $V(\vec{r}-\vec{r}_j)$ and $J(\vec{r}-\vec{r}_j)$ are, respectively, the orbital potential and the exchange integral, ε denotes the electron charge, and $\vec{S} = (\hbar/2)\vec{\sigma}$, $\vec{\sigma}$ being the Pauli spin vector.

The (2x2) scattering matrix of the form

$$M = A1 + \vec{B}.\vec{\sigma}$$

which describes completely the system in spin-space, is given in B.A. for the above Hamiltonian by

$$M = -\frac{\mu}{2\pi\hbar^2}\sum_j \{<\vec{k}'f|\varepsilon V(\vec{r}-\vec{r}_j)|\vec{k}i> 1 + \frac{\hbar}{2}<\vec{k}'|J(\vec{r}-\vec{r}_j)|\vec{k}> <f|\vec{S}_j|i> . \vec{\sigma}\} \qquad (2)$$

where

$$|\vec{k}> = e^{i\vec{k}.\vec{r}} \quad , \quad |\vec{k}'> = e^{i\vec{k}'.\vec{r}}$$

and μ is the electron mass; $|i>$ and $|f>$ represent respectively the initial and final states of the target.

We supose that the initial state of the target is the ferromagnetic ground state.

We use the Holstein Primakoff (H.P.) operators[7]

$$S_j^{(+)} = (2S)^{1/2} f(n_j)\, a_j$$
$$S_j^{(-)} = (2S)^{1/2} a_j^{*} f(n_j) \qquad (3)$$
$$S_j^{(z)} = S - n_j$$

where

$$f(n_j) = \left(1 - \frac{n_j}{2S}\right)^{1/2} \quad ; \; n_j = a_j \, a_j^* \quad ; \; [a, a^*] = 1 \tag{4}$$

The destruction and creaction operators in the $\vec{r}$-space are connected with the corresponding operators in the $\vec{q}$-space by Fouries transforms

$$a_{\vec{q}} = \left(\frac{1}{N}\right)^{1/2} \sum_j e^{i\vec{q}.\vec{r}_j} a_j \quad ; \quad a^*_{\vec{q}} = \left(\frac{1}{N}\right)^{1/2} \sum_j e^{-i\vec{q}.\vec{r}_j} a^*_j \tag{5}$$

where N is the number of spins up in the target.

The H.P. operators for one-magnon are

$$S_j^{(+)} = (2S)^{1/2} a_j \; ; \; S_j^{(-)} = (2S)^{1/2} a_j^* \; ; \; S_j^{(z)} = S \tag{6}$$

The amplitudes A and $\vec{B}$ of the scatterlng matrix written in eq. (2) are respectively

$$A = -\frac{\mu}{2\pi\hbar^2} \sum_j \langle \vec{k}' f | \varepsilon V(\vec{r}-\vec{r}_j) | \vec{k} i \rangle$$

$$\vec{B} = -\frac{\mu}{2\pi\hbar^2} \sum_j \frac{\hbar}{2} \langle \vec{k}' | J(\vec{r}-\vec{r}_j) | \vec{k} \rangle \langle f | \vec{S}_j | i \rangle \tag{7}$$

To perform the calculations in the context of one-magnon process we use the one-magnon H.P. operators (6). In this way there are two possible final states: the ferromagnetlc ground state and the exclted state with the creaction of one magnon. We have

$$a_{\vec{q}} | i \rangle = 0 \quad ; \quad a^*_{\vec{q}} | i \rangle = | \vec{q}_1 \rangle \tag{8}$$

where $|\vec{q}_1\rangle$ denotes the one-magnon state.

Substituting (6) and (5) into (7) and performing the calculations, we obtain for the z-component of the vectors

$$\vec{P} = \frac{\mathrm{tr}(MM^+\vec{\sigma})}{\mathrm{tr}(MM^+)}$$

$$\vec{A} = \frac{\mathrm{tr}(M^+M\vec{\sigma})}{\mathrm{tr}(MM^+)}$$

and

$$(\vec{P}-\vec{A}) = \frac{\mathrm{tr}\{[M,M^{+}]\vec{\sigma}\}}{\mathrm{tr}(MM^{+})}$$

the results

$$P_{\perp=z} = \frac{R_{\vec{K}} + |J_{\vec{K}}|^2 S}{\Delta}$$

$$A_{\perp=z} = \frac{R_{\vec{K}} - |J_{\vec{K}}|^2 S}{\Delta} \qquad (9)$$

and

$$(P-A)_{\perp=z} = \frac{2|J_{\vec{K}}|^2\, S}{\Delta}$$

where

$$R_{\vec{K}} = 2\varepsilon\,(V_{\vec{K}} J^{*}_{\vec{K}} + J_{\vec{K}} V^{*}_{\vec{K}})$$

$$\Delta = 4\varepsilon^2 |V_{\vec{K}}|^2 + |J_{\vec{K}}|^2 \;(1 + S)S \qquad (10)$$

and

$$J_{\vec{K}} = \int d^3r\; e^{i\vec{K}.\vec{r}}\; J(\vec{r}) \;;\; V_{\vec{K}} = \int d^3r\; e^{i\vec{K}.\vec{r}}\; V(\vec{r})$$

with

$$\vec{K} = \vec{k} - \vec{k}'$$

The important result here is that the difference between the spin-flip amplitudes is proportional to $|J_{\vec{K}}|^2$. In terms of cross sections this difference can be expressed by

$$(P_{\perp}-A_{\perp}) - \sigma_{\downarrow\uparrow} - \sigma_{\uparrow\downarrow}$$

For the ferromagnetic ground state the scattering amplitude $\sigma_{\uparrow\downarrow}$ is equal to zero. In this way the above result becomes

$$(P_{\perp}-A_{\perp}) \sim \sigma_{\downarrow\uparrow}$$

The total asymmetry between these spin-flip transitions .is a characteristic of the ferromagnetic ground state at 0^0K.

3. FERRIMAGNETIC TARGET

The extension of this approach to a ferrimagnetic target requires further considerations. Indeed, we now have both spins up and spins down In the target. We then consider the H.P. operators for spins down

$$S_{\ell}^{(+)} = (2S)^{1/2}\, b_{\ell}^{*} f(n_{\ell})$$

$$S_{\ell}^{(-)} = (2S)^{1/2}\, f(n_{\ell}) b_{\ell} \qquad (11)$$

$$S_{\ell}^{(z)} = -S + n_{\ell}$$

where

$$f(n_{\ell}) = \left(1 - \frac{n_{\ell}}{2S}\right)^{1/2} \; ; \; n_{\ell} = b_{\ell}^{*} b_{\ell} \; ; \; [b, b^{*}] = 1 \qquad (12)$$

The creation and destruction operators in $\vec{r}$-space are connected withthe corresponding operators in $\vec{q}$-space through the Fourier transforms

$$b_{\vec{q}} = \left(\frac{1}{Q}\right)^{1/2} \sum_{\ell} e^{-i\vec{q}.\vec{r}_{\ell}}\, b_{\ell} \quad ; \quad b_{\vec{q}}^{*} = \left(\frac{1}{Q}\right)^{1/2} \sum_{\ell} e^{i\vec{q}.\vec{r}_{\ell}}\, b_{\ell}^{*} \qquad (13)$$

where Q is the number of spins down in the target.

Taking the H.P. operators for one magnon we have

$$S_{\ell}^{(+)} = (2S)^{1/2}\, b_{\ell}^{*} \; ; \; S_{\ell}^{(-)} = (2S)^{1/2}\, b_{\ell} \; ; \; S_{\ell}^{(z)} = -S \qquad (14)$$

The Hamiltonian for a ferrimagnetic target is

$$H = \varepsilon \sum_{j;\ell} V(\vec{r}-\vec{r}_{j;\ell}) + \sum_{j;\ell} J(\vec{r}-\vec{r}_{j;\ell})\vec{S}.\vec{S}_{j;\ell} \qquad (15)$$

and the corresponding scattering matrix in B.A. is

$$M = -\frac{\mu}{2\pi\hbar^{2}} \sum_{j;\ell} \{<\vec{k}'f|\varepsilon V(\vec{r}-\vec{r}_{j;\ell})|\vec{k}i> 1 + \frac{\hbar}{2} <\vec{k}'|J(\vec{r}-\vec{r}_{j;\ell})|\vec{k}><f|\vec{S}_{j;\ell}|i>\cdot\vec{\sigma}\} \qquad (16)$$

where the Indices j and R correspond, respectively, to spIns up and down in the target.

We now consIder that the initial state of the target is the ferrimagnetic ground state. The calculations cannot be performed at this stage because the ferrirnagnetic ground state does not correspond to the magnon operators $a_{\vec{q}}$, $a_{\vec{q}}^{*}$, $b_{\vec{q}}$ and $b_{\vec{q}}^{*}$. This difficulty is solved through the introduction of the Bogoliubov transformations[7]

$$a_{\vec{q}} = x_{\vec{q}} \cosh U_{\vec{q}} + y^*_{\vec{q}} \sinh U_{\vec{q}} \; ; \qquad a^*_{\vec{q}} = x^*_{\vec{q}} \cosh U_{\vec{q}} + y_{\vec{q}} \sinh U_{\vec{q}}$$

$$b_{\vec{q}} = x^*_{-\vec{q}} \sinh U_{\vec{q}} + y_{\vec{q}} \cosh U_{-\vec{q}} \; ; \qquad b^*_{\vec{q}} = x_{-\vec{q}} \sinh U_{\vec{q}} + y^*_{-\vec{q}} \cosh U_{\vec{q}} \tag{17}$$

The ferrimagnetic ground state correspond to the Bogoliubov operators $x_{\vec{q}}$, $x^*_{\vec{q}}$, $y_{\vec{q}}$ and $y^*_{\vec{q}}$ and so the calculations can be performed. Restricting to one-magnon process, there are two possible final states - the ferrimagnetic ground state and another one with one spin-wave.

We have

$$x_{\vec{q}}|i\rangle = 0 \qquad x^*_{\vec{q}}|i\rangle = |\vec{q}_1\rangle$$

$$y_{\vec{q}}|i\rangle = 0 \qquad y^*_{\vec{q}}|i\rangle = |\vec{q}_2\rangle \tag{18}$$

where $|i\rangle$ is the ferrimagnetic ground state and $|\vec{q}_1\rangle$ and $|\vec{q}\rangle$ are two degenerate one-magnon final states.

Using the Bogoliubov transformations and the above considerations, we obtain for the difference between the spin-flip amplitudes, in units $\hbar=1$,

$$i(\vec{B} \wedge \vec{B}^*)_{\perp=z} = \frac{\mu^2}{16\pi^2} |J_{\vec{K}}|^2 \, S(|g_1|^2 N - |g_2|^2 Q)$$

where $|g_1|^2$ and $|g_2|^2$ are two geometric factors, non explicited in the calculations as they are not important for our purposes. The important reçult is that $(\vec{P}-\vec{A})$ is proportional to $|J_{\vec{K}}|^2$ for a ferrimagnetic target in B.A.

The effect for ferrimagnetic target is smaller than for ferromagnetic target because the contributions from up and down spins have different signs.

4. CONCLUSION

We have calculated that for one magnon process treated in the B.A. the difference between the spin-flip amplitudes is proportional to $|J_{\vec{K}}|^2$. The above result shows that the competition between the two spins states in the target determine the magnitude of the effect.

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Resumo

Estudamos o espalhamento magnético de eletrons por rnagnons no contexto da aproximação de Born. Obtemos, para materiais ferro e ferrimagnéticos no alvo, que a diferença entre as amplitudes de spin-flip é proporcional ao quadrado do valor absoluto da integral de intercâmbio.