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Migdal-Kadanoff Renormalization Group for the Z(5) Model

V. L. V. BALTAR, G.M. CARNEIRO, M. E. POL and N. ZAGURY

Departamento de Física, Pontifícia Universidade Católica, Caixa Postal 38071, Rio de Janeiro, 22453, RJ, Brasil

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Abstract The Migdal-Kadanoff renormalization group methods isused to calculate the phase diagram of the AF Z(5) model. We find that this scheme simulates a fixed line which we interpret as the locus of attraction of a critical phase. This result is in reasonable agreement with the predictions of Monte Carlo simulations.

Recently we have reported our studied of the phase diagram of the Z(4) and Z(5) models on a square lattice[1,2] using a combination of Monte Carlo and renormalization group techniques. In the case of the Z(4) model a simple renormalization group transforrnations based on the Migdal-Kadanoff scheme (MKRG) was shown to reproduce accurately the phase diagram obtained by Monte Carlo simulations[1], both in the ferromagnetic (F) and antiferromagnetic (AF) regions.

In this paper we report on the results of the MKRG calculations for the phase diagram of the Z(5) model on a square lattice. We find, in agreement with previous calculations, that the MKRG results predict only two phases in the F region, separated by the self-dual line. This result is in disagreement with other calculations[3,4,5] and with Monte Carlo simulations[2,6]. However, in the AF region the MKRG scheme simulates a fixed line.

In what follows we give some details of the calculations and discuss the interpretation of its results.

The Z(5) rnodel is defined as follows. Consider a lattice whose sites are occupied by classical spins which can take values $e^{i\theta}$, $\theta=2\pi k/5$ $k=0,1,...,4$. Assuming nearest-neighbour interactions only, the total energy of the Z(5) model on a square lattice is defined as

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$$E = -J_1 \sum_{\vec{n},\vec{\mu}}' \{\cos[\theta(\vec{n})-\theta(\vec{n}+\vec{\mu})]-1\} + \qquad (1)$$
$$- J_2 \sum_{\vec{n},\vec{\mu}}' \{\cos 2[\theta(\vec{n})-\theta(\vec{n}+\vec{\mu})]-1\}$$

In Eq. (1) $\vec{n}$ is a vector that labels the lattice sites, $\vec{\mu}$ represents the conventional primitive vectors of the square lattice, and J_1 and J_2 are the coupling constants.

The MKRG method consists, as usual, of a bond-shifting operation followed by decimation[7], as shown in Fig. 1. The transformation is made in such a way as to preserve the symmetry of two interpenetrating sublattices. This means that an even number of spins is decimated. We also work in the limit of infinitesimal scale change, inorder to preserve the self-duality of the model. The application of this scheme to the Z(N) models has been discussed in the Appendix of Ref. 1. Here we merely quote the results obtained for the Z(5) model.

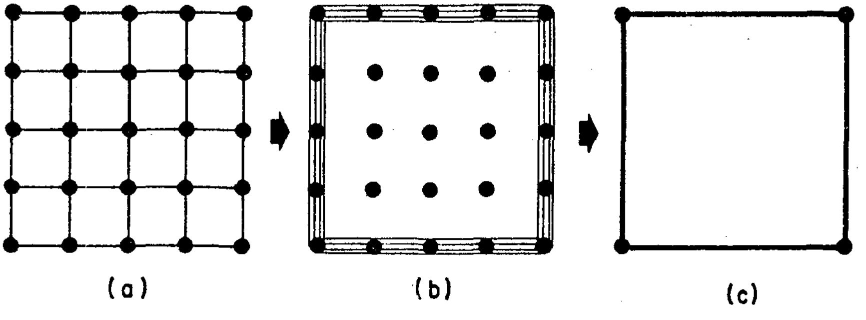

Fig. 1 - Schematic representarion of the Migdal-Kadanoff renormalization scheme: a) original interaction; b) interaction after bond shifting; c) new interaction.

In terms of the Boltzmann weights

$$x_1 = \exp\{-\beta E_1\} \text{ and } x_2 = \exp\{-\beta E_2\}$$

where $\beta = 1/kT$ and

$$E_1 = a\,J_1 + b\,J_2$$
$$E_2 = b\,J_1 + a\,J_2$$
$$a = 1 - \cos 2\pi/5$$
$$b = 1 - \cos 4\pi/5$$

the infinitesimal MKRG transformation is given by

$$x_1'' = x_1 + \varepsilon G_1(x_1,x_2)$$
$$x_2'' = x_2 + \varepsilon G_2(x_1,x_2)$$

where $\varepsilon = \lambda - 1$, λ being the scale factor of the transformation.

According to Ref. 1 (c.f. Eq. (A.18)),

$$G_1(x_1,x_2) = x_1 \ln x_1 + \frac{1}{5}\{(1-x_1)(1+2x_1+2x_2)\ln|1+2x_1+2x_2|$$
$$+ 2[1-a-x_1][1+2x_1(1-a) + 2x_2(1-b)]\ln|1+2x_1(1-a) + 2x_2(1-b)|$$
$$+ 2[1-b-x_1][1+2x_1(1-b) + 2x_2(1-a)]\ln|1+2x_1(1-b) + 2x_2(1-a)|\}$$

$$G_2(x_1,x_2) = x_2 \ln x_2 + \frac{1}{5}\{(1-x_2)(1+2x_1+2x_2)\ln|1+2x_1+2x_2|$$
$$+ 2[1-b-x_2][1+2x_1(1-a) + 2x_2(1-b)]\ln|1+2x_1(1-a) + 2x_2(1-b)|$$
$$+ 2[1-a-x_2][1+2x_1(1-b) + 2x_2(1-a)]\ln|1+2x_1(1-b) + 2x_2(1-a)|\}$$

This transformation, which is valid for both the F and AF regions, coincides with that obtained by Rujan *et al.*[8] in the F region, where the arguments of the $\ln$ are all positive. These authors show that this transformation predicts a phase diagram with two phases, the ferromagnetic and the disordered ones. We concentrate our discussion on the AF region where at least one of x_1 and x_2 are >1.

The fixed points of the transformation are defined as $G_1(x_1^*,x_2^*) = G_2(x_1^*,x_2^*) = 0$. In the AF region we find only one fixed point $x_1^* = x_2^* = 1$, which is the sink of the disordered phase.

However, if we look at the lines of zeros of G_1 and G_2, defined as

$$G_1(x_1,x_2) = 0 : \text{line of zeros of } G_1$$
$$G_2(x_1,x_2) = 0 : \text{line of zeros of } G_2$$

we find that for x_1 and x_2 large they come very close to each other. It can be shown that they do not cross and that they both approach asimptotically the line $x_2 = 0.1696\ x_1$ as x_1 and $x_2 \to \infty$, as is shown in Fig. 2. The region where these two lines come very close to each other simulates, as far as the RG flows are concerned, a fixed line since the RG flows slow down considerably there[9].

The MKRG flows are shown in Fig.3 in terms of the variables $y_1 = 1/x_1$ and $y_2 = 1/x_2$, which are more convenient to plot. The thick line in the figure, which we interpret as the fixed line, is such that $y_2 = 5.896\ y_1$ as y_1 and $y_2 \to 0$. Along the fixed line, the lines of zeros of G_1 and G_2 are very close to each other, diverging considerably at the point where the fixed line ends[9].

The MKRG flows indicate the existence of two phases, a critical phase which is the region of the parameters space attracted by the fixed line (I in Fig.3) and a disordered phase which is attracted by the sink H (II in Fig. 3). The phase boundary, *a* in Fig. 3, separates these two phases.

This result is in agreement with recent Monte Carlo simulations performed in this region[10] which also predict the existence of a critical phase. The MKRG results further support the existence of a critical phase in the AF region of the Z(5) model.

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9. The $y_1 > y_2$ region of the figure follows from the $y_2 > y_1$ region using the symmetry of the partition function with respect to the exchange of x_1 and x_2, which is preserved by the MKRG transformation.

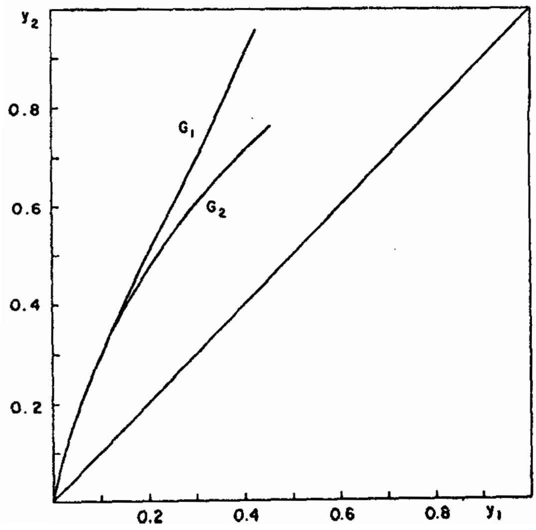

Fig. 2 - Lines of zeros of G_1 and G_2.

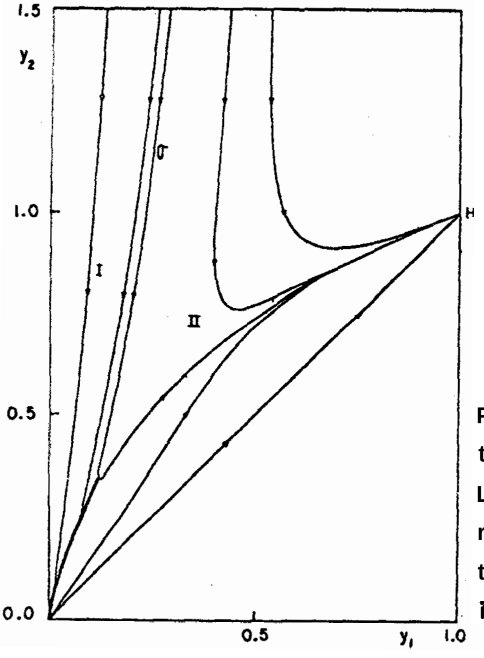

Fig. 3 - MKRG flows in the AF region: the thick line is the fixed line. Line a is the boundary between the region where the MKRG flows towards the fixed line and the region where it flows towards the fixed point H.

10. V.L.Baltar, G.M.Carneiro, M.E.Pol and N.Zagury, to be published.

Resumo

O método do grupo de renormalização de Migdal-Kadanoff é usado para a determinação do diagrama de fase do modelo Z(5) AF na rede quadrada. Encontramos que este método simula uma linha fixa, a qual interpretamos como sendo o foco de atração de uma fase crítica. Este resultado concorda qualitativamente com as predições das simulações de Monte Carlo.