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Numerical Simulation of $h\phi_4^4$

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Abstract The purpose of this paper is to study the behaviour of a lattice theory where spontaneous symmetry breaking occurs. We calculate and discuss the effective potential and obtain the phase diagram of an Euclidean $\lambda\phi_4^4$ theory using Monte Carlo methods on a hypercubic lattice with 4^4 sites and periodic boundary conditions. We haveexamined the vacuum expectation value $\langle\theta\rangle_j$ in the presence of an external source $j(x)$, keeping the parameter $\beta=1/\lambda$ fixed. We call ϕ_c the limit of $\langle\theta\rangle_j$ as $j\to 0$. We have found that for values of β larger than $\beta_{critical}$, we have $\phi_c\neq 0$, while for values of β smaller than $\beta_{critical}$, we obtain $\phi_c=0$. We thus observe that the $\lambda\phi_4^4$ field theory exhibits two phases, adisordered phase in which the vacuum expectation value of the field vanishes, and an ordered phase characterized by spontaneous magnetization. The value of $\beta_{critical}$ is estimated through the behaviour of the specific heatand susceptibility as functions of β.

1. INTRODUCTION

The simplest of interesting field theories, $\lambda\phi^4$, has eluded a complete analytical understanding for quite some time. Long ago, questions about the existence of a renormalized $\lambda\phi_4^4$ were raised[1]. Recently, much work has been devoted to the clarification of the renormalized structure of this theory. A rigorous upperbound on the renormalized coupling of its continuum limit was established[2]. There are many indications that $\lambda\phi_4^4$ is a trivial (non-interacting) theory, that is, its renormalized coupling constant is zero[3]. They come from a lattice strong coupling series expansion, where either the effective potential[4], or, by using Padé approximants, the renormalized coupling constant is analyzed[5]. Also, a Monte Carlo numerical simulation of this theory shows that the renormalized coupling constant goes to zero inthe continuum limit[6]. More rigorous arguments using the random walk representation of scalar lattice field[7,8], also support the notion of triviality of $\lambda\phi_4^4$. In contrast with the triviality of the theory in 4

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dimensions, it can be rigorously shown that $\lambda\phi^4$ is an interacting (non-trivial) theory in 3 dimensions[3].

On the other hand, there are claims that $\lambda\phi^4_4$ can be a non trivial theory when the bare coupling constant is small and negative[10]. There, a variational calculation of the effective potential is done and the phenornenon of "quantum ressucitation"[11], i.e., the renormalized coupling constant becoming positive, is shown to occur. These statements are derived from studies of the theory in the symmetrlc phase, when the quadratic bare mass is positive.

The effective potential in the broken phase, i.e., when the quadratic bare mass is negative, has been studied using the loop expansion technique[12]. However, this perturbative derivations leads to a compiex expression for the effectlve potential[13]. The complex contribution arises whenever the classical potential is non-convex[14], and a perturbatiive expansion of the effective potential is performed. Actualiy, it is easy to show that the effective potential (in Euclidean space) must always be convex[15]. Callaway and Maloof gave a prescription on how to construct the effective potential even on regions where it is undefined, while maintaining the convexity property[16], which is reminiscent of the thermodynamic Maxwell construction used in phase diagrams derived from Van der Waals forces, like that of water[14]. The numerical simulatiori of the effective potential gives support to the appropriateness of this Maxwell construction[16].

In this paper we describe the complete phase diagram of $\lambda\phi^4_4$, with spontaneous symmetry breaking at the tree level, doing a numerical simulatiori using the Monte Carlo method. This numerical sirnulation is performed on a lattice of dimension 4^4, applying an external current (which plays the role of a magnetic field) j to drive the order parameter (ttie average field or, equivalently, the magnetization). We construct the phase diagram in the space of coupling constant, which plays the role of temperature in this rnodel, and external current j. This external current j is instrumental in the computation of theeffective potential, as will be shown later.

This paper is organized in the following way: in section 2 we give the details of the model and the definitions of quantities which we will later compute numerically. Section 3 has a brief review of the

Monte Carlo method of numerical simulation and discusses the way we implement it in this problem. We describe the numerical results in the section 4, where we estimate the critical "temperature" of the model, discuss the sources of error, the field distribution in the lattice and the hysteresis curve generated by our algorithm. We present the computation of the effective potential in section 5 and present a summary and conclusions in section 6.

2. $\lambda\phi^4_4$ IN THE LATTICE

We will study the lattice version of the Euclidean Lagrangian:

$$L = \frac{1}{2}\{(\partial_0\Phi)^2 + (\nabla_i\Phi)^2 - m^2\Phi^2 + \frac{1}{4}(\lambda\Phi^4 + \frac{m^4}{\lambda})\} \qquad (1)$$

We scale up the field, defining

$$\varphi = \sqrt{\lambda}\,\frac{\Phi}{m} \qquad (2)$$

making it dimensionless, as well as the space variables, redefined by $y_i = m\chi_i$. The partition function,

$$Z(J) = N\int [\mathcal{D}\Phi]\ \exp\{-\int d^4\chi\,[L(\Phi,\partial_i\Phi) + J\Phi]\} \qquad (3)$$

written in terms of the scaled variables and with the scaled current defined by

$$j = \frac{\sqrt{\lambda}}{m^3}J \qquad (4)$$

is now

$$Z(j) = N'\int [\mathcal{D}\theta]\ \exp\{-\frac{1}{\lambda}\int d^4y\ [\frac{1}{2}(\partial_i\theta)^2 + \frac{1}{4}(\theta^2-1)^2 + j\theta]\} \qquad (5)$$

In the discrete version of this theory, the fields $\theta(\vec{n})$ are defined at the lattice sites, labelled by the number vector $\vec{n}$. The lattice sites are separated by a distance a and we impose periodic boundary conditions over the fields on the surface of the finite lattice volume. The lattice version of the derivative is defined by

$$\partial_i\theta(\vec{\chi}) = \frac{\theta(\vec{n}+\hat{e}_i) - \theta(\vec{n})}{a} \qquad (6)$$

where $\vec{e}_i$ is the unit vector in the direction i. We will use

$$\beta \equiv 1/\lambda \tag{7}$$

to be in accord with notation used in statistical mechanics. The lattice vers'ion of the partition function,

$$Z(j) = N' \int [\mathcal{D}\theta(\vec{n})] \exp\{-\beta H\} \tag{8}$$

has the Eiiclidean Hamiltonian defined by

$$H = \frac{a^4}{2} \sum_n \{ - \frac{1}{a^2} \theta(\vec{n}) \sum_{i=1}^{d} [\theta(\vec{n}+\vec{e}_i) + \theta(\vec{n}-\vec{e}_i)] + \frac{2d}{a^2} \theta^2(\vec{n}) + \frac{1}{2} [\theta^2(\vec{n}) - 1]^2 + 2j(\vec{n})\theta(\vec{n}) \} \tag{9}$$

in a space of dimension d.

The classical potential

$$V(\theta)/\beta = \frac{1}{4} (\theta^2 - 1)^2 + j\theta \tag{10}$$

has its extrema at the solutions of the equation

$$\theta^3 - \theta + j = 0 \tag{11}$$

This equation has either three real roots, being two of them minima of the potential and the third a maximum, or one real root corresponding to a minimum. The nature of the rootr is controlled by $A = \frac{j^2}{4} - \frac{1}{27}$, insuch a way that when $\Delta \leq 0$ the equation has three real roots, while for $\Delta > 0$ there is only one real root. So the classical potential will have a single minimum if $|j| > 2/3\sqrt{3}$.

The average of a functional of the fields in the presence of the curreiit j is

$$\langle A(\theta(\vec{n})) \rangle_j \equiv \frac{\int [\mathcal{D}\theta(\vec{n})] A(\theta(\vec{n})) e^{-\beta H}}{\Gamma [\mathcal{D}\theta(\vec{n})] e^{-\beta H}} \tag{12}$$

We define the connected partition function in the usual way

$$Z(J) = e^{-W(J)} \tag{13}$$

In this work we use a constant current j over the whole lattice, so that the derivative in j yields

$$\frac{\delta W(j)}{\delta j} = \beta N a^4 \sum_{\vec{n}} < \frac{\theta(\vec{n})}{N} >_j \qquad (14)$$

where N is the number of sites in the lattice and $\sum_{\vec{n}} <\theta(\vec{n})/N>_j$ is the average value of the field over the entire volume.

The effective action, the Legendre transform of W, is defined by

$$\Gamma(\phi) = W(j) - \beta N a^4 j\phi \qquad (15)$$

with the boundary condition over ϕ,

$$\frac{\delta W(j)}{\delta j} = \beta N a^4 \phi \qquad (16)$$

The variable ϕ of the effective action is equivalent to the average value of the field over the entire volume.

The effective potential is derived from the effective action by extracting the volume factor

$$V(\phi) \equiv \frac{1}{Na^4} \Gamma(\phi) \qquad (17)$$

In general the effective potential is derived from the effective action by making an expansion in terms of constant (in space) values of the variable ϕ, however, the effective action contains more than that, namely terms which dependon spatial variationsof ϕ (i.e., terms in derivatives of ϕ). In our case, as $W(j)$ is a function of a constant j, this implies that the effective potential coincides with the effective action.

From the definition of the effective potential we write the equation

$$\frac{\delta V(\phi)}{\delta \phi} = - \beta j \qquad (18)$$

The effective potential can be integrated using the inversion of the function $\phi(j)$. However this can only be done in an unambiguous way if j is a continuous and single valued function of ϕ. However, one of the signs of a phase transition is a discontinuity in this function, and this is the cause of trouble in the construction of the effectivepotential. In our numerical simulation we plot j against $\sum_{\vec{n}} <\theta(\vec{n})/N>_j$, do a fitting of the curve and then integrate nuinerically eq.(18). In a fi-

nite lattice, strictly speaking there is no phase transition so the problem with $j(\phi)$ is non existent and the effective potential can be constructed without ambiguity.

The second derivative of the effective potential can be derived from

$$\frac{\delta^2 V(\phi)}{\delta\phi^2} = -\beta \frac{\delta j}{\delta\phi}$$

$$= \{ < \left(\sum_{\vec{n}} \frac{\theta(\vec{n})}{N} \right)^2 >_j - \phi^2 \}^{-1} / Na^4 \qquad (19)$$

This quantity is analogous to the inverse magnetic susceptibility in a spin system

$$\chi(j,\beta) \equiv -\frac{\delta\phi}{\delta j}$$

$$= \beta N a^4 \{ < \left(\sum_{\vec{n}} \frac{\theta(\vec{n})}{N} \right)^2 >_j - \phi^2 \} \qquad (20)$$

The term inside the bracket in equation (19) is positive definite, which implies that the effective potential must be convex. However, some caution should be taken with this statement, for it is true when ϕ is a continuous function of j. Had this function a discontinuity, as is the case when the system undergoes a phase transition, then the effective potential is undefined at the gap in ϕ.

The energy density at a site

$$h(\vec{n}) \equiv \frac{1}{2} \{ -\frac{1}{a^2} \theta(\vec{n}) \sum_{i=1}^{d} \left[\theta(\vec{n}+\hat{e}_i) + \theta(\vec{n}-\hat{e}_i) \right]$$

$$+ \frac{2d}{a^2} \theta^2(\vec{n}) + \frac{1}{2} \left[\theta^2(\vec{n}) - 1 \right]^2 + 2j\theta(\vec{n}) \} \qquad (21)$$

can be used in the definition of the internal energy density

$$\varepsilon = < \frac{H}{Na^4} >_j = -\frac{1}{Na^4} \frac{\partial \ln Z(j)}{\partial \beta} \qquad (22)$$

where H is given by eq. (19), and

$$\varepsilon = \frac{1}{N} \sum_{\vec{n}} < h(\vec{n}) >_j \qquad (23)$$

The specific heat is derived from the energy density

$$C = -\beta^2 \frac{\partial \varepsilon}{\partial \beta}$$

$$= \beta^2 N a^4 \{ \langle \left(\frac{1}{N} \sum_{\vec{n}} h(\vec{n}) \right)^2 \rangle_j - \varepsilon^2 \} \qquad (24)$$

This quantity is very useful, for a peak in its diagram as a function of the temperature, is an indication of a phase transition in the system.

3. THE MONTE CARLO METHOD

To evaluate functional integrals, like that defined in eq. (12), is a prohibitive task if some method of "cutting down" the phase space is not used. The Monte Carlo method[17], which was first applied to problems in statistical mechanics by Metropolis and collaborators'[18], is based on the idea of importance sampling of the phase space. In this method a set of configurations from phase space is chosen in such a way that its distribution is related to the density of the phase space. The estirnate $\tilde{A}_{mc}$ to the average $\langle A \rangle$ is given by a weighted average over the set w of m states sampled

$$A = \frac{\sum_{\omega=1}^{m} A(\theta_\omega) \; P_\omega^{-1} \; e^{-\beta H(\theta_\omega)}}{\sum_{\omega=1}^{m} P_\omega^{-1} \; e^{-\beta H(\theta_\omega)}} \qquad (25)$$

Here, $(\theta_{(\omega)})$ is the value of A computed for the configuration w. Motivated by statistical mechanics we choose the states 0_ω according to the Boltzman distribution

$$P_{eq} = P_\omega = \frac{e^{-\beta H(\theta_\omega)}}{\sum_{\omega=1}^{m} e^{-\beta H(\theta_\omega)}} \qquad (26)$$

Then, the Monte Carlo estimate of $\langle A \rangle$ reduces to the arithmetic average.

$$\bar{A}_{mc} = \frac{1}{m} \sum_{\omega=1}^{m} A(\theta_\omega) \qquad (27)$$

This choice of distributions is particularly convenient for numerical computation, for when a new state is generated, only its relative probability to the preceding one needs to be computed, i.e.,

$$\rho_{\omega\omega'} = \frac{P_\omega}{P_{\omega'}} = e^{-\beta(H(\theta_\omega) - H(\theta_{\omega'}))} \tag{28}$$

In this Monte Carlo simulation, we set the system on a hypercubic lattice with 4^4 sites and impose periodic boundary conditions at the surface of the lattice volume. Although, in most of our runswehave started from a "hot" (randomly chosen) configuration, we have checked the consistency of our results by starting some runs from a "cold" (ordered) configuration.

When generating a new field, instead of using the random walk algorithm[6], defined by

$$\theta_{new} = \theta_{old} + (2s-1)*\Delta \tag{29a}$$

with s beiny a random number in the interval $[0,1]$ and A some empirical constant, we have chosen to use the random field

$$\theta_{new} = (2s-1)*\Delta' \tag{29b}$$

where A' is an empirical constant. We have used A' = 3 below the critical temperature (so that θ_{new} is a random and evenly distributed number in the interval $(-3,3)$) and A' = 5 above it. We have used the pseudo--random sequence generated by the machine, but at each run we changethe seed number of the random number generator.

The acceptance criteria for a new field, θ_{new}, is governed by the Metropolis algorithm[18]. If the eriergy H of the system is lowered by the replacement of θ_{old} by θ_{new} (i.e., $P_{eq}(\theta_{new}) > P_{eq}(\theta_{old})$) at the site j, then the field at this site is replaced by the new value. If $\Delta H \gtrsim 0$, where $\Delta H = H(\theta_{new}) - H(O_{old})$, then a random number, r, with uniform distribution in the interval $[0,1]$ is generated, and the new field accepted only if $\exp[-\beta\Delta H] > r$, otherwise, the field retains its previous value. It is this occasional acceptance of configurationswhich do not lower the energy which simulates the effects of quantum fluctuations. We update the value of the fields at each site once at a time, and run over the sites in a sequential way, until we complete a Monte

Carlo sweep. We do 8 trials (sometimes 10) to change the field at a single site, before proceeding to the next site. Thls procedure reduces the correlation among different configurations, increases the rate of change of the fields at each iteration (sweep) and in addition makes the system converge to equilibrium at a faster rate. With 8 trials per update, 92% of the sites has its field value change between sequential sweeps.

Our typical run for a pair of parameters (β, j) has 6000 Monte Carlo sweeps, each sweep being an entire update of the whole lattice. We wait for about 600 Monte Carlo sweeps, allowing the system to thermalyze, before collecting data. We separate the sequence of thermalyzed configurations in islands, with an interval of 20 sweeps between them. We collect the central configuration of each island and use the others to do statistics and have some çontrol over correlations in the sample.

We estimate the statistical errors by computing the variance σ of the quantity being measured and then using the formula[19]

$$\langle A \rangle = \bar{A}_{mc} \pm \frac{\sigma(A)}{\sqrt{m}} \qquad (30)$$

where m is the number of configurations used.

This Monte Carlo program required 8.4×10^{-5} sec for a single site update, in a CDC-CYBER 170-835.

4. NUMERICAL RESULTS

We have investigated the behaviour of the model $\lambda\phi_4^4$, in the space of parameters $\beta(=1/\lambda)$ and j, while keeping the lattice spacing constant and equal to 1.0. We have measured the expectation valueofthe field θ, in thepresenceof anexternal source $\langle\theta\rangle_j$, its variance, the effective mass and the susceptibility, for values of j in the interval $(-3.0,3.0)$, for several values of β. In addition, we havemeasured the interval' energy density and the specific heat at $|j| = 0.01$ and $|j| = 0.001$ for values of $1/\beta$ ranging between 0.2 up to 3.0. In the following we will comment on the behaviour of each of the quantities we measured, the numerical results obtained and eventual sources of errors.

We can identify the limit $j\to0$ of the vacuum expectation value

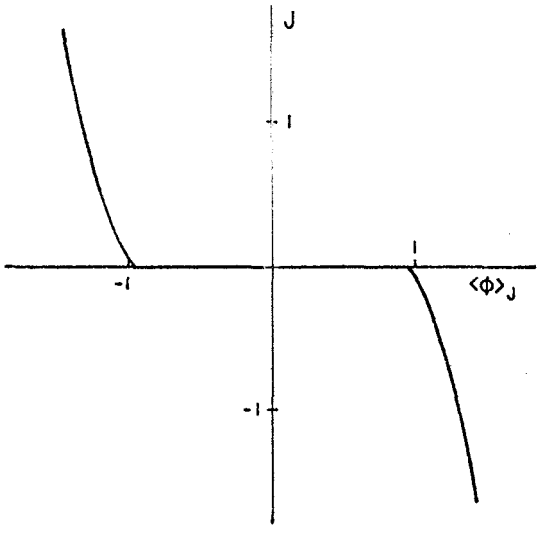
of the field, $\langle\theta\rangle$ as theorder parameter of the theory, in analogy with statistical mechanics. We expect $\langle\theta\rangle_{j=0} \neq 0$ for $\beta > \beta_{critical}$, while $\langle\theta\rangle_{j=0}$ for $\beta < \beta_{critical}$. When generating the sample configurations, which will be used in the measurement of various quantities, at low temperature (large β) and small j, the sequence of configurations may get trapped in a metastable state (or in a false vacuum), with a very long Monte Carlo relaxation time. This can be checked right away, by inspecting the value of the average field of each configuration. We have avoided the contamination of our data from spurious metastable state, by imposing a cut over the average field $\langle\theta\rangle_j$ before accepting a new configuration in the Monte Carlo sample. We have checked theconsistency of this procedure by repeating the runs at some specific set of parameters, with a different starting configuration and pseudo-random number sequence.

In figure 1, we show the diayram of the external current j against the average field $\langle\theta\rangle_j$, for temperatures below, near and above the critical temperature. The diagram for temperatures below the critical one shows very clearly the gap in $\langle\theta\rangle_j$ as the limit $j \to 0_+$ and $j \to 0_-$ is crossed, signaling a phase transition. The smallest value of at which we take rneasurements is $|j| = 0.001$, so, to obtain the value of $\langle\theta\rangle_{j=O} = \theta_c$ we do an extrapolation. These diagrams will be very useful when constructing the effective potential.

In figure 2, we plot the measured value of $\langle\theta\rangle_j$ at different temperaturesat $|j| = 0.001$ alongwith the infinite lattice curve (broken line), which is obtained from the extrapolation of $j4$.

The magnetic susceptibility defined in eq. (20) shows a very clear cusp at the transition temperature, as it is exhibited in figure 3. This quantity is quite sensitive to the cuts we impose (to eliminate metastable states) upon the Monte Carlo sample. The value of $\langle\theta^2\rangle$ isnot affected be the cuts, however $\langle\theta\rangle$ can change a lot; nevertheless, the peak at the critical temperature is insensitive to the cuts imposed.

The specific heat defined by eq. (24) also shows a very clear peak at the transition temperature, consistent wlth the peak at the magnetic susceptibility, which is measured at $|j| = 0.001$ and shown in figure 4. In contrast to the magnetic susceptibility, the specific heat



Flg. 1.a

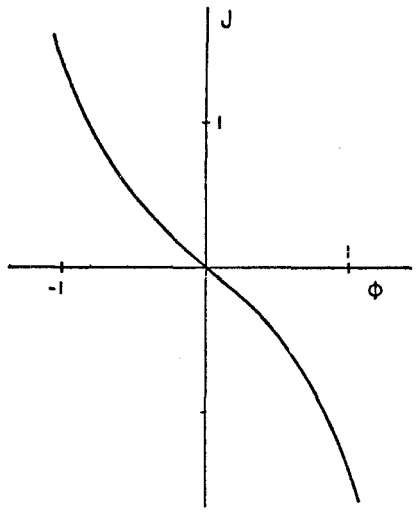


Fig. 1.b

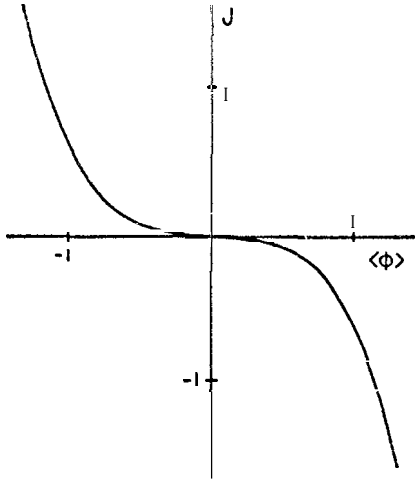


Fig. 1.c

Figure 1. Average fields for different values of j: a) low "temperature" (β=10); b) near the critical "temperature" (β=0.75); c) high "temperature" (β = 0.30).

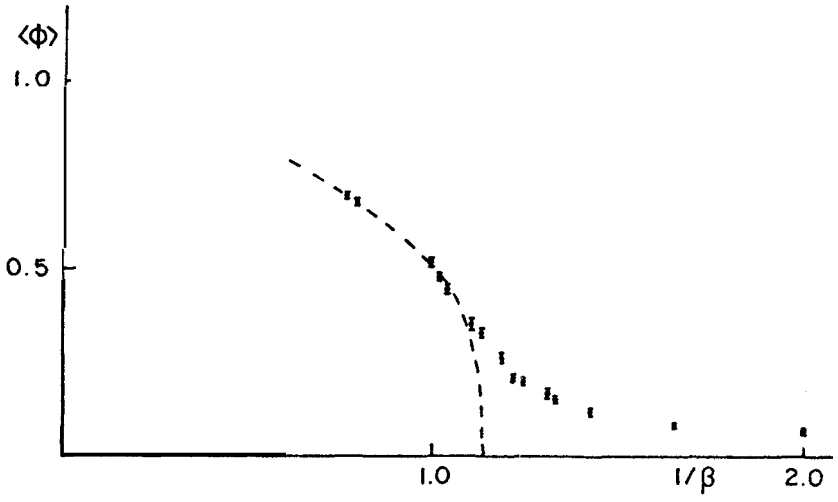


Figure 2 - Average field as a function of "temperature" for j = -0.001. The broken line is the result of the extrapolation to $j = 0$.

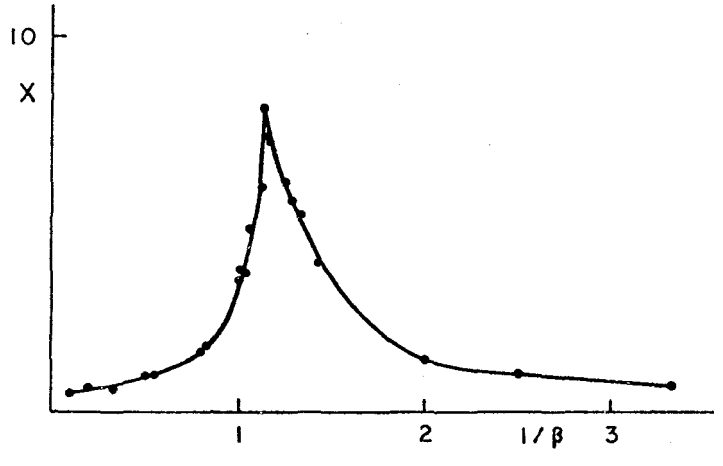


Figure 3 - Susceptibllity as a function of the temperature" for j = 0.01.

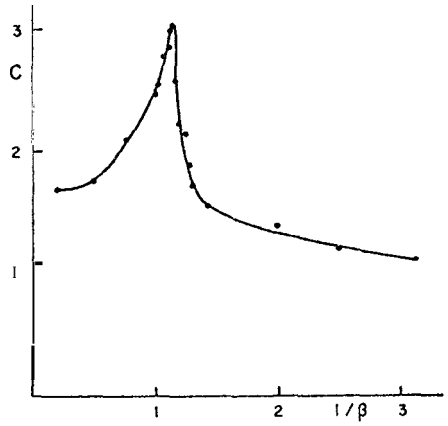


Figure 4 - Specific heat as a function of the "temperature" for $j = -0.001$.

and, also, the internal energy density, are insensitive to the cuts which are imposed upon the Monte Carlo sarnple.

The critical temperature of the $\lambda\phi_4^4$ model which we rneasure, either from the magnetic susceptibility or the specific heat diagram is

$$\lambda = (1/\beta) = 1.14 \pm 0.01 \tag{31}$$

We have estimated the correlation lenght using the expression

$$\xi^2 = \frac{\sum_{\vec{x}} x^2 G(\vec{x},0)}{\sum_{\vec{x}} G(\vec{x},0)} \tag{32}$$

where $\vec{x} = a\vec{n}$ and the connected propagator is given by

$$G(\vec{x},0) = <\theta(\vec{x})\theta(0)>_c \tag{33}$$

We have found that the correlation lenght is smaller than the lattice dirnension even near the critical temperature, exhibiting a "bump" when this ternperature is crossed. This is due to the finite size of the lattice for, strictly speaking, there is nophase transition once the systern is finite and so the correlation lenght is also finite.

The field distribution over the entire sample for a pair of parameters (j,β) is related to the quantum mechanical wave functional of the systern. Below the critical temperature, the field distribution is peaked around the rninirnum of the classical potential with a gaussian

like shape, as is shown in figure 5. Had we not imposed a cut on the samples we collected, diagrams like that of figure 5 would show another much smaller gaussian peak around the bottom of the "false" vacuum (the other local minimum of the classical potential), near the critical temperature. This field distribution is very sensitive to j, figure 5 being a measurement taken at $j = -0.001$ though the effect of this small value of j is imperceptible at the scale of the classical potential, the quantum mechanical system only spends very little Monte Carlo time at the false "vacuum".

In contrast with the field distribution below the critical temperature, above it the field is spread all over the potentlal, as is shown in figure 6, where the measurement was taken for $\beta = 0.30$ and $j = -0.001$. Here the field distribution is quite symmetric around both classical minima.

We have run hysteresis loop with $j = -0.001$ which is shown in figure 7. To run this loop, we first thermalize the system at low temperature and take measurements of the internal energy density. Then we change the value of the temperature very slowly (in Monte Carlo "time"). In the run shown in figure 7, we change the temperature by $\Delta T = 2.5 \times 10^{-3}$ at each Monte Carlo sweep. The temperature is increased until somemaximum value (dots in the figure) and then decrease to the thermalization temperature (the crosses in the figure). The area of the hysteresis loop becomes smaller if we reduce the step ΔT, however the fluctuations in the values of rhe internal energy density, prevents using this diagram to make a good meaturement of the critica1 temperature.

We have tested the stability of our results against the variation of the lattice size, by making a few runs for larger volumes, 6^4 and 8^4. The results we obtain are consistent with those reported above.

5. THE EFFECTIVE POTENTIAL

The effective potential can be constructed by integration eq. (18). The field ϕ is thevacuumexpectation value $\sum_{\vec{n}} \left\langle \frac{\theta(\vec{n})}{N} \right\rangle_j$, so we have to iiivert this function and compute $j(\phi)$. We have fitted the diagrams shown in figure 1 with polynomials and then we integrate eq. (18) analytically.

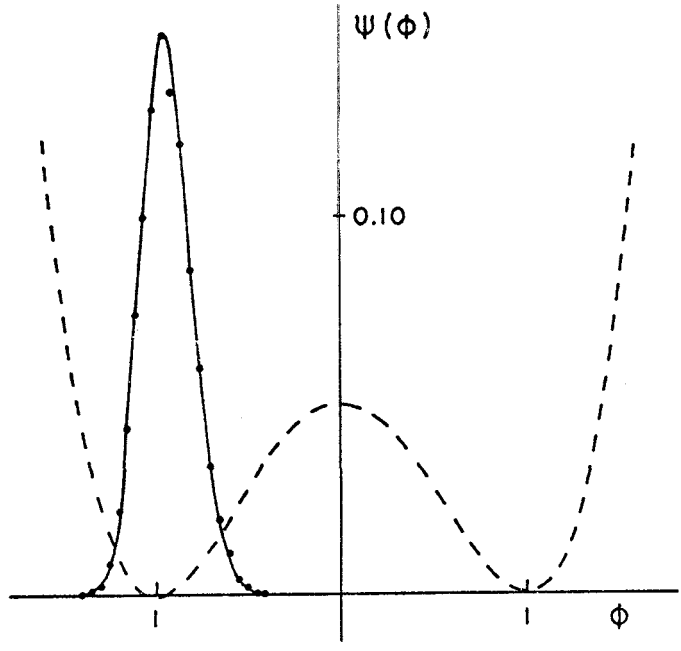


Figure 5 - Field distribution in the entire sample for $\beta = 10.0$ and $j = -0.001$.

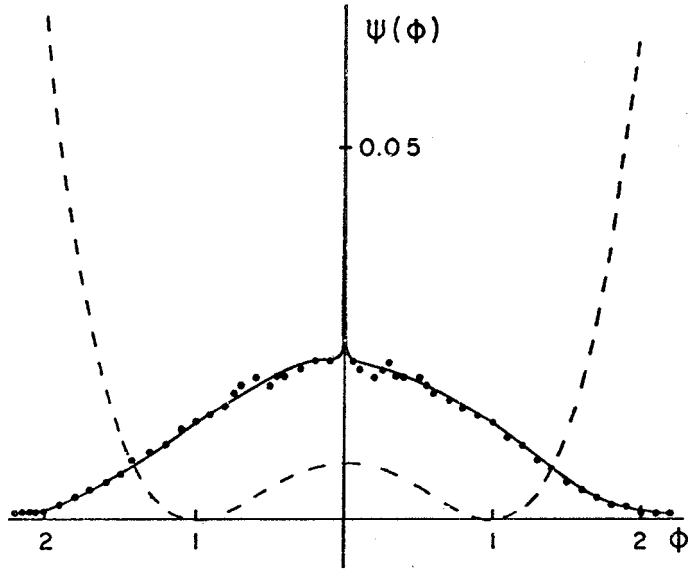


Figure 6 - Field distribution in the entire sample for $\beta = 30.0$ and $j = -0.001$.

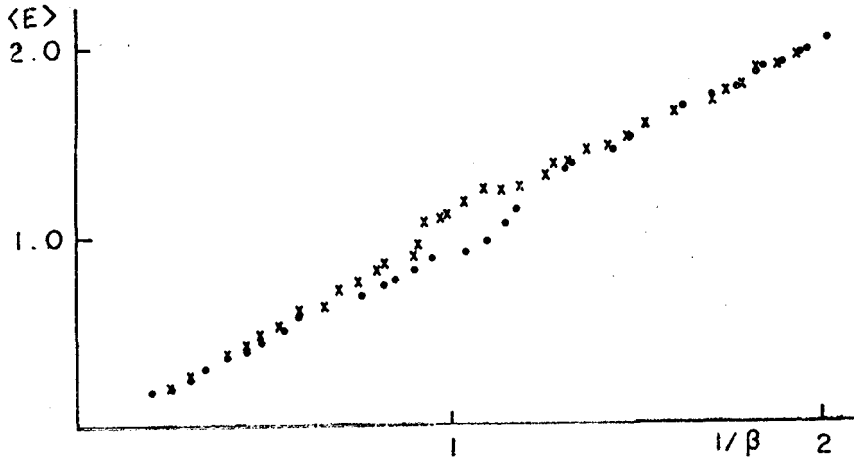


Figure 7 - The hysteresis diagram for $j = -0.001$.

In figure 8 weshow the effective potential at high temperature, ie, above the critical temperature, computed for $\beta=0.30$. This is a well beliaved potential without spontaneous symmetry breaking, inspite of the fact that we started with a bare potential where spontaneous symmetry breaking occur. This effective potential can be contrasted with the one shown in figure 9, constructed with $\beta=10.0$, ie, much below the critical temperature. Here, the regions of ϕ which interpolate the two values of the field which are attained at $j=0$ coming from above and from below, is simply connected by a straight line. This is the Maxwell construction proposed by Callaway and Maloof[16]. If the Monte Carlo sample is long enough, and if we take smaller and smaller values of j, the diagram of figure 1.a, will eventually bend and the line goes through zero, thus avoiding the necessity to impose the Maxwell construction; it will come naturally in the limit of an infinite volume lattice. However, even at a small lattice like ours, the appropriateness of the Maxwell construction is evident.

6. SUMMARY

We have studied the phase diagram of $\lambda\phi_4^4$ and the shape of the effective potential for different values of A. This theory exhibit two distinct phases: an ordered phase with $\beta>\beta_{critical}(\beta=1/\lambda)$, where the

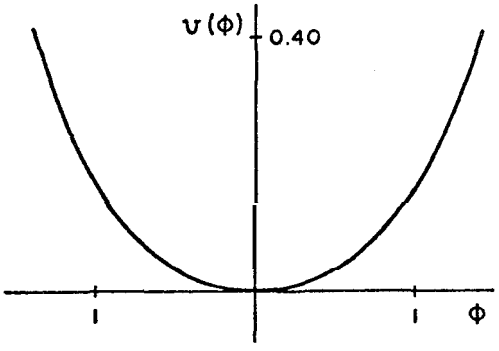


Figure 8 - The effective potential at high "temperature" ($\beta = 0.30$).

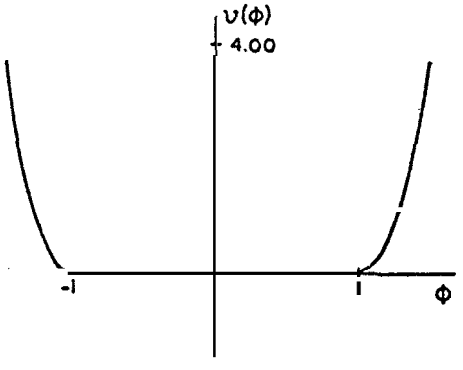


Figure 9 - The effective potential at high "temperature" ($\beta = 10.0$)

symmetry of the vacuum is spontaneously broken and a disordered phase, with $\beta<\beta_{critical}$, with unbroken symmetry. Thus, as expected, a symmetry which is broken at low "temperature", is restored at high "temperature"!

We determined the critica1 temperature, analyzing themagnetic susceptibility and the specific heat diagrams, with the result

$$\beta_{critical} \quad 0.88 \pm 0.01$$

We have also constructed the effective potential at "temperatures" below and above the critical one. Below the critical "temperature" we obtain a result which is consistent with the Maxwell construction procedure sugested by Callaway and Maloof[16].

Our Monte Carlo samples are not rich enough yet, near the critical ternperature to study the limit to the continuum of the theory. It is particularly interesting to study the behaviour of the renormalized coupling constant, once we expect that it should show

indications of the triviality of the theory. Work on this issue is in progress.

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Resumo

O objetivo deste trabalho é estudar o comportamento de uma teoria na rede na qual ocorra quebra espontânea de simetria. Calculamos o potencial efetivo e obtemos o diagrama de fase para uma teoria $\lambda\phi^4$ através do uso do método de Monte Carlo. Utilizamos para este cálculo uma rede hipercúbica com 4^4 sítios e impomos condições de contorno periódicas. Examinamos o comportamento do valor esperado no vácuo $\langle\theta\rangle_j$, em presença de uma fonte externa $j(x)$, mantendo fixo o parâmetro $\beta=1/\lambda$. Chamamos de ϕ_c o limite de $\langle\theta\rangle_j$ quando $j\to 0$. Observamos que para valores de β maiores que $\beta_{crítico}$ temos $\phi_c\neq 0$, enquanto que para valores de β menores que $\beta_{crítico}$ temos $\phi_c=0$. Verificamos então que a teoria $\lambda\phi^4_4$ exibe duas fases, uma desordenada na qual o valor esperado no vácuo do campo é nulo, e uma fase ordenada que é caracterizada por uma magnetizaçao espontânea. O valor de $\beta_{crítico}$ é estimado com auxílio do comportamento do calor específico e da susceptibilidade como funções de β.