Revista Brasileira de Física, Volume 13, nº 4, 1983

A Derivation of the Renormalized Charge Using the Wilson Loop

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Recebido em 13 de fevereiro de 1984

Abstract The expression for the charge renormalization (up to second order) is obtained in a simple way by using the Abelian Wilson Loop.

As is well known we can obtain the renormalized charge in Quantum Electrodynamics (QED) by calculating the electron scattering by an external field whose interaction is appropriately described by QED. For this it is necessary to consider systematically[1] all posçible diagrams that contribute corrections (known as radioactive corrections) to the scattering amplitude for the process under consideration. The computation of the contribution of these so-called radioactive corrections due to the electron self-energy, vertex modification and photon self-energy, must of course be combined to obtain the charge renormalization. It is verified, then, that the charge renormalization is due exclusively to photon self-energy effects[2], i.e., to vacuum polarization modifications inserted into photons lines. In this procedure there appear two different types of divergences: the ultraviolet divergence, that can be eliminated by the renormalization theory, and the infrared divergence. The difficulties associated with the infrared divergence can be overcome by applying the functional method developed by Bloch and Nordsieck[3], which involves non-trivial calculations.

Here we obtain the expression for the charge renormalization (up to second order) by a straightforward way. For this, we make use of the Wilson Loop for Quantum Electrodynamics by considering its perturbation expansion up to the second order term. In this procedure, processes such as the electron self-energy and the vertex modification do not appear, and therefore we take into account only photon self-energy correction. Then, we obtain that the Wilson Loop with correction due to the photon self-energy effects, in the lowest order approximation, has as coupling constant, the renormalized charge of Quantum Electrodynamics.

Now, consider the Wilson Loop average, which in the Abelian Theory, like QED, is defined by the following Euclidean functional integral

$$W(C) = \int DA_\mu \exp(ie \oint_C A_\alpha dx_\alpha) e^{-S} \Big/ \int DA_\mu e^{-S} \qquad (1)$$

where S is the action of the free electromagnetic field without gauge fixing terms and is given by

$$S = -\frac{1}{4}\int F^2_{\mu\nu}(x) d^4x \qquad (2)$$

with $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$.

The perturbative series corresponding to eq. (1) is[4]

$$W(C) = \sum_n W_n(C) = 1 + \sum_{n=2}^{\infty} (ie)^n \frac{1}{n!} \int dx_1^{\alpha_1} \ldots \int dx_n^{\alpha_n} \langle A_{\alpha_1}(x_1) \ldots A_{\alpha_n}(x_n)\rangle \qquad (3)$$

where $\langle A_{\alpha_1}(x_1)\ldots A_{a_n}(x_n)\rangle \equiv D_{\alpha \ldots \alpha_n}(x_1, \ldots$ X are the n-point Green's functions (in this case, the propagator of the electromagnetic field) and can be computed according to the usual diagram technique.

The lowest-order diagram that corresponds to series (3) isgiven by

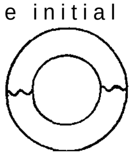
$$W_2^{(2)}(C) = x$$  $$y = -\frac{1}{2} e^2 \int dx_\alpha \int dy_\beta \langle A_\alpha(x) A_\beta(y)\rangle$$

$$\equiv -\frac{1}{2} e^2 \int dx_\alpha \int dy_\beta D_{\alpha\beta}(x-y) \qquad (4)$$

where $D_{\alpha\beta}(x-y)$ is the photon Green's function in the zero-order approximation[5].

At this point we introduce a modification in the lowest-order diagram by the addition of a fermionic loop, that is, taking into account the photon self-energy.

Then, the initial diagram becomes

$$\bar{W}_2^{(2)} = x$$  $$y = -\frac{1}{2} e^2 \int dx_\alpha \int dy_\beta \bar{D}_{\alpha\beta}(x-y) \qquad (5)$$

where $\bar{D}_{\alpha\beta}(x-y)$ is the new photon propagator which includes the correction to $D_{\alpha\beta}(x-y)$ due to the insertion of the second-order photon self-energy.

As is well known[1,5], the relation between the Fourier transforms of $\bar{D}_{\alpha\beta}(x-y)$ and $D_{\alpha\beta}(x-y)$ is

$$\bar{D}_{\alpha\beta}(k) = D_{\alpha\beta}(k) \left[1 - \frac{e^2}{12\pi^2} \log\left(\frac{M^2}{m^2}\right)\right] \tag{6}$$

where M^2 is a cut-off parameter such that $M^2 \gg k^2$. The relation expressed by eq. (6) is obtained by the use of current conservation in QED and by considering that the photon propagator lies between two currents (vertex), and the momentum transfer is small, that is, $k^2 \sim 0$.

Now, we take the inverse Fourier transform of expression (6) to obtain a relation between $\bar{D}_{\alpha\beta}(x-y)$ and $D_{\alpha\beta}(x-y)$. Evidently, this relation is valid in the limit of long wavelenght (i.e.: x far removed from x), beoause expression (6) is valid only for low frequencies. However this is all we need for our purpose, as the renormalized charge can be defiried by the coefficient of the (long distance) Coulomb law''. Putting the expression for $\bar{D}_{\alpha\beta}(x-y)$ in terms of $D_{\alpha\beta}(x-y)$ in expression (5) and comparing it with eq.(4), we conclude that, in order that both equations give the same result, e^2 that appears in eq.(5) must be replaced by e_R^2, given by

$$e_R^2 = e^2 \left[1 - \frac{e^2}{12\pi^2} \log \left(\frac{M^2}{m^2}\right)\right] \tag{7}$$

Eq. (7) is the well-known expreçsion for the charge renormalization[5] in the lowest order approximation In Quantum Electrodynamics, obtained here without the need of treating the infrared divergence as In the usual procedure. Such a divergence does not appear in this method because it is associated with the electron self-energy and the vertex modifications and these processes do not occur in the present procedure.

It is probable that the charge renormalization with higher-order corrections can be obtalned in this simple way by using the Wilson Loop perturbation expansion rather than the traditional methods, and therefore, the Abelian Wilson Loop, with these corrections due to the self-energy effects of the photon, would be given by eq. (1) with the coupling constant (electric charge) repiaced by the renormalized electric charge.

We are grateful to profs. J.J.Giambiagi and C.C. Bolini from Centro Brasileiro de Pesquisas Físicas (CBPF), Rio de Janeiro, Brasil, for useful dlscussions on this subject.

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Resumo

Obtém-se, de modo simples, a expressão para a carga renormalizada (até segunda ordem), usando-se o Loop de Wilson Abeliano.