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Influence of Light-Quark Masses in Dynamical Scale Breaking

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Abstract We demonstrate that light quark masses may significantly contribute to the logarithmic scale breaking in deep inelastic electromagnetic lepton-nucleon scattering. This is mainly due to the combination of scale variables together with large "current" masses for u and d quarks, recently reported in the literature. We also estimate upper limits for current masses of u and d quarks, using positivity properties of the forward electromagnetic structure function F_2 of the nucleon.

1. INTRODUCTION

It is generally believed that quantum chromodynamics (QCD)[1], based on the non-abelian colour-gauge symmetry $SU_C(3)$, is the field theory of strong interactions. In contrast with abelian gauge theories and other field theories such as $\lambda\phi^4$, QCD is asymptotically free[2] and possibly confines colour. This property of colour confinement, presumably responsible for the absence of free quarks and gluons from the physically observable spectra renders the theory difficult to betested experimentally. Only at the kinematic region of high energy and momentum transfer, perturbative calculations are applicable. Little isknown about tht? long-distance nonperturbative domain of QCD.

The common tests of perturbative QCD are (i) the logarithmic breaking of scale invariance in deep inelastic lepton-nucleon scattering, (ii) production of hadronic jets with large transverse momenta in high energy nucleon-nucleon collisions, (iii) radiative corrections in e^+e^- hadrons and Drell-Yan processes and (iv) heavy particle spectroscopy. In the first two cases, distributions of quarks withinhadrons introduce: inaccuracies, especially for low values of the scaling variable x[3]. Hadronic jet production in photon-photon interaction already considered in the literature[4] may provide relatively clean tests at

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LEP energies[5]. In a recent work[6], we have called attention to the importance of heavy quark masses in these processes, and to the need for their isolation before a meaningful confrontation with QCD estimates may be carried out.

In this work we analyse the influence of light quark masses in deep inelastic processes contributing to scale breaking. We shall see that these effects, usually ignored, may be comparable to the QCD corrections under certain clrcumstances. The non-negligible contribution of light quark masses in logarithmic scale breaking is due to the combination of scale variables and masses, appearing in eq. (3.18), together with larger "current" masses for u and d quarks, recently reported in literature[7-9]. On shell heavy quark contributions in nucleon targets are negligible.

As we shall show by eq. (3.18), this contribution doesn't depend on $1/Q^2$ terms, such as high-twist arid kinematic scale breaking contributions; hsnce, for sufficiently high Q^2 values, the scale--breaking given by eq. (3.18) will survive in comparison with these contributions. Because of this, we believe that light quark masses provide an important correction not included in earlier calculations, which must be taken into account in the confrontation of QCD estimates and experiments.

Our paper is organized as follows. In sec.2 we detail the general formalism and present some definitions. In sec.3 we carry out the calculations. In section 4 we estimate upper limits of m_u and m_d. Section 5 contains some final observations.

2. QUARK-PARTON MODEL AND DEFINITIONS

The asymptotic freedom of QCD implies that at the large momentum transfer limit, quarks are free and the naive quark-parton model (QPM) should be valid[2,10]. Sincedynamics is switched-off in this limit, any scale breaking has to be kinematic in origin[11]. This section introduces notations and definitions to be used later and also shows how to include quark masses in perturbatíve QCD estimates.

Let $P_{a/b}(x)$ be. the probabílity of finding a parton (quark or gluon) aoriginating froma given parton b, x being the fraction of longitudinal momentum carried by b. By the convolution theorem, we can write

$$P_{a/b}(x) = \int_x^1 \frac{dy}{y} P_{b/c}(y)\, P_{a/b}\left(\frac{x}{y}\right) \qquad (2.1)§$$

Let us apply this relation to the electromagnetic deep inelastic lepton nucleon scattering in the domain of validity for the QPM (see fig.1). In this case $P_{b/c}(y)$ represents the probability of encountering a parton with momentum fraction y within the nucleon; we call this quantity $q(y)$. $P_{a/\tilde{}}$ and $P_{a/b}$ are related to the transition probabilities for the processes $N\gamma^* \to f$ and $b\gamma^* \to f$, respectively, where f represents any final state parton and N the target nucleon. Thus we have (see ref.1, E. Reya, Sec.6)

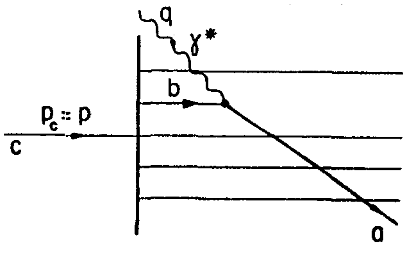

Fig. 1

Fig.1 - Virtual photon-nucleon interaction in Parton Model,

$$p_{f/N}(x) = \bar{\Sigma}\Sigma_{if} \int (2\pi)^4 \delta^4 (p_f - p - q) |\bar{M}|^2_{N\gamma^*\to f} \frac{1}{4E\omega} \frac{d^3\vec{p}_f}{(2\pi)^3 2E_f} \qquad (2.2)$$

where $|\bar{M}|^2_{N\gamma^*\to f}$ represents the squared matrix element averaged over the polarlzation states of the incident virtual photon and the spins of the target nucleon and is given by

$$|\bar{M}|^2_{N\gamma^*\to f} = \frac{e^2}{6} \sum_{spin} \varepsilon^\mu \varepsilon^\nu \langle N|J^+_\mu|f\rangle \langle f|J_\nu|N\rangle \qquad (2.3)$$

---

§ Eq.(2.1) is only correct if all three P functions have support between the values 0 and 1 in thelr arguments. In the presence of target and quark masses, this is no longer the case. But, the mass terms in these variables depend on $1/Q^2$. For sufficiently high Q^2 values we can consider the mass terms as negligible, and the use of this equation is justified.

The symbol $\bar{\Sigma}$ represents the average over the initial states of colour and spins of incident particles; ε's are the polaritation vectors of the virtual photon. p, q and p_f are specified in figure 1 and their components are

$$p^\mu = (E,\vec{p})$$
$$q^\mu = (\omega,\vec{q}) \tag{2.4}$$
$$p_f^\mu = (E_f,\vec{p}_f)$$

For the transition $b\gamma^* \to f$, we have

$$p_{f/b}(z) = \bar{\sum_{i}}\sum_{f} \int (2\pi)^4\delta^4(p_f - p_b - q)\,|\bar{M}|^2_{b\gamma^*\to f}\,\frac{1}{4E_b\omega}\,\frac{d^3\vec{p}_f}{(2\pi)^3 2E_f} \tag{2.5}$$

where

$$|\bar{M}|^2_{b\gamma^*\to f} = \frac{e^2}{6}\sum_{\text{spin}} \varepsilon^\mu\varepsilon^\nu [\bar{u}(f)\gamma_\mu u(b)]\,[\bar{u}(f)\gamma_\nu\, u(b)]^+ \tag{2.6}$$

Substituting eq. (2.6) into eq. (2.5) and using eqs. (2.1)-(2.3) we obtain

$$W_1 = q(x) \tag{2.7}$$

$$W_2 = \frac{2M^2 x q(x)}{p\cdot q} \tag{2.8}$$

$$x = -\frac{q^2}{2p.q} = \frac{Q^2}{2p.q} \tag{2.9}$$

where W_1 and W_2 are the nucleon electromagnetic structure functions, defined by

$$\bar{\Sigma}\sum_{if} \int (2\pi)^4\delta^4(p_f-p-q)\,\langle N|J^+_\mu|f\rangle\langle f|J_\nu|N\rangle\,\frac{d^3\vec{p}_f}{(2\pi)^3 2E_f} = 2(2\pi)^4\left(-g_{\mu\nu}W_1 + \frac{p_\mu p_\nu}{M^2}W_2\right)$$

(2.10)

Here M is the mass of the target nucleon.

Thus we recover the well-known result of QPM, that the nucleon structure functions are functions of x only (scale invariance). Using the usual definitions

$$F_1(x) = W_1(x) \tag{2.11}$$

$$F_2(x) = \frac{p \cdot q}{M^2} W_2(x) \tag{2.12}$$

together with relations (2.7) and (2.8) we immediately recover the Callan-Gross relation[12]

$$F_2(x) = 2xF_1(x) \tag{2.13}$$

3. COMPUTATION OF DYNAMICAL SCALE BREAKING

QPM is the zero-order approximation in perturbative QCD. In first order approximation, the scale invariance is broken and such breaking provides information about the quark-gluon interaction. Consider the simplest QCD correction due to gluon emission by quark which interacts with the virtual photon. Figure 2 defines this process.

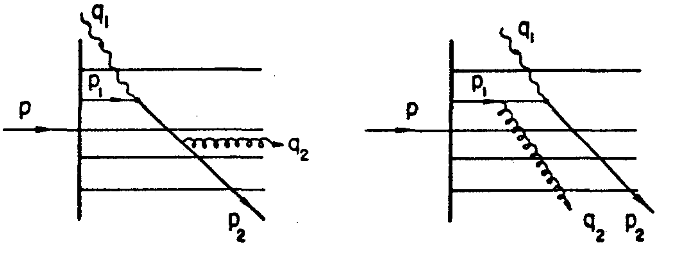

Fig.2 - QCD correction to scaling due to gluon emission by the quark which interacts withthe virtual photon.

By previous definitions, relations (2.1) give

$$\frac{e^2}{6} \varepsilon_1^\mu \varepsilon_1^\nu \, 2(2\pi)^4 \left(-g_{\mu\nu} F_1' + \frac{p_\mu p_\nu}{p \cdot q} F_2'\right) = \int_x^1 \frac{dy}{y^2} q(y) (2\pi)^4 \delta^4 (p_1 + q_1 - p_2 - q_2) \, |\overline{M}|^2_{q_1 p_1 \to q_2 p_2} \frac{d^3\vec{q}_2}{(2\pi)^3 2\omega_2} \frac{d^3\vec{p}_2}{(2\pi)^3 2E_2} \tag{3.1}$$

where $F_1'(x,Q^2)$ and $F_2'(x,Q^2)$ are corrections to $F_1(x)$ and $F_2(x)$ defined previously. Eq. (3.1) may be rewritten as[13]

$$\varepsilon_1^\mu \varepsilon_1^\nu R_{\mu\nu} = \frac{3}{256\pi^5 e^2} \int_x^1 \frac{dy}{y^2} q(y) \int |\overline{M}|^2_{q_1 p_1 \to q_2 p_2} \frac{dt}{[(p_1 q_1)^2 - m^2 q_1^2]^{1/2}} \tag{3.2}$$

where t is the Mandelstam variable (see fig.2), m is the struck quark mass and

$$R_{\mu\nu} = - g_{\mu\nu} F_1' + \frac{p_\mu p_\nu}{M\omega} F_2' \tag{3.3}$$

with

$$F_1' = \frac{(p^\mu p^\nu - M^2 g^{\mu\nu}) R_{\mu\nu}}{3M^2} \tag{3.4}$$

$$F_2' = \frac{(4p^\mu p^\nu - M^2 g^{\mu\nu}) \omega R_{\mu\nu}}{3M^3} \tag{3.5}$$

The corrections F_1' and F_2' are calculated by replacing $\varepsilon_1^\mu \varepsilon_1^\nu$ that appears in the right hand side of eq. (3.2) respectively by

$$\frac{p^\mu p^\nu - M^2 g^{\mu\nu}}{3M^2} \quad \text{and} \quad \frac{(4p^\mu p^\nu - M^2 g^{\mu\nu})\omega}{3M^3}$$

Let us determine F_2' . The quantity $|\bar{M}|^2_{q_1 p_1 \to q_2 p_2}$ is given by

$$|\bar{M}|^2_{q_1 p_1 \to q_2 p_2} = \frac{e^2 g^2}{18} \sum_{a,b} \sum_{\substack{i,j \\ k,\ell}} \sum_{\text{spin}} T^a_{ij} T^{b*}_{\ell k} \varepsilon_1^\mu \varepsilon_1^\rho \varepsilon_{2a}^\nu \varepsilon_{2b}^\lambda [\bar{u}_j(p_2) Q_{\mu\nu} u_i(p_1)] [\bar{u}_k(p_2) Q_{\rho\lambda} u_\ell(p_1)]^+ \tag{3.6}$$

The indices of polarizations vectors etc. are specified in figure 3. The term $Q_{\mu\nu}$ is given by

$$Q_{\mu\nu} = \frac{1}{t-m^2} \gamma_\mu (\not{p}_1 - \not{q}_2 + m) \gamma_\nu + \frac{1}{s-m^2} \gamma_\nu (\not{p}_1 + \not{q}_1 + m) \gamma_\mu \tag{3.7}$$

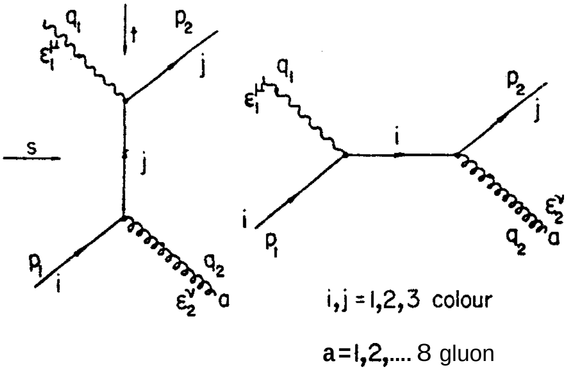

Fig.3 - Lowest order Feynman diagram for gluon emission in electromagnetic deep inelastic scattering.

The Mandelstam varlables in this case are

$$s = (p_1 + q_1)^2 = (p_2 + q_2)^2$$

$$t = (q_1 - p_2)^2 = (q_2 - p_1)^2 \qquad (3.8)$$

$$u = (q_1 - q_2)^2 = (p_2 - p_1)^2$$

and ε^{ν}_{2a} and $\varepsilon^{\lambda}_{2b}$ are the polariration of the gluons, a and b being the colour indices. We shall use

$$\sum_{\text{spin}} \varepsilon^{\nu}_{2a}\, \varepsilon^{\lambda}_{2b} = -\, g^{\nu\lambda}\, \delta_{ab} \qquad (3.9)$$

Substituting eqs.(3.7) and (3.9) into eq. (3.6), replacing $\varepsilon^{\nu}_{1}\varepsilon^{\rho}_{1}$ by $(4p^{\rho}p^{\nu} - M^2 g^{\rho\nu})\omega/3M^3$, and performing some of algebraic transformations we obtain

$$|\bar{M}|^2_{q_1 p_1 \to q_2 p_2} = \frac{2\omega e^2 g^2}{27M^3} \left\{ \frac{1}{(t-m^2)^2} \left[\frac{128}{y^2} m^4 p_1\cdot p_2 - \frac{128}{y^2} m^2 p_1\cdot p_2\, p_1\cdot q_2 \right.\right.$$

$$\left. - 32m^2M^2 p_1\cdot p_2 + 32m^2M^2 p_2\cdot q_2 - \frac{128}{y^2} p_1\cdot p_2 p_1\cdot q_2 p_1\cdot q_2 + 32M^2 p_1\cdot q_2\, p_2\cdot q_2 \right]$$

$$+ \frac{1}{(s-m^2)^2} \left[\frac{256}{y^2} m^6 + \frac{256}{y^2} m^4 p_1\cdot q_1 - 64m^4M^2 - 64m^2M^2 p_1\cdot q_1 - \frac{128}{y^2} m^4 p_1\cdot p_2 \right.$$

$$- \frac{128}{y^2} m^4 p_2\cdot q_1 - \frac{128}{y^2} m^2 p_1\cdot q_1\, p_2\cdot q_1 + \frac{64}{y^2} m^2 q_1^2 p_1\cdot p_2 + 32m^2M^2 p_1\cdot p_2$$

$$\left. + 32m^2M^2 p_2\cdot q_1 + 32M^2 p_2\cdot q_1\, p_1\cdot q_1 - 16M^2 q_1^2\, p_1\cdot p_2 \right] + \frac{1}{(t-m^2)(s-m^2)}$$

$$\left[\frac{128}{y^2} m^6 - \frac{256}{y^2} m^4 p_1\cdot q_2 - \frac{128}{y^2} m^2 p_1\cdot q_1\, p_1\cdot q_2 + \frac{128}{y^2} m^4 p_1\cdot p_2 - 32m^2M^2 p_1\cdot p_2 \right.$$

$$+ 32m^2M^2 p_2\cdot q_2 + 64m^2M^2 p_1\cdot q_2 + 32m^2M^2 p_2\cdot q_1 - 32m^4M^2 + 32m^2M^2 q_1\cdot q_2$$

$$+ \frac{128}{y^2} m^2 p_1\cdot p_2\, p_1\cdot q_2 + \frac{128}{y^2} p_1\cdot p_2 p_1\cdot q_1 p_1\cdot q_2 - \frac{128}{y^2} m^4 p_2\cdot q + \frac{128}{y^2} m^2 p_1\cdot q_1 p_1\cdot p_2$$

$$- \frac{128}{y^2} m^2 p_1\cdot q_1 p_2\cdot q_2 - \frac{128}{y^2} m^4 p_2\cdot q_1 + \frac{128}{y^2} m^2 p_1\cdot q_2 p_1\cdot p_2$$

$$+ \frac{128}{y^2} m^2 p_1\cdot q_2\; p_2\cdot q_1 + \frac{128}{y^2} m^2 p_1\cdot p_2\; p_1\cdot q_1 - \frac{128}{y^2} p_1\cdot p_2\; p_1\cdot q_2\; p_1\cdot q_1$$

$$- 64M^2 p_1\cdot q_1\; p_1\cdot p_2 + 64M^2 p_1\cdot q_1\; p_2\cdot q_2 - 64M^2 p_1\cdot p_2\; p_1\cdot q_2 - 64M^2 p_2\cdot q_1\; p_1\cdot q_2 \Big]\Big\} \tag{3.10}$$

In the above relation we have used $p_1 = yp$ §

$$\text{Since } x = \frac{-q_1^2}{2p\cdot q_1}$$

$$z = \frac{x}{y} = -\frac{q_1^2}{2p_1\cdot q_1} \tag{3.11}$$

Eqs. (3.8) and (3.11) lead to

$$2p_1\cdot q_1 = -\frac{q_1^2}{z}, \qquad 2p_1\cdot q_2 = m^2 - t$$

$$2p_2\cdot q_2 = q_1^2 - \frac{q_1^2}{z}, \quad 2q_1\cdot q_2 = t + q_1^2 - \frac{q_1^2}{z} - m^2 \tag{3.12}$$

$$2p_2\cdot q_1 = q_1^2 + m^2 - t, \quad 2p_1\cdot p_2 = m^2 - \frac{q_1^2}{z} + t$$

Substituting eq. (3.12) into eq. (3.10) we obtain

$$|\bar{M}|^2_{q_1p_1\to q_2p_2} = \frac{2\omega e^2 g^2}{27M^3} \Big\{ \frac{1}{(t-m^2)^2} \Big[\frac{64}{y^2} m^6 - \frac{64}{xy} m^4 q_1^2 + \frac{64}{y^2} m^4 t - 16m^4M^2$$

$$- 16m^2M^2 t + 16m^2M^2 q_1^2 \Big] + \frac{1}{q_1^4(1-\frac{y}{x})^2} \Big[\frac{128}{y^2} m^6 - \frac{36}{xy} m^4 q_1^2 - 32m^4M^2$$

$$+ \frac{8}{x} m^2M^2 q_1^2 y - \frac{32}{y^2} m^4 q_1^2 - \frac{32}{xy} m^2 q_1^2 t + \frac{32}{y^2} m^2 q_1^2 t + 8m^2M^2 q_1^2 + \frac{8}{x} M^2 q_1^2 t y - 8M^2 q_1^2 t \Big]$$

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§ The quantity m which appears in eq. (3.10) is the mass of u and d (valente quarks). We have used $p_1^2 = m^2$ for these quarks. For sea quarks we can not do the same. The usual parametrization $p_1 = yP$ was used in substituting P by p_1/y to obtain eq.(3.10), but in no place we have used the approximation $m^2 \simeq y^2M^2$ valid only for vaience quarks, when we do not consider their transverse momenta inside the nucleon.

$$+ \frac{1}{q_1^2(1-\frac{y}{x})(t-m^2)} \Big[\frac{128}{y^2} m^6 - 48m^4M^2 - \frac{128}{xy} m^4q_1^2 + \frac{128}{y^2} m^4t + \frac{16}{x} m^2M^2q_1^2y$$

$$- 16m^2M^2t + 32m^2M^2q_1^2 + \frac{64}{x^2} m^2q_1^4 - \frac{64}{xy} m^2q_1^2t - \frac{64}{y^2} m^4q_1^2 - \frac{16}{x^2} M^2q_1^4y^2$$

$$+ \frac{16}{x} M^2q_1^2ty\Big] + \frac{1}{t-m^2} \Big[-\frac{32}{y^2} m^4 + \frac{32}{y^2} m^2t - 8M^2q_1^2 - \frac{8}{x} M^2q_1^2y + 16m^2M^2\Big]$$

$$+ \frac{1}{q_1^2(1-\frac{y}{x})} \Big[\frac{32}{y^2} m^4 + \frac{32}{xy} m^2q_1^2 - 16m^2M^2 - \frac{32}{y^2} m^2t - \frac{32}{y^2} m^2q_1^2 - \frac{16}{x} M^2q_1^2y$$

$$+ 16M^2q_1^2 + 16m^2M^2\Big] - \frac{16}{y^2} m^2 + \frac{16}{xy} q_1^2 - \frac{16}{y^2} t \} \qquad (3.13)$$

Now, we substitute eq. (3.13) into eq. (3.2) and integrate over t in the range given by

$$t_{max} = m^2 - \mu^2 \qquad (3.14)$$

$$t_{min} = m^2 + \frac{yq_1^2}{x} \qquad (3.15)$$

A non-vanishing μ^2 is introduced to parametrise confinement effects. The non-applicability of Bloch-Nordsieck theorem in the infrared region of QCD[14] points to the necessity of inclusion of nonperturbative effects (effective gluon mass at large distance scale). The magnitude of the cut-off parameter is chosen to be of the order of inverse hadronic size, i.e., $\mu \approx 100$ MeV should not be too unreasonable.

Finally we obtain

$$F_2'(x,Q^2) = \frac{\omega g^2}{1152\pi^5M^3} \int_x^1 \frac{dy}{y^2} \frac{q(y)}{\left[\frac{y^2q_1^4}{4x^2} - m^2q_1^2\right]^{1/2}} \{ \frac{128}{y^3q_1^2} m^6x - \frac{64}{y^2} m^4$$

$$- \frac{32}{yq_1^2} m^4M^2x + \frac{16}{y} m^2M^2x + \frac{128}{\mu^2y^2} m^6 - \frac{64}{\mu^2xy} m^4q_1^2 - \frac{32}{\mu^2} m^4M^2 - \frac{16}{\mu^2} m^2M^2q_1^2$$

$$- \frac{16}{xy} \mu^2q_1^2 - \frac{8}{y^2} \mu^2 + (\frac{64}{y^2} m^4 - 8M^2q_1^2 - \frac{8}{x} M^2q_1^2y) \ln \frac{\mu^2x}{-yq_1^2} + \frac{1}{q_1^4(1-\frac{y}{x})}$$

$$\Big[-\frac{128}{y^2} m^6\mu^2 + \frac{64}{xy} m^4\mu^2q_1^2 + 32m^4M^2\mu^2 - \frac{16}{x} m^2M^2\mu^2q_1^2y - \frac{128}{xy} m^6q_1^2$$

$$+ \frac{64}{x^2} m^4 q_1^4 + \frac{32}{x} m^4 M^2 q_1^2 y - \frac{16}{x^2} m^2 M^2 q_1^4 y^2 - \frac{16}{xy} m^2 \mu^4 q_1^2 + \frac{16}{y^2} m^2 \mu^4 q_1^2$$

$$+ \frac{4}{x} M^2 q_1^2 \mu^4 y - 4M^2 \mu^4 q_1^2 + \frac{32}{x^3} m^2 q_1^6 y - \frac{32}{x^2} m^2 q_1^6 - \frac{8}{x^3} M^2 q_1^6 y^3 + \frac{8}{x^2} M^2 q_1^6 y^2 \Big]$$

$$+ \frac{1}{q_1^2 (1 - \frac{y}{x})} \Big[(\frac{256}{y^2} m^6 - 64 m^4 M^2 - \frac{192}{xy} m^4 q_1^2 + \frac{32}{x} m^2 M^2 q_1^2 y + 32 m^2 M^2 q_1^2$$

$$+ \frac{64}{x^2} m^2 q_1^4 - \frac{64}{y^2} m^4 q_1^2 - \frac{16}{x^2} M^2 q_1^4 y^2) \ln \frac{\mu^2 x}{-y q_1^2} - \frac{128}{y^2} m^4 \mu^2 + 16 m^2 \mu^2 M^2 + \frac{32}{xy} m^2 \mu^2 q_1^2$$

$$- \frac{128}{xy} m^4 q_1^2 + \frac{32}{x} m^2 M^2 q_1^2 y + \frac{64}{x^2} m^2 q_1^4 + \frac{32}{y^2} m^2 \mu^2 q_1^2 - 16 \mu^2 M^2 q_1^2 + \frac{16}{x} m^2 M^2 q_1^2 y$$

$$- \frac{16}{x} M^2 q_1^4 y - \frac{16}{y^2} m^2 \mu^4 + \frac{32}{xy} m^2 q_1^4 \Big] \} \qquad (3.16)$$

Using our assumptions, $-q_1^2 = Q^2 >> M^2$, m^2, μ^2, we obtain

$$F'(x,Q^2) = \frac{\omega g^2}{72\pi^5 M} \int_x^1 \frac{dy q(y)}{y^2} \{ \Big[\frac{x^2 + y^2}{y(y-x)} - \frac{8m^2}{y(y-x)M^2} \Big] \ln \frac{yQ^2}{x\mu^2}$$

$$+ \frac{8m^4}{\mu^2 M^2 y^2} - \frac{2\mu^2}{y^2 M^2} + \frac{4m^2(x+y)}{M^2 y^2 (y-x)} - \frac{2x+y}{y-x} \} \qquad (3.17)$$

At the large Q^2 limit the terms with power law behavior in μ become non-leading. In the usual prescription of setting $m = 0$ one does not have such terms. Because of confinement the $\mu \to 0$ limit is not allowed (indeed $\mu > m$) and the dominant terms are

$$F_2'(x,Q) = \frac{\omega g^2}{72\pi^5 M} \int_x^1 \frac{dy q(y)}{y^2} \Big[\frac{x^2 + y^2}{y(y-x)} - \frac{8m^2}{y(y-x)M^2} \Big] \ln \frac{yQ^2}{x\mu^2} \qquad (3.18)$$

We see that when we take $m^2 \to 0$, the terms within square brackets tend to the usual relation[15] §

$$P_{q/q}(z) = \frac{z^2 + 1}{1 - z} \qquad (3.19)$$

Little is known theoretically about the quark distribution inside nucleon $q(y)$. Considering that the gluons carry approximatelly 50% of nucleon momentum[10], it is reasonable to assume that $q(y)$ must have a peak near y = 1/6 for valence quarks. For example, taking y = 1/6 and x = $y/2$ we have $x^2 + y^2 \simeq .03$, a likely value for $x^2 + y^2$.

There is a controversy about the light current quark masses in the literature. Ear! ier works[16], using current algebra, give small values for u and d quarks masses, which are near 4 and 8 MeV, respectively. For such low values, the quark mass corrections are irrelevant in eq. (3.18).

A set of interesting works, using arguments other than current algebra, obtain substantlally higher values for the light current quark masses. For example, in ref.7, the inequality

$$m_u + m_d \gtrsim 33 \text{ MeV} \qquad (3.20)$$

is obtained.

In recent works[8,9], using among other things the Shifman--Vainshtein-Zakharov technique[17] the following values for the current masses of the u and d quarks

$$m_u \simeq 12 \text{ MeV} \quad \text{and} \quad m_d \simeq 22 \text{ MeV} \qquad (3.21)$$

and[9]

---

§ Eq. (3.19) is correct for $z<1$. For $z=1$ we must include high order diagrams, not considered in the present work, taking into account vertex and self-mass corrections[15]. The general result is

$$P_{q/q}(z) = \frac{z^2 + 1}{(1 - z)_+} + \frac{3}{2}\,\delta(1-z)$$

On theothsr hand, $z=1$ must give a very small contribution in eq. (3.18) because $z=1$ corresponds to $x=y$ and this is very unlikely to occur. Although little is known about $q(y)$, it must be highly suppressive for eq. (3.18). Then, the $z=1$ value is not important in this analysis and, because of this, we did not consider the above mentioned high order diagrams in our calculations.

$$m_u \quad m_d \cong 250 \text{ MeV} \tag{3.22}$$

are obtained. These relatively high values for current quarkmasseswere a motivation for the present work. For example, wesee that if m 6 0 MeV, the term $8m^2/M^2$ is approximately .03, which is , as we observed before, one of the best possible values of x^2+y^2.

4. DETERMINATION OF UPPER LIMITS FOR CURRENT MASSES OF LIGHT QUARKS

As we mentioned in the preceding section, the current masses for the light quarks have been estimated by various authors using a number of methods and techniques. Now, using eq. (3.18), we present a single way of estimating upper limits for m_u and m_d, using positivity properties of the nucleon forward electroniagnetic structure function F_2. An interesting consequence of eq.(3.18) follows immediately as a consequence of $F_2^{\prime} \geqslant 0$. SInce $x^2 + y^2 - 8m^2/M^2$ has to be posltive, and as the range of integration in y lies between x and 1, x = 0 is incompatible with a non-vanishing value for m^2/M^2.

The constraint of positivity of $F_2^{\prime}$ together with the minimum experimental values of x for a given Q^2 [18] yield upper limits of the struck quark mass, which are displayed in table 1. Observing the data in this table, we see that the upper mass limits are, on the whole, consistent with the results quoted in literature for the current masses of

Table 1 - Upper limits of the light quark masses from the positivity of $F_2^{\prime}(x,Q^2)$.

0^2 GeV^2	x_{min}	m MeV
4.50	0.01000	$\lesssim$ 4.7
7.50	0.01900	$\lesssim$ 8.9
9.00	0.02500	$\lesssim$ 12.0
22.50	0. 03636	$\lesssim$ 17.0
40.00	0.12500	$\lesssim$ 59.0
65.00	0. 28570	$\lesssim$ 130.0

the light quarks, (with the exception of that of ref.9) leading to eq. (3.22). Better determination of these upper mass limits could be obtained if experiments were done to obtain the least possIble x value at sufficieritly high Q^2 values, so that eq. (3.10) could be considered a goos approximation.

5. CONCLUSION

We have shown that light current quark masses lead to a significant influence in the usual calculations of perturbative QCD. Our result is mainly contained in eq. (3.18). The special case of eq.(3.19), and the upper limits for current masses of quarks u and d which are, on the whole, consistent with the results quoted in the literature, show an internal consistency of our calculations.

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Resumo

Demonstramos que as massas de quarks leves podem contribuir significantemente para a quebra de escala logarítmica no espalhamento lepton-nucleon profundamente inelástico. Isto é devido, principalmente, à combinação de variáveis de escala com maiores valores das "massas de corrente" dos quarks u e d, recentemente referidos na literatura. Também estimamos limites superiores para as massas de corrente dos quarks u e d, usando as propriedades de positividade da função de estrutura F_2 do nucleon.